MATH 306 Linear Algebra Name________________________________
Test #1 Spring 2007

I. **Definitions** (5 points each). Write definitions of the specified terms. Be sure to use complete English sentences in all cases.

1. Define what it means for a set of indexed vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ to be *linearly independent*.

2. Define: *linear transformation*.

II **Exercises** (5 points each).

3. Suppose the matrix below is the augmented matrix for a linear system. Show, step-by-step, how to produce the unique reduced (row) echelon form of that matrix. State whether the associated linear system is consistent or inconsistent. If the system is consistent, specify whether or not there is a unique solution.

$$\begin{bmatrix} 1 & 2 & -3 & 6 \\ 1 & 3 & 1 & 4 \\ 2 & 5 & -2 & 11 \end{bmatrix}$$

4. Suppose the matrix below is the reduced (row) echelon form for the augmented matrix of a linear system. Is the system consistent? Justify your response, and if you think the system is consistent specify either the unique solution or the general form for all solutions in parametric vector form.

$$\begin{bmatrix} 1 & -2 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

5. Consider the set of vectors $S = \left\{ \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} \right\}$. Is S a linearly independent set? Carefully justify your answer by appealing directly to the definition of linear independence.

6. Suppose we define the *3 x 3* matrix A so that the columns of A are the vectors in set S above.

That is $A = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 2 \\ 0 & 2 & 2 \end{bmatrix}$.

a. Specify the span of the columns of A.

b. Is the mapping the mapping $\mathbf{x} \mapsto A\mathbf{x}$ from R^3 into R^3 onto R^3?

c. Is the mapping the mapping $\mathbf{x} \mapsto A\mathbf{x}$ from R^3 into R^3 one-one?

d. What is the image of $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$ under the mapping $\mathbf{x} \mapsto A\mathbf{x}$ from R^3 into R^3?

e. What is the image of the zero vector under the mapping $\mathbf{x} \mapsto A\mathbf{x}$ from R^3 into R^3?

7. Is the transformation T: $R^2 \to R^2$ defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ |x_2| \end{bmatrix}$ a linear transformation?

Justify your answer by appealing directly to the definition of a linear transformation.

8. Given T: $R^3 \rightarrow R^2$ defined by $T(\mathbf{x}) = A\mathbf{x}$ where A is the *2 x 3* matrix specified below, and suppose that **b** is the *3 x 1* column vector defined below. Find an **x** in R^3 such that $T(\mathbf{x}) = \mathbf{b}$. If no such **x** exists, justify that response.

$$A = \begin{bmatrix} 2 & 0 & 6 \\ 0 & 3 & 12 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -2 \\ 15 \end{bmatrix}$$

9. Is the mapping of Exercise #8 above onto R^2? (Justify your answer.) Is the mapping of Exercise #8 above one-one? (Justify your answer.)

10. Suppose $T: R^2 \rightarrow R^2$ is a linear transformation and also

$$T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \quad \text{and} \quad T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} 7 \\ 3 \end{bmatrix}.$$

a. Specify the standard matrix for the linear transformation T.

b. Is $\{ T(\begin{bmatrix} 3 \\ 0 \end{bmatrix}), T(\begin{bmatrix} 0 \\ 2 \end{bmatrix})\}$ a linearly independent set? Justify your answer.

III True-False (2 points each.) If a statement is true, circle the "T." Otherwise circle the "F."

T F 11. If an augmented matrix [A **b**] is transformed into [C **d**] by elementary row operations, then the equations A**x** = **b** and C**x** = **d** must have exactly the same solution sets.

T F 12. If A is an $m \times n$ matrix and the equation A**x** = **b** is consistent for some b, then the columns of A must span R^m.

T F 13. If an $m \times n$ matrix A has a pivot in each row, then the equation A**x** = **b** has a unique solution for each **b** in R^m.

T F 14. If none of the vectors in the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ in R^3 is a linear combination of the other two, then S is a linearly independent set.

T F 15. If an augmented matrix [A **b**] has a pivot in each column, then the equation A**x** = **b** must be inconsistent.