Exercise 16 on page 55

Describe the solution set of the following system in parametric vector form, and provide a geometric comparison with the solution set in exercise 6.

$$\begin{cases} x_1 + 3x_2 - 5x_3 = 4 \\ x_1 + 4x_2 - 8x_3 = 7 \\ -3x_1 - 7x_2 + 9x_3 = -6 \end{cases}$$

Let A be the matrix of coefficients of the system, and lets row reduce the augmented matrix $[A \; \vec{B}]$ to reduced echelon form, where $\vec{B} = \begin{bmatrix} 4 \\ 7 \\ -6 \end{bmatrix}$

$$\begin{bmatrix} 1 & 3 & -5 & 4 \\ 1 & 4 & -8 & 7 \\ -3 & -7 & 9 & -6 \end{bmatrix} \xrightarrow[R_2 + (-1)R_1 \to R_2]{3*R_1 + R_3 \to R_3} \begin{bmatrix} 1 & 3 & -5 & 4 \\ 0 & 1 & -3 & 3 \\ 0 & 2 & -6 & 6 \end{bmatrix} \xrightarrow[R_3 \cdot \frac{1}{2} + (-1) \cdot R_2 \to R_3]{R_1 + (-3)*R_2 \to R_1}$$

$$\sim \begin{bmatrix} 1 & 0 & 4 & -5 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The general solution set has a form as follows:

$$\begin{cases} x_1 = -5 - 4x_3 \\ x_2 = 3 + 3x_3 \\ x_3 \text{ is free} \end{cases}$$

The parametric vector form of the solution set of the given system is as follows:

$$\vec{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 - 4x_3 \\ 3 + 3x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} \checkmark$$

So, geometrically the solution of the given system is a line through $\begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$ parallel to the solution set of the homogeneous system in exercise 6, which is $x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} \checkmark$

Exercise 22 on page 55

Find a parametric equation of the line $\mathcal{M}$ through $\vec{p}$ and $\vec{q}$ ($\mathcal{M}$ is parallel to vector $\vec{q} - \vec{p}$)

$$\vec{p} = \begin{bmatrix} -6 \\ 3 \end{bmatrix} \qquad \vec{q} = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$