Section 1.3 Exercises

18. By row reducing the augmented matrix

$$\begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & -3 \end{bmatrix} \overset{R_1 \leftrightarrow R_3}{\sim} \begin{bmatrix} -2 & 8 & -3 \\ 0 & 1 & -5 \\ 1 & -3 & h \end{bmatrix} \overset{-\frac{1}{2}R_1 \to R_1}{\sim} \begin{bmatrix} 1 & -4 & \frac{3}{2} \\ 0 & 1 & -5 \\ 1 & -3 & h \end{bmatrix}$$

$$\overset{-R_1+R_3 \to R_3}{\sim} \begin{bmatrix} 1 & -4 & \frac{3}{2} \\ 0 & 1 & -5 \\ 0 & 1 & h-\frac{3}{2} \end{bmatrix} \underset{-R_2+R_3 \to R_3}{\overset{4R_2+R_1 \to R_1}{\sim}} \begin{bmatrix} 1 & 0 & -\frac{37}{2} \\ 0 & 1 & -5 \\ 0 & 0 & h+\frac{7}{2} \end{bmatrix}$$

The system is consistent when $h+\frac{7}{2}=0$.
$\therefore \bar{y}$ is in the plane generated by $\bar{v}_1$ and $\bar{v}_2$ when $h=\frac{-7}{2}$. ✓

26. Let $A=\begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$, let $\bar{b}=\begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$, and let W be the set of all linear combinations of the columns of A.

a. Is $\bar{b}$ in W?

$$\begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$$ represents the vector equation

$$x_1\bar{A}_1 + x_2\bar{A}_2 + x_3\bar{A}_3 = \bar{b}$$

x_1, x_2 & x_3 make the vector equation true if and only if x_1, x_2 & x_3 satisfy the system

$$\begin{cases} 2x_1 + 0x_2 + 6x_3 = 10 \\ -x_1 + 8x_2 + 5x_3 = 3 \\ x_1 - 2x_2 + x_3 = 3 \end{cases}$$

By row reducing the augmented matrix

$$\begin{bmatrix} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{bmatrix} \overset{\frac{1}{2}R_1 \to R_1}{\sim} \begin{bmatrix} 1 & 0 & 3 & 5 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{bmatrix} \underset{-R_1+R_3 \to R_3}{\overset{R_1+R_2 \to R_2}{\sim}} \begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 8 & 8 & 8 \\ 0 & -2 & -2 & -2 \end{bmatrix} \underset{\frac{1}{8}R_2 \to R_2}{\overset{-\frac{1}{2}R_3 \to R_3}{\sim}}$$

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \overset{-R_2+R_3 \to R_3}{\sim} \begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{cases} x_1 + 3x_3 = 5 \\ x_2 + x_3 = 1 \\ 0x_3 = 0 \end{cases}$$

$$\begin{cases} x_1 = 5 - 3x_3 \\ x_2 = 1 - x_3 \\ x_3 \text{ is free} \end{cases}$$

So our system is consistent and has infinitely many solutions of the form $(5-3t, 1-t, t)$ where t is any real number.

$\therefore \bar{b}$ is in W.

Or we can appeal to theorem 2.

b. Show that the 3rd column of A is in W.

$$A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix} \quad \bar{A}_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \bar{A}_2 = \begin{bmatrix} 0 \\ 8 \\ -2 \end{bmatrix}, \bar{A}_3 = \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$$

✓

$$0\bar{A}_1 + 0\bar{A}_2 + 1\bar{A}_3 = \bar{A}_3$$

$$= 0\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + 0\begin{bmatrix} 0 \\ 8 \\ -2 \end{bmatrix} + 1\begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$$

$\therefore$ the 3rd column of A. is a linear combination of the columns of A.