Solving Linear Systems

Given a linear system:

$1x_1 + 2x_2 + 3x_3 = 10$
$1x_1 + 4x_2 + 5x_3 = 16$
$2x_1 + 6x_2 + 8x_3 = 26$

Form the augmented matrix for the system:

$$\begin{bmatrix} 1 & 2 & 3 & 10 \\ 1 & 4 & 5 & 16 \\ 2 & 6 & 8 & 26 \end{bmatrix}$$

Apply elementary row operations to reduce the augmented matrix to row reduced echelon form:

Replace **a row by the sum of itself and a multiple of another row.** $(kR_i + R_j \rightarrow R_j)$
Interchange **two rows.** $(R_i \leftrightarrow R_j)$
Scale **a row by multiplying its entries by a nonzero constant.** $(kR_i \rightarrow R_i)$

$$\begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Return to equation notation:
$x_1 + x_3 = 4$
$x_2 + x_3 = 3$

Interpret and state conclusion:

The general solution of the system is

$x_1 = 4 - x_3$
$x_2 = 3 - x_3$
x_3 **is free**

That is, the set of solutions for the system consists of all 3-tuples of the form $(4 - t, 3 - t, t)$ **where t is any real number.**