Assignment #11

4.5 #20

a) False — See Example 4
b) False — Page 260 (top)
c) False — See definition on p. 257
d) False — See Basis Theorem
e) True — See Example 4

4.6 #2

rank $A = 3$

dim Nul $A = 2$

A basis for Col $A = \left\{ \begin{bmatrix} 1 \\ -2 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ -6 \\ 4 \end{bmatrix}, \begin{bmatrix} 9 \\ -10 \\ -3 \\ 0 \end{bmatrix} \right\}$

A basis for Row $A = \{ [1,-3,0,5,-7], [0,0,2,-3,8], [0,0,0,0,5] \}$

A basis for Nul $A = \left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 3/2 \\ 1 \\ 0 \end{bmatrix} \right\}$

4.6 #27

These subspaces are in $\mathbb{R}^m$: Col A, Nul A^T, Row A^T

These subspaces are in $\mathbb{R}^n$: Row A, Nul A, Col A^T

There are <u>four</u> distinct subspaces because Row A^T = Col A and Col A^T = Row A

4.7 #2

a) $\underset{C \leftarrow B}{P} = \begin{bmatrix} -1 & 5 \\ 4 & -3 \end{bmatrix}$ by Theorem 15

b) $\underset{C \leftarrow B}{P} [\vec{x}]_B = [\vec{x}]_C \Rightarrow \begin{bmatrix} -1 & 5 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \end{bmatrix} = [\vec{x}]_C$

4.7 #8

$\left[\begin{array}{cc:cc} 1 & 1 & -1 & 1 \\ 4 & 1 & 8 & -5 \end{array}\right] \sim \begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & -4 & 3 \end{bmatrix} \Rightarrow \underset{C \leftarrow B}{P} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$

$\left[\begin{array}{cc:cc} -1 & 1 & 1 & 1 \\ 8 & -5 & 4 & 1 \end{array}\right] \sim \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 4 & 3 \end{bmatrix} \Rightarrow \underset{B \leftarrow C}{P} = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$

4.7 #14

$${}_{C\leftarrow B}P = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ -3 & -5 & 0 \end{bmatrix} \; ; \; {}_{C\leftarrow B}P^{-1} =$$

$${}_{C\leftarrow B}P\,[\bar{p}]_B = [\bar{p}]_C \Rightarrow [\bar{p}]_B = P_B^{-1}[\bar{p}]_C$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ -3 & -5 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right] \Rightarrow [\bar{p}]_C = [t^2]_C = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

So, $t^2 = 3(1-3t^2) - 2(2+t-5t^2) + 1(1+2t)$