Sarah Curran

Section 1.5

6. We need to solve the solution set of the given homogeneous system

$$x_1 + 3x_2 - 5x_3 = 0$$
$$x_1 + 4x_2 - 8x_3 = 0$$
$$-3x_1 - 7x_2 + 9x_3 = 0$$

We can put the set into a matrix then row reduce it to row reduced echelon form

$$\begin{bmatrix} 1 & 3 & -5 & 0 \\ 1 & 4 & -8 & 0 \\ -3 & -7 & 9 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & -5 & 0 \\ 0 & 1 & -3 & 0 \\ -3 & -7 & 9 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & -5 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 2 & -6 & 0 \end{bmatrix}$$

$-R_1 + R_2 \to R_2$ $\qquad$ $3R_1 + R_3 \to R_3$

$$\begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 2 & -6 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 1 & -3 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is the row reduced echelon form

$-3R_2 + R_1 \to R_1$ $\qquad$ $\frac{1}{2}R_3 \to R_3$ $\qquad$ $-R_2 + R_3 \to R_3$

The solution set of the reduced matrix is

$$\begin{aligned} x_1 + 4x_3 &= 0 \\ x_2 - 3x_3 &= 0 \\ 0 &= 0 \end{aligned} \qquad \begin{cases} x_1 = -4x_3 \\ x_2 = 3x_3 \\ x_3 \text{ is free} \end{cases}$$

So, the solution set in parametric vector form is:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4x_3 \\ 3x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$$

8. We are given a matrix and supposed to describe all solutions of $Ax = 0$ in parametric vector form.

$$\begin{bmatrix} 1 & -2 & -9 & 5 \\ 0 & 1 & 2 & -6 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & -5 & -7 \\ 0 & 1 & 2 & -6 \end{bmatrix}$$

This is the row reduced echelon form

$2R_2 + R_1 \to R_1$

The solution set is:

$$\begin{cases} x_1 = 5x_3 + 7x_4 \\ x_2 = -2x_3 + 6x_4 \\ x_3 \text{ is free} \\ x_4 \text{ is free} \end{cases}$$

The parametric vector form of solution set is:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5x_3 + 7x_4 \\ -2x_3 + 6x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 7 \\ 6 \\ 0 \\ 1 \end{bmatrix}$$