Lesson: Developing Pascal's Triangle

Prerequisites

- Finding area of rectangle
- Multiplying monomials
- Knowledge of exponents
- Combining Like Terms
- Writing polynomials in descending order by degree

## Vocabulary

- ♦ Expression
- ♦ Equation
- ♦ Dimensions
- ♦ Monomials
- Binomial
- ♦ Polynomial
- ♦ Coefficient

## Materials

- ♦ Algebra Tiles (optional)
- Index cards could replace algebra tiles

**Time Allowance: Two 50-minute periods** 

## **Developing Pascal's Triangle**

Drawing models of problems can help visualize a process that is used in algebra. Binomials can be modeled using dimensions of rectangles and hence, multiplication of binomials can be represented using the area of the rectangles.

1. Find the *total area* of each figure below. Use the chart provided to help you organize your work.



|   | Length | Width | Expression for<br>Area | Total Area |
|---|--------|-------|------------------------|------------|
| a |        |       |                        |            |
| b |        |       |                        |            |
| С |        |       |                        |            |
| d |        |       |                        |            |
| е |        |       |                        |            |

Note: Be sure to combine like terms when appropriate and write the polynomials representing the total area in descending order by degree.

2. Find the *total area* of each figure below. Use the chart provided to help you organize your work.



- 3. Repeated multiplication with binomials can be written using exponents. For example, the area of *a* in Chart II can be written  $(a+b)^2$ . Rewrite the expression for the area of *b* and *c* in Chart II using the binomial a + b.
- 4. Copy the <u>coefficients</u> of each total area from Chart II into the boxes below.



- 5. What patterns do you see?
- 6. Each line of the triangle represents the coefficients of the binomial a + b raised to a power. Make a prediction for the coefficients of  $(a+b)^5$ .
- Draw a model of (a + b)<sup>5</sup> to verify your prediction. (Hint: Build onto a previously given model.)
- 8. If your prediction was incorrect, what went wrong? What should you do to correct your mistake? If your prediction was correct, proceed to the next question.

- 9. Examine all the exponents on the variables for the previous areas. What patterns do you see?
- 10. Draw a model to help you find  $(3x+2y)^2$ .

11. Could you use the triangle above to help you answer this without drawing a model? Explain.

12. The following gives you answers for when binomials are subtracted.

$$(a-b)^2 = a^2-2ab+b^2$$
  
 $(a-b)^3 = a^3-3a^2b+3a^2b-b^3$   
 $(a-b)^4 = a^4-4ab^3+6a^2b^2-4a^3b+b^4$ 

Study these examples. How does the subtraction affect the pattern in the triangle?

- 13. Use the triangle to find  $(2x y)^3$ .
- 14. What does the triangle suggest  $(a + b)^0$  is equal to?