Section 2.3

E4.) The matrix $\begin{bmatrix} -7 & 0 & 4 \\ 3 & 0 & -1 \\ 2 & 0 & 9 \end{bmatrix}$ is not invertible.

One of its columns is the zero vector which makes the set of its columns linearly dependent by Theorem 9 (Chapter 1). Therefore the matrix is not invertible by the Invertible Matrix Theorem (statement e).

E8.) The matrix $\begin{bmatrix} 1 & 3 & 7 & 4 \\ 0 & 5 & 9 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 10 \end{bmatrix}$ is invertible.

The matrix is in echelon form with a pivot in each row and each column. It is therefore row equivalent to I_4 and thus is invertible by the Invertible Matrix Theorem (statement b).

E10.)
$$\begin{bmatrix} 5 & 3 & 1 & 7 & 9 \\ 6 & 4 & 2 & 8 & -8 \\ 7 & 5 & 3 & 10 & 9 \\ 9 & 6 & 4 & -9 & -5 \\ 8 & 5 & 2 & 11 & 4 \end{bmatrix} \overset{\text{Method?}}{\sim} \begin{bmatrix} 1 & \frac{2}{3} & \frac{4}{9} & -1 & -\frac{5}{9} \\ 0 & 1 & \frac{11}{3} & -36 & -\frac{106}{3} \\ 0 & 0 & 1 & -\frac{87}{4} & -\frac{37}{2} \\ 0 & 0 & 0 & 1 & 33.999 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The matrix is invertible. When it is reduced to echelon form, we can tell that it is row equivalent to I_5. Therefore the matrix is invertible by the Invertible Matrix Theorem (statement b).

Section 2.3

E12.) a.) This statement is true by the IMT (statements j and k).

b.) This statement is true by the IMT (statements e and h).

c.) This statement is true. In an $n \times n$ matrix, A, if the equation $A\bar{x} = \bar{b}$ has at least one solution for each $\bar{b}$ in $\mathbb{R}^n$ then A is an invertible matrix. This is verified by the IMT (statements g and a). Since A is an invertible $n \times n$ matrix then for each $\bar{b}$ in $\mathbb{R}^n$, the equation $A\bar{x} = \bar{b}$ has a unique solution by Theorem 5.

d.) This statement is true by the IMT (statements i and c). Must be ONTO

e.) This statement is true by the IMT (statements g and f)

E14.) A square lower triangular matrix is invertible iff its diagonal entries are all non-zero. Elementary row operations can reduce all entries below the diagonals to zero while keeping all diagonal entries non-zero. Therefore the matrix has n pivot positions and is thus invertible by the IMT (statements c and a).

E22.) If the equation $H\bar{x} = \bar{c}$ is inconsistent for some $\bar{c}$ in $\mathbb{R}^n$ then H does not have a pivot position in every row by Theorem 4 (Chapter 1). Therefore H has at least one free variable. Therefore the equation $H\bar{x} = 0$ has at least one nontrivial solution by the blue-highlighted statement on page 50.

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E32) If A is an $n \times n$ matrix and the equation $A\bar{x} = 0$ has only the trivial solution then the equation $A\bar{x} = 0$ has no free variables. Therefore the matrix has n pivot positions, one in every row and every column. Therefore the equation $A\bar{x} = \bar{b}$ has a solution for each $\bar{b}$ in $\mathbb{R}^n$ by Theorem 4 (Chapter 1).

E34) $T(x_1, x_2) = (6x_1 - 8x_2, -5x_1 + 7x_2)$

The standard matrix for $T = A = \begin{bmatrix} 6 & -8 \\ -5 & 7 \end{bmatrix}$.
A is invertible by Theorem 4 because...

$ad - bc = 6 \cdot 7 - (-8)(-5) = 42 - 40 = 2 \neq 0$.

$A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 8 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 7/2 & 4 \\ 5/2 & 3 \end{bmatrix}$

$T^{-1}(x_1, x_2) = (\frac{7}{2}x_1 + 4x_2, \frac{5}{2}x_1 + 3x_2)$

E36) If T is a linear transformation that maps $\mathbb{R}^n$ onto $\mathbb{R}^n$ then the columns of its standard matrix A span $\mathbb{R}^n$ by Theorem 12. Therefore A has a pivot position in every row by Theorem 4 (Chapter 2). Since T maps $\mathbb{R}^n$ onto $\mathbb{R}^n$ then A is an $n \times n$ matrix. Since A is an $n \times n$ matrix with a pivot position in every row then it has n pivot positions. Therefore A is invertible by the IMT (statements c and a). Therefore T^{-1} exists by Theorem 9. Since A is invertible then A^{-1} is invertible by Theorem 6. Therefore the linear transformation $\bar{x} \mapsto A^{-1}\bar{x}$ (a.k.a. $T^{-1}\bar{x}$) is one-to-one by the IMT (statements a and f).

Section 2.8

Picture ?

E2.) The vectors $\begin{bmatrix} 0 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$ are in H. However $\begin{bmatrix} 0 \\ 4 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ is not in H. Therefore H violates the definition of a subspace and is not a subspace of $\mathbb{R}^2$.

E8.)

$$6\ 3\ 3\ -9$$
$$-6\ -4\ 0\ 2$$
$$0\ 2\ -6\ 14$$
$$0\ -2\ 6\ -14$$

$$\left[\begin{array}{ccc|c} -3 & -2 & 0 & 1 \\ 0 & 2 & -6 & 14 \\ 6 & 3 & 3 & -9 \end{array}\right]$$

The system $A\bar{x} = \bar{p}$ is consistent. Therefore $\bar{p} \in \text{Col } A$.

$$\begin{bmatrix} -3 & -2 & 0 & 1 \\ 0 & 2 & -6 & 14 \\ 0 & -1 & 3 & -7 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -2 & 0 & 1 \\ 0 & -1 & 3 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

E10.)

$$\begin{bmatrix} -3 & -2 & 0 \\ 0 & 2 & -6 \\ 6 & 3 & 3 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 + (-6) + 0 \\ 0 + 6 + (-6) \\ -12 + 9 + 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The product $A\bar{u} = \bar{0}$. Therefore $\bar{u} \in \text{Nul } A$.