MATH 306 Linear Algebra Name________________________________
Test #3 Fall 2006

1. (5 points) Complete the definition of a vector space.

A **vector space** is a nonempty set V of objects, called *vectors*, on which are defined two operations, called *addition* and *multiplication by scalars* (real numbers), subject to the ten axioms listed below. The axioms below must hold for all vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in V and for all scalars c and d.

2. (5 points) Write a complete definition of the term "subspace of a vector space."

3. (5 points) State the Basis Theorem.

4. (5 points) Consider P_4 the vector space of polynomials of degree 4 or less. Is $H = \{\mathbf{p}: \mathbf{p}(t) = a_0 + a_1t + a_3t^3 \text{ for some } a_0, a_1, a_3 \in R\}$ a subspace of P_4? Carefully express your answer using a complete sentence, and then carefully justify your answer.

5. (5 points) Consider the matrix A defined below. Show how to use row reduction to help you determine both a basis for the column space of A and a basis for the null space of A. Clearly state your answers using complete sentences.

$$A = \begin{bmatrix} 1 & -2 & 0 & -1 & 5 \\ -2 & 4 & 4 & 2 & -2 \\ 0 & 0 & 4 & 5 & 0 \end{bmatrix}$$

6. (10 points) Consider the linear transformation T: $R^3 \to R^2$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_2 - 6x_3)$.

a. Show how to find Ker T. Describe Ker T explicitly using parametric vector form.

b. Show how to find the range of T. Describe the range explicitly using parametric vector form.

7. (5 points) Consider the matrix A below , the linear transformation T: $R^m \to R^n$ defined by $T(\mathbf{v}) = A\mathbf{v}$, and the linear system defined by $A\mathbf{x} = \mathbf{0}$

$$A = \begin{bmatrix} 2 & 0 & 4 & 6 & 0 & -4 \\ 0 & 1 & 0 & 3 & -1 & 0 \\ 0 & 1 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

Specify each of the following:

Rank A =	Dim Domain of T =	Rank A + Dim Nul A =
Dim Nul A =	Dim Range of T =	$A\mathbf{x} = \mathbf{0}$ has ____ free variables.
Dim Col A =	Dim kernel of T =	A has ____ pivot columns.

8. (10 points) $B = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix} \right\}$ and $C = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$, are both bases for $\mathbb{R}^2$.

a. If $[\mathbf{v}]_B = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$, show how to determine the coordinates of **v** relative to the standard basis.

b. If $u = \begin{bmatrix} 5 \\ 13 \end{bmatrix}$, specify $[\mathbf{u}]_B$.

c. Show how to apply an algorithm to find the change-of-coordinates matrix from *B* to *C*, and clearly identify that matrix.

d. Show how to use the change-of-coordinates matrix you found in part c. to obtain $[\mathbf{v}]_C$ if $[\mathbf{v}]_B = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$.