1.4 Exercises

18. $B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix} \xrightarrow{-R_1+R_3 \Rightarrow R_3} \sim \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & -1 & -1 & 5 \\ -2 & -8 & 2 & -1 \end{bmatrix} \xrightarrow{2R_1+R_4 \Rightarrow R_4} \sim \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & -1 & -1 & 5 \\ 0 & -2 & -2 & 3 \end{bmatrix}$

$\xrightarrow{-3R_2+R_1 \Rightarrow R_1} \sim \begin{bmatrix} 1 & 0 & -5 & 17 \\ 0 & 1 & 1 & -5 \\ 0 & -1 & -1 & 5 \\ 0 & -2 & -2 & 3 \end{bmatrix} \xrightarrow{R_2+R_3 \Rightarrow R_3} \sim \begin{bmatrix} 1 & 0 & -5 & 17 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & -2 & 3 \end{bmatrix} \xrightarrow{2R_2+R_4 \Rightarrow R_4} \sim \begin{bmatrix} 1 & 0 & -5 & 17 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix}$

$\xrightarrow{R_3 \Rightarrow R_4} \sim \begin{bmatrix} 1 & 0 & -5 & 17 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{-1/7 R_3 \Rightarrow R_3} \sim \begin{bmatrix} 1 & 0 & -5 & 17 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{5R_3+R_2 \Rightarrow R_2} \sim \begin{bmatrix} 1 & 0 & -5 & 17 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\xrightarrow{-17R_3+R_1 \Rightarrow R_1} \sim \begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Since there is not a pivot position in every row, the columns of B do not span $\mathbb{R}^4$.
No, $Bx=y$ does not have a solution of each y in $\mathbb{R}^4$.
I used theorem 4 to determine my answer.