1.3 12, 14, 18, 26, 28, 29, 30, 32

12) $\bar{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$ $\bar{a}_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}$ $\bar{a}_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}$ $\bar{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$

I must determine if $\bar{b}$ is a linear combination of $\bar{a}_1$, $\bar{a}_2$, and $\bar{a}_3$. This means I must determine weights x_1, x_2, x_3 such that

$$x_1\bar{a}_2 + x_2\bar{a}_2 + x_3\bar{a}_3 = \bar{b}.$$

My vector equation would then be

$$x_1\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix} + x_3\begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$$

x_1, x_2, and x_3 make the vector equation true iff x_1, x_2, and x_3 satisfy the system:

$$\begin{cases} x_1 + 2x_3 = -5 \\ -2x_1 + 5x_2 = 11 \\ 2x_1 + 5x_2 + 8x_3 = -7 \end{cases}$$

Reducing the system...

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{bmatrix} \underset{R_2+R_3 \to R_3}{\overset{2R_1+R_2 \to R_2}{\sim}} \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 10 & 8 & 4 \end{bmatrix} \overset{\frac{1}{2}R_3 \to R_3}{\sim} \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 5 & 4 & 2 \end{bmatrix} \overset{-R_2+R_3 \to R_3}{\sim}$$

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

The system is inconsistent because there is a pivot in the last column of the augmented matrix. Therefore vector $\bar{b}$ cannot be written as a linear combination of $\bar{a}_1$, $\bar{a}_2$ and $\bar{a}_3$.

good

14.) $A = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$

Again I must determine if b is a linear combination of the vectors formed from the column of the matrix A.

The augment matrix for the associated linear system would be:

$$\begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{bmatrix} \overset{-R_1+R_3 \to R_3}{\sim} \begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 0 & 0 & 11 & -2 \end{bmatrix}$$

At this point, it is clear that there will be a pivot in each column of the coefficient matrix. This means that as long as b is not larger than a vector of R^3, then it will be a linear combination of the vectors formed by matrix A.

18.) $v_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ $v_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}$ $y = \begin{bmatrix} h \\ -5 \\ 3 \end{bmatrix}$

Rewriting these 3 vectors into an augmented matrix:

$\begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & 3 \end{bmatrix}$. I must find h so that y is in the plane generated by v_1 and v_2.

First I must row reduce the matrix.

$$\begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & 3 \end{bmatrix} \overset{2R_1+R_3 \to R_3}{\sim} \begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 2 & 2h-3 \end{bmatrix} \overset{\substack{3R_2+R_1 \to R_1 \\ -2R_2+R_3 \to R_3}}{\sim} \begin{bmatrix} 1 & 0 & h-15 \\ 0 & 1 & -5 \\ 0 & 0 & 2h+7 \end{bmatrix}$$

The system is consistent when $2h+7=0$.

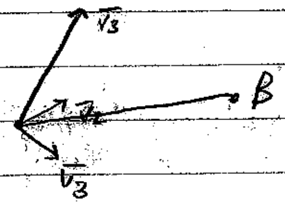
Therefore y is in the plane generated by v_1 and v_2 when $h = -7/2$.

1.3

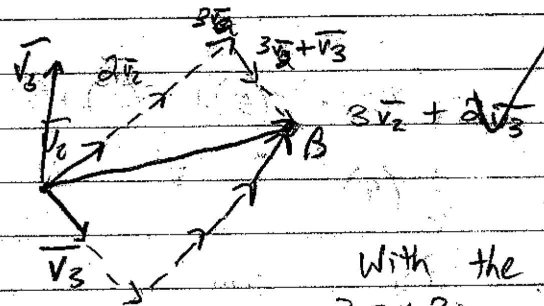
32) In $\mathbb{R}^2$

Solution?

Unique Solution.

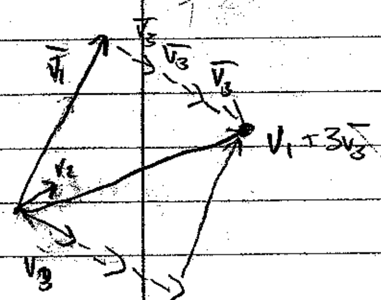


With scalor multiplication and vector addition, there would be many solutions, for example:



With the communative property of addition $3\vec{v_2} + 2\vec{v_3}$ would be a solution as well as $2\vec{v_3} + 3\vec{v_2}$.

Another solution might look like:



This just shows there are many solutions not one unique solution.