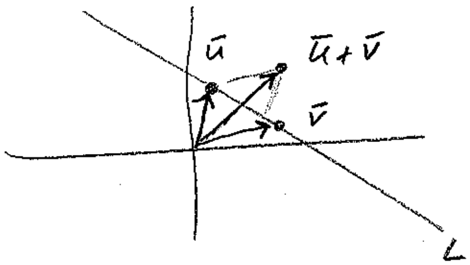
ASSIGNMENT 9

4.1 #4



Consider line L in $\mathbb{R}^2$
$\bar{u}, \bar{v} \in L$ but $\bar{u}+\bar{v} \notin L$.
So, L is not closed under addition.

4.1 #8

The set of all polynomials in $\mathbb{P}_n$ such that $\bar{p}(0)=0$ is a subspace of $\mathbb{P}_n$.

Take $\bar{p}, \bar{q} \in \mathbb{P}_n$, $(\bar{p}+\bar{q})(0) = \bar{p}(0) + \bar{q}(0) = 0+0=0$

So $\bar{p}+\bar{q}$ is in the set $H = \{\bar{p} \in \mathbb{P}_n : \bar{p}(0)=0\}$

Take $\bar{p} \in \mathbb{P}_n$ and $c \in \mathbb{R}$, $(c\bar{p})(0) = c(\bar{p}(0)) = c\,0 = 0$.

So, $c\bar{p} \in H$. Hence, H is closed under addition and scalar multiplication.

So, by the definition of subspace H is a subspace of $\mathbb{P}_n$.

4.1 #10

$$H = \left\{ \bar{x} : \bar{x} = \begin{bmatrix} 2t \\ 0 \\ -t \end{bmatrix} \text{ for } t \in \mathbb{R} \right\}$$

$H = \text{span}\left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \right\}$. So by Theorem 1 (4.1) H is a subspace of $\mathbb{R}^3$.

4.1 #14

To see if $\bar{w} = \begin{bmatrix} 8 \\ 4 \\ 7 \end{bmatrix}$ is in the subspace $\text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix} \right\}$ we solve the system with augmented matrix M below

$$M = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 2 & 4 \\ -1 & 3 & 6 & 7 \end{bmatrix} \overset{rref}{\sim} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Since the system has no solution $\bar{w} \notin \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix} \right\}$

4.1 / #16

$$W = \left\{ \bar{x} : \bar{x} = \begin{bmatrix} -a+1 \\ a-6b \\ 2b+a \end{bmatrix} \right\} = \left\{ \bar{x} : \bar{x} = a\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 0 \\ -6 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

So, $\bar{0} \notin W$. Hence, W is not a vector space.

4.1 / #18

$$W = \left\{ \bar{x} : \bar{x} = \begin{bmatrix} 4a+3b \\ 0 \\ a+b+c \\ c-2a \end{bmatrix}, a,b,c \in \mathbb{R} \right\}$$

$$= \left\{ \bar{x} : \bar{x} = a\begin{bmatrix} 4 \\ 0 \\ 1 \\ -2 \end{bmatrix} + b\begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, a,b,c \in \mathbb{R} \right\}$$

$$= \text{span}\left\{ \begin{bmatrix} 4 \\ 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

So, W is a subspace of R^4 by Theorem 1 (4.1)

4.1 / #20

a) To show that $C[a,b]$ is a subspace we must show

- ✓ $\bar{f}$ defined by $\bar{f}(t) = 0$ for all $t \in [a,b]$
- ✓ For any functions $\bar{g}, \bar{h} \in C[a,b]$, $\bar{g} + \bar{h} \in C[a,b]$
- ✓ For any $\bar{g} \in C[a,b]$ and $c \in \mathbb{R}$, $c\bar{g} \in C[a,b]$

b) We will show $H = \{ \bar{f} \in C[a,b] : f(a) = f(b) \}$ is a subspace of $C[a,b]$.

- ✓ $\bar{h}(t) = 0$, the zero function is in H because $h(a) = h(b) = 0$.
- ✓ Suppose $\bar{f}, \bar{g} \in H$. Then $\bar{f}(a) = \bar{f}(b)$ and $\bar{g}(a) = \bar{g}(b)$.
  $(\bar{f}+\bar{g})(a) = \bar{f}(a) + \bar{g}(a) = \bar{f}(b) + \bar{g}(b) = (\bar{f}+\bar{g})(b)$
- ✓ Suppose $\bar{f} \in H$ and $c \in R$. Then $(c\bar{f})(a) = c(\bar{f}(a)) = c(\bar{f}(b)) = (c\bar{f})(b)$

So, H is a subspace of $C[a,b]$.

4.1 #24

a) True - Definition of vector space
b) True - See p. 217 (above blue box)
c) True - See p. 220 (above Example 6)
d) False - See Example 8
e) False - $\bar{u}$ and $\bar{v}$ must be <u>arbitrary</u> elements in H

4.1 #26

a) Axiom 3
b) Axiom 5
c) Axiom 4

4.1 #28

a) Axiom 4
b) Axiom 7
c) Axiom 3
d) Axiom 5
e) Axiom 4

4.2 #16

$$H = \left\{ \begin{bmatrix} b-c \\ 2b+c+d \\ 5c-4d \\ d \end{bmatrix} : b,c,d \in R \right\} = \left\{ \bar{x} : \bar{x} = b\begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} + c\begin{bmatrix} -1 \\ 1 \\ 5 \\ 0 \end{bmatrix} + d\begin{bmatrix} 0 \\ 1 \\ -4 \\ 1 \end{bmatrix}; b,c,d \in R \right\}$$

So $H = \text{Col}\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 5 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

4.2 #24

$\bar{w} \in \text{Col}\,A$ provided the system with augmented matrix $[A | \bar{w}]$ is consistent.

$$\begin{bmatrix} A & \bar{w} \end{bmatrix} = \begin{bmatrix} -8 & -2 & -9 & 2 \\ 6 & 4 & 8 & 1 \\ 4 & 0 & 4 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & -1/2 \\ 0 & 0 & 1/2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$, so the system is consistent and $\bar{w} \in \text{Col}\,A$.

$$\begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \bar{w} \in \text{Nul}\,A$$

4.2 #26
a) True, Thm 2
b) True, Thm 3
c) False, Col A is all $\bar{b}$ such that the system is consistent.
d) True, See p. 232 (below green box)
e) True, See Fig 2
f) True, See Example 8

4.2 #32
Given $\bar{p} \in \mathbb{P}_2$, $\bar{p}(t) = a_0 + a_1 t + a_2 t^2$ for some $a_0, a_1, a_2 \in \mathbb{R}$.
So, for $\bar{p}(0) = 0$ we must have $a_0 = 0$. Hence,
$\operatorname{Ker} T = \{ \bar{p} \in \mathbb{P}_2 : \bar{p}(t) = a_1 t + a_2 t^2 \text{ for } a_1, a_2 \in \mathbb{R} \}$

So, $\operatorname{Ker} T = \operatorname{span}\{t, t^2\}$

$\operatorname{Range} T = \left\{ \begin{bmatrix} a_0 \\ a_0 \end{bmatrix} : a_0 \in \mathbb{R} \right\}$

4.2 #34
Let $\bar{f}, \bar{g} \in C[0,1]$ and $c \in \mathbb{R}$. Then $T(\bar{f}) = \bar{F}$ where $\bar{F}' = \bar{f}$ and $\bar{F}(0) = 0$ and $T(\bar{g}) = \bar{G}$ where $\bar{G}' = \bar{g}$ and $\bar{G}(0) = 0$. By the rules for antidifferentiation
$(\bar{F} + \bar{G})' = \bar{f} + \bar{g}$ and $(\bar{F} + \bar{G})(0) = \bar{F}(0) + \bar{G}(0) = 0 + 0 = 0$
and $(c\bar{F})' = c\bar{f}$ and $(c\bar{F})(0) = c\bar{F}(\bar{0}) = c(0) = 0$.
So, $T(\bar{f} + \bar{g}) = T(\bar{f}) + T(\bar{g})$ and $T(c\bar{f}) = cT(\bar{f})$
and T is a linear transformation.

$\operatorname{Ker} T = \{ \bar{f} \in C[0,1] : \int \bar{f} = 0 \} = \{ \bar{0} \}$
where $\bar{0}$ is the zero function.