MATH 306 Linear Algebra — Name________________________________

Test #2 — Spring 2002

I. Definitions Write definitions of the following terms. Be sure to use complete sentences in each case.

1. Define: linearly independent
2. Define: linear transformation
3. Define: inverse of a matrix
4. Define: subspace of R^n
5. Define: column space of a matrix
6. Define: null space of a matrix
7. Define: basis of a subspace of R^n

II. Computations Given the matrices and vectors defined below, perform the indicated computations if possible. If a particular computation cannot be performed, say so and explain why it cannot be performed.

$$A = \begin{matrix} 2 & 0 & 4 \\ 0 & 1 & 1 \\ 4 & 1 & 9 \end{matrix}\ ; \quad B = \begin{matrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 2 & 4 & 7 \end{matrix}\ ; \quad C = \begin{matrix} 1 & 0 \\ 2 & 3 \\ 0 & 4 \end{matrix}$$

8. A + 2B

9. AC

10. CB

11. C^TA^T

III. Exercises

12. Consider the function T: R^3 into R^2 defined by $T(x_1, x_2, x_3) = (x_1 + 4x_3, 3x_2 - 6x_3)$

    a. Find the standard matrix for T.

    b. Specify using parametric vector form all **v** in R^3 such that $T(\mathbf{v}) = (0,0)$.

13. Show, step-by-step, how to employ the row reduction algorithm to find the inverse of the matrix below.

$$\begin{matrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 2 & 4 & 7 \end{matrix}$$

14. Suppose T: R^3 R^3 is defined by $T(\mathbf{v}) = D\mathbf{v}$ where D is defined below.

	1	0	-3
D =	0	-1	1
	0	2	-4

a. Does the inverse of T, T^{-1}: R^3 into R^3 exist? If so, specify a rule for T^{-1}.

b. Is T onto R^3? Justify your answer.

c. Is T one-to-one? If so, specify the unique **v** in R^3 such that $T(\mathbf{v}) = [1\ \ 0\ \ 0]^T$.

15. Suppose the matrix A is defined as follows.

	1	0	2
A =	0	1	2
	0	2	5

a. Specify a basis for the column space of A.

b. Characterize the vectors in the null space of A in parametric vector form.

c. Specify a basis for Nul A.

IV. True-False Mark each statement true or false. You do not need to justify your answers. Assume that the matrices mentioned in the statements below have the appropriate sizes. Objects named by capital letters A, B, C, and D are matrices.

16. If AB = C and B has 3 rows, then C must have 3 rows.

17. If AB = AC, then it must follow that B = C.

18. It is possible for a square matrix A to be invertable even if its columns are not lineraly independent.

19. If A and B are both n x n and they each have a pivot in each row, then AB is invertable.

20. If the columns of A are linearly independent, then the rows of A are linearly independent also.