Exercise 12 on page 38

$$\vec{a} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \quad \vec{a}_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \quad \vec{a}_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$$

Determine if $\vec{b}$ is a linear combination of $\vec{a}_1, \vec{a}_2$, and $\vec{a}_3$

That means, I need to determine weights x_1, x_2, x_3 such that

$$x_1\vec{a} + x_2\vec{a}_2 + x_3\vec{a}_3 = \vec{b}$$

lets rewrite the vector equation

$$x_1\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix} + x_3\begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$$

x_1, x_2, x_3 make the vector equation true iff x_1, x_2, x_3 satisfy the system below:

$$\begin{cases} x_1 + 2x_3 = -5 \\ -2x_1 + 5x_2 = 11 \\ 2x_1 + 5x_2 + 8x_3 = -7 \end{cases}$$

Lets try to solve the system by row reduction operations on the augmented matrix.

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{bmatrix} \underset{R_2 + R_3 \to R_3}{\overset{2R_1 + R_2 \to R_2}{\sim}} \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 10 & 8 & 4 \end{bmatrix} \sim$$

$$\underset{1/2 \cdot R_3}{\sim} \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 5 & 4 & 2 \end{bmatrix} \underset{R_2 + (-1)R_3}{\sim} \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

The system is inconsistent, which means that vector $\vec{b}$ can not be written as a linear combination of $\vec{a}_1, \vec{a}_2$, and $\vec{a}_3$

Answer: NO