# Assigment #10

4.3/#20) We seek a basis for $H = \text{span}\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ where $\bar{v}_1 - 3\bar{v}_2 + 5\bar{v}_3 = \bar{0}$.

Of course $\bar{v}_1 = 3\bar{v}_2 - 5\bar{v}_3$. So, if $\bar{x} = \alpha_1\bar{v}_1 + \alpha_2\bar{v}_2 + \alpha_3\bar{v}_3$ is any element of $H$, then $\bar{x} = (3\alpha_1\bar{v}_2 - 5\alpha_1\bar{v}_3) + \alpha_2\bar{v}_2 + \alpha_3\bar{v}_3$
$= (3\alpha_1 + \alpha_2)\bar{v}_2 + (-5\alpha_1 + \alpha_3)\bar{v}_3$.

So, $\bar{x}$ is in span $\{\bar{v}_2, \bar{v}_3\}$. Hence $H = \text{span}\{\bar{v}_2, \bar{v}_3\}$. We can also see that $\{\bar{v}_2, \bar{v}_3\}$ is a linearly independent set. Consequently $\{\bar{v}_2, \bar{v}_3\}$ is a basis for $H$.

4.3/#22)

a) False. The set must also span $H$
b) True. Theorem 5
c) True. Page 242
d) False Page 242
e) False Page 242

4.3/#26) $\{\sin t, \sin 2t\}$ is a basis for span $\{\sin t, \sin 2t, \sin t \cos t\}$ because $\sin 2t = 2 \sin t \cos t$.

4.3/#34) $\bar{p}_1(t) = 1+t$, $\bar{p}_2(t) = 1-t$, and $\bar{p}_3(t) = 2$.
A linear dependence relation ~~on~~ $\bar{p}_1, \bar{p}_2, \bar{p}_3$ is

$$1\cdot\bar{p}_1(t) + 1\cdot\bar{p}_2(t) - 1\cdot\bar{p}_3(t) = \bar{0}$$

$\{\bar{p}_1, \bar{p}_2\}$ is a basis for span $\{\bar{p}_1, \bar{p}_2, \bar{p}_3\}$

4.4/#4) $P_B[\bar{x}]_B = \bar{x} \Rightarrow \begin{bmatrix} -1 & 3 & 4 \\ 2 & -5 & -7 \\ 0 & 2 & 3 \end{bmatrix}\begin{bmatrix} -4 \\ 8 \\ -7 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}$

**4.4 #8**

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & 8 & 2 \end{bmatrix} [\bar{x}]_B = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 0 & 1 & -1 & : & -5 \\ 3 & 8 & 2 & : & 4 \end{bmatrix} \overset{rref}{\sim} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix} \Longrightarrow [\bar{x}]_B = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$$

**4.4 #12**

$$P_B = \begin{bmatrix} 4 & 6 \\ 5 & 7 \end{bmatrix} \Longrightarrow P_B^{-1} = \begin{bmatrix} -\frac{7}{2} & 3 \\ \frac{5}{2} & -2 \end{bmatrix}$$

$$P_B [\bar{x}]_B = \bar{x} \Longrightarrow [\bar{x}]_B = P_B^{-1} \begin{bmatrix} 2 \\ 0 \end{bmatrix} \Longrightarrow [\bar{x}]_B = \begin{bmatrix} -7 \\ 5 \end{bmatrix}$$

**4.4 #14**

Give $B = \{1-t^2,\ t-t^2,\ 2-2t+t^2\}$ is a basis for $\mathbb{P}_2$.

we seek $[\bar{p}]_B$ where $\bar{p}(t) = 3 + t - 6t^2$.

that is, we seek $x_1, x_2, x_3$ such that

$$x_1(1-t^2) + x_2(t-t^2) + x_3(2-2t+t^2) = 3+t-6t^2$$
$$(x_1 + 2x_3) + (x_2 - 2x_3)t + (-x_1 - x_2 + x_3)t^2 = 3+t-6t^2$$

So,

$$\begin{cases} x_1 + 2x_3 = 3 \\ x_2 - 2x_3 = 1 \\ -x_1 - x_2 + x_3 = -6 \end{cases} \Longleftrightarrow \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & 1 \\ -1 & -1 & 1 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$\Longrightarrow [\bar{p}]_B = \begin{bmatrix} 7 \\ -3 \\ -2 \end{bmatrix}$$

**4.4 #32**

$\bar{p}_1(t) = 1+t^2 \longleftrightarrow \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$; $\bar{p}_2(t) = 2-t+3t^2 \longleftrightarrow \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$; $\bar{p}_3(t) = 1+2t-4t^2 \longleftrightarrow \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 1 & 3 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Longrightarrow \{\bar{p}_1, \bar{p}_2, \bar{p}_3\}$ is a basis for $\mathbb{P}_2$. (IMT)

b) $P_B[\bar{q}]_B = \bar{q} \Longrightarrow \bar{q}(t) = -3\bar{p}_1(t) + 1\bar{p}_2(t) + 2\bar{p}_3(t)$

$\Longrightarrow \bar{q}(t) = 1 + 3t - 8t^2 \longleftrightarrow \begin{bmatrix} 1 \\ 3 \\ -8 \end{bmatrix}$