

Assignment #11

4.5
#20

- a) False See Example 4
 b) False Page 260 (top)
 c) False See definition on p. 257
 d) False See Basis Theorem
 e) True See Example 4

4.6
#2

$$\text{rank } A = 3 + 1 + (5 - 3 + 5) - (5 - 1) = 8$$

$$\dim \text{Nul } A = 2$$

$$A \text{ basis for Col } A = \left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ -6 \\ 4 \end{bmatrix}, \begin{bmatrix} 9 \\ -10 \\ -3 \\ 0 \end{bmatrix} \right\}$$

$$A \text{ basis for Row } A = \left\{ [1, -3, 0, 5, -7], [0, 0, 2, -3, 8], [0, 0, 0, 0, 5] \right\}$$

$$A \text{ basis for Nul } A = \left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 9 \\ 3/2 \\ 1 \\ 0 \end{bmatrix} \right\}$$

4.6
#27

These subspaces are in \mathbb{R}^m : $\text{Col } A, \text{Nul } A^T, \text{Row } A^T$
 These subspaces are in \mathbb{R}^n : $\text{Row } A, \text{Nul } A, \text{Col } A^T$
 There are four distinct subspaces because $\text{Row } A^T = \text{Col } A$
 and $\text{Col } A^T = \text{Row } A$

4.7
#2

a) $P_{C \leftarrow B} = \begin{bmatrix} -1 & 5 \\ 4 & -3 \end{bmatrix}$ by Theorem 15

b) $P_{C \leftarrow B} \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix}_C \Rightarrow \begin{bmatrix} -1 & 5 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \end{bmatrix} = \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix}_C$

4.7
#8

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 4 & 1 & 8 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & -4 & 3 \end{bmatrix} \Rightarrow P_{C \leftarrow B} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 1 & 1 \\ 8 & -5 & 4 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 4 & 3 \end{bmatrix} \Rightarrow P_{B \leftarrow C} = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$$

4.7
#14

$$P = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ -3 & -5 & 0 \end{bmatrix}; P^{-1} =$$

$$P [P]_c = [P]_c \Rightarrow [P]_c = P^{-1} [P]_c$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ -3 & -5 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow [P]_c = [t^2]_c = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

Sol, $t^2 = 3(1-3t^2) - 2(2+t-5t^2) + 1(1+2t)$

A basis for Null A = { [0, 0, 0, 1]^T, [0, 0, 0, 0]^T, [0, 0, 0, 0]^T } = A Null of A

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These subspaces are in \mathbb{R}^4 : Col A, Null A, Row A
 These subspaces are in \mathbb{R}^4 : Row A, Null A, Col A
 They are four distinct subspaces because Row A = Col A and Col A = Row A

$$[x] = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ -3 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ -3 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$