Session 6

Suppose $\bar{a}_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$, $\bar{a}_2 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $\bar{a}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\bar{b} = \begin{bmatrix} 8 \\ 17 \\ 27 \end{bmatrix}$

Is $\bar{b}$ in Span $\{\bar{a}_1, \bar{a}_2, \bar{a}_3\}$?

Can $\bar{b}$ be written as a linear combination of $\bar{a}_1, \bar{a}_2, \bar{a}_3$?

Definition If $A = [a_{ij}]_{m \times n}$ with columns $\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n$ and if $\bar{x} \in \mathbb{R}^n$ then the product of A and $\bar{x}$ is the linear combination of the columns of A using the corresponding entries of $\bar{x}$ as weights, that is

$$A\bar{x} = [\bar{a}_1 \ \bar{a}_2 \ \cdots \ \bar{a}_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = x_1\bar{a}_1 + x_2\bar{a}_2 + \cdots + x_n\bar{a}_n .$$

Equivalently

$$A\bar{x} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

Examples

1) Find the product

$$\begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 1 \\ 6 & 4 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} =$$

2) Solve: $\begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 1 \\ 6 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$