MATH 306 Linear Algebra
Test #1

Name______________________________
Fall 2002

I. **Definitions** (3 points each). Write definitions of the specified terms. Be sure to use complete English sentences in all cases.

1. Define what it means for a set of indexed vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ to be *linearly independent*.

2. Define: *linear transformation.*

II **Exercises** (4 points each).

3. Suppose the matrix below is the augmented matrix for a linear system. Show, step-by-step, how to produce the unique reduced (row) echelon form of that matrix. State whether the associated linear system is consistent or inconsistent.

$$\begin{bmatrix} 3 & 6 & 6 & 9 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & 4 & 0 \end{bmatrix}$$

4. Suppose the matrix below is the reduced (row) echelon form for the augmented matrix of a linear system. Is the system consistent? Justify your response, and if you think the system is consistent specify either the unique solution or the general form for all solutions.

$$\begin{bmatrix} 1 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & -2 & -2 \end{bmatrix}$$

5. Suppose we define the 3 x 4 matrix A as below. Are the columns of A linearly independent? Justify your answer. If you think the columns form a linearly dependent set of vectors, write one column as a linear combination of the others.

$$A = \begin{bmatrix} 2 & -2 & 0 \\ 0 & 3 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

6. Suppose we define the *3 x 3* matrix B as below. Solve the homogeneous equalion $B\mathbf{x} = \mathbf{0}$ and specify the general form of the solutions in parametric vector form.

$$B = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

7. Is the transformation T: $R^2 \rightarrow R^2$ a linear transformation? Justify your answer by appealing directly to the definition of a linear transformation.

$T([x_1, x_2]) = [2x_1 + 3x_2,\ |x_2|]$

For example, $T([3,-4] = [-6, 4]$

8. Given T: $R^2 \rightarrow R^2$ defined by $T(\mathbf{v}) = A\mathbf{v}$ where A is the *2 x 2* matrix specified below, and suppose that **b** is the *2 x 1* column vector defined below. Show how to find an **x** in R^2 such that $T(\mathbf{x}) = \mathbf{b}$. If no such **x** exists, justify that response.

$A = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

9. Is the mapping of Exercise #8 above onto R^2? (Justify your answer.) Is the mapping of Exercise #8 above one-one? (Justify your answer.)