PascGalois Project

Self-Similarity and the Klein-4 Group

In this section we will characterize the self-similar growth structure of $(P_{Z_2\times Z_2}, a, b)$ where $a = (0, 1)$ is placed down the left side of the triangle and $b = (1, 0)$ is placed down the right. The self-similarity studied in this and the earlier lessons is different from that of fractal geometry. Instead of "zooming in" to see copies of a pattern, we "zoom out". Now, you may have already studied $Z_2 \times Z_2$ under a different name - the Klein-4 group. Both groups are non-cyclic of order 4 and, hence, are isomorphic. Before starting the exercises you should construct the group table for the Klein-4 group and write down its subgroup lattice.

Remark: *PascalGT* uses the notation 0 = (0,0), 1 = (1,0), 2 = (0,1), and 3 = (1,1) to represent $Z_2 \times Z_2$.

Exercises:

1. Construct $(P_{Z_2\times Z_2}, a, b)$ as described above. Try the first 4, 8, 16, 32, and 64 rows of the triangle. What is the dominant pattern emerging? What sub-triangles of the triangle are repeated at different scales?

2. Give a description of the first 2^n rows of $P_{Z_2\times Z_2}$ that applies for *any* $n \geq 3$. Draw a diagram that provides the overall structure as part of your answer. This diagram is called a *growth* triangle - it reveals the self-similar structure of the PascGalois triangle. How does the group structure determine this growth triangle? <u>Note</u>: You should pay close attention to the boundaries of the sub-triangles as you try to understand the growth triangle in terms of the structure of the Klein-4 group. It may also help to write out the first 16 rows by hand to understand how the geometry of the growth triangle is determined by the group multiplication.

3. Determine the growth triangles for P_{Z_n} for $n = 2, 3, 4, 5$ and 6. Compare the patterns you see with the subgroup lattice of each group. Then compare the orders of the group elements for each group versus the number of rows needed to repeat the self-similar structure. Does the value of n, whether it is prime or composite, seem to affect the corresponding PascGalois triangle?

What about the number of relatively prime factors? (The answers to these last two questions are addressed in more detail in the projects on quotient groups).