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Learning Number Sense from a Broken Calculator

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Introduction >>

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Calculators weaken students' computational skills and number sense—or do they? Many parents and educators worry that students will become overly reliant on machines to perform calculations and that students' computational facility will be weakened. What would happen if students were asked to perform calculations on a calculator that is broken? A broken calculator has selectively disabled numerical or operations keys. It is a fertile number laboratory in which students can develop number sense and insight into the structure of the number system. The calculator is now not a crutch but a significant challenge for students.

The *Broken Calculator* program offers a versatile medium for these computation exercises. Students might attempt problems with a real calculator, pretending keys or operations are nonfunctional, or use a flexible computer model. They must figure out ways to express numbers without relying on the many routine shortcuts calculators provide. They must decompose and rewrite numbers for entry, find alternative paths for composing numbers and seek out equivalent operations. As one teacher states, "*Broken Calculator* forces kids into every nook and cranny of every single number and operation."

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Broken Calculator offers an example of a powerful pedagogic strategy: To gauge how well someone understands the use of a set of tools (in this case, numbers and operations), deny them the use of some of the tools and challenge them to use the remaining ones to accomplish the task at hand.

Kelly Goorevich reflects on the students' work on *Broken Calculator* and the ways the software supports mathematical dialogue.

Broken Calculator is, in effect, an inexpensive, versatile number laboratory. Using *Broken Calculator* to solve problems forces students to explore operations on numbers and representations much more deeply than does rote application of algorithms. Most important, working with *Broken Calculator* shifts the responsibility of finding and expressing the answer to the student. Solving problems means inventing a wide variety of strategies to work around the limits set by the disabled keys or functions. Students deepen their number sense, problem-solving capacity, and understanding of the number system—and have fun at the same time!

The material described in this article was developed by staff at the Concord Consortium as part of a series of online courses now offered by TeachScape as Seeing Math Elementary™. Seeing Math was funded partly under grant No. R286A000006 from the U.S. Department of Education.

The site http://www.seeingmath.concord.org gives more information about the Seeing Math courses.

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Challenges with *Broken Calculator* >>

Use the Broken Calculator to solve the followwing challenges.

Challenge 1: Solve 35 × 42 with the 2, 3, 4 and 5 keys disabled.

Set as the goal the problem 35 × 42. Disable the 2, 3, 4 and 5 keys.

How does a multiplication problem change your approach?

After you have worked on the problem yourself, take a look at Kelly Goorevich's fourth-grade students from Watertown, Massachusetts, working on this problem.

Challenge 2: Now use the Leading Digit mode to consider 35 × 42 with the 2, 3, 4 and 5 keys disabled.

Set up as with challenge 1, but use the Leading Digit mode. Take mental notes on how you adjusted your strategies had to accommodate this new constraint.

On this video segment, Dr. Susan Empson of the University of Texas discusses the challenges the students faced and resolved. Dr. Empson highlights decomposition of numbers as well as the rote nature of the standard algorithm.

Challenge 3: Consider 358 ÷ 8 with the ÷ operation key disabled.

How would other operations guide you to a solution?

Challenge 4: Make up your own grade level challenge.

Operations on two-digit numbers may better suit younger students.

Upper elementary school students may benefit from challenges that use decimals or fractions. Students should make up their own challenge and discuss how they would use the disable key function.

Challenge 5: Understanding the power of place value

Disable the following keys: 2, 3, 4, 5, 6, 7, 8, 9, ×, and ÷. Set a goal of some four-digit number, say 1776. Use the remaining keys to get that number. Explain your strategies.

Challenge 6: Exploring Number Theory

Suppose you have a four-function calculator modeled by the Broken Calculator. Because it can display only a limited number of digits, not all the numbers on the real number line can appear in the calculation window. For example, there is no way to either enter or compute the correct value for the number 1/3. Here are some explorations that help students think about how calculators can display the real numbers and how a calculator fails to represent real numbers accurately.

- What is the largest number in absolute value that your calculator can display?
- What is the smallest positive number that your calculator can display?
- What is the total number of distinct numbers that your calculator can display?
- What is the smallest separation between "adjacent" numbers on the calculator?
- What is the largest separation between "adjacent" numbers on the calculator?
- How many calculator numbers are included in an interval along the number line of length 2—for example, between 7.3 and 9.3, or 543.1 and 545.1? How does the answer to this question depend on where the chosen interval is located?
- Make up three addition problems that the calculator computes incorrectly and three that it computes correctly. What general statements, if any, can you make about calculator addition? Repeat the process for subtraction, multiplication, and division.

Challenge 7: More Number Theory

Consider problems related to constructing even and odd numbers and prime numbers.

- If all the even keys are disabled, can you construct an odd number?
- If all the odd keys are disabled, can you construct an even number?
- If the keys for 3, 5, and 7-the prime numbers besides 2-are disabled and multiplication is disabled, can you construct the prime numbers from 3 to 50? Are there any strategies that help in constructing them?

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Using *Broken Calculator* in the Classroom >>

Broken Calculator works well with small groups of students because their familiarity with the calculator encourages dialogue. The "broken" number keys or functions pose problems for the group to overcome. Successful problem solving demands attention to the basic structure of the number system and builds on skills and factual knowledge students bring to the problem. Students weaker in computation are not at a disadvantage when using *Broken Calculator,* while students with rote mastery of algorithms are challenged to go beyond procedures to deepen their knowledge of relationships among numbers and plan computations where standard methods do not apply.

Warm-Up Activities

Take some time to become acquainted with *Broken Calculator* software. You will need the Flash plug-in installed on your computer. If you do not have Flash, download the free player here.

Sample 1: Try to solve the addition problem with the 1, 2, 3, 4 and 5 keys disabled.

> *Using the "Set a Goal" screen, enter* 312 + 514. *Use the "Disable Keys" screen to disable the 1, 2, 3, 4 and 5 keys.*

As you solve the problem, make mental notes about how you had to compose the numbers you needed and how the broken keys made you think differently about computation. Did you find any tricks that might work on other problems?

Younger students may benefit from problems involving only one- or two- digit numbers. Older students may be more challenged by using problems that involve decimals or fractions.

Sample 2: Consider a similar problem: 512 – 312. Try to solve this one with the 1, 2, 3, 4 and 5 keys disabled. Again, make mental notes of your thought process and any strategies you learned.

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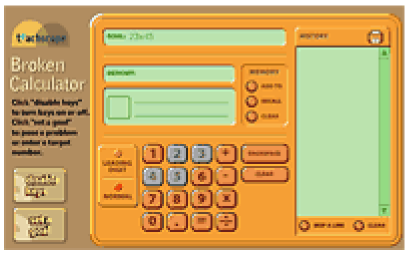

Figure 1 The Calculation Window

Dr. Judah Schwartz of Tufts University first published the Broken Calculator software twenty years ago under the title "What Do You Do with a Broken Calculator?" The Web-based Flash version used here is similar to the original design published by Sunburst Inc. The interface is simple, appearing much like the keypad of a familiar calculator. The calculation screen **(fig. 1)** shows a goal window on the top and a history tape on the left that records a printable record of progress toward the goal. If the goal is reached, the user is informed in the scrolling tape section of the calculation window. A separate window enables the user to set a goal, or problem, for a computational exercise (see **fig. 2**). Another enables the user to disable selected keys or functions. Selected numerical keys or operators are grayed out and made unusable in the computation window (see **fig. 3**).

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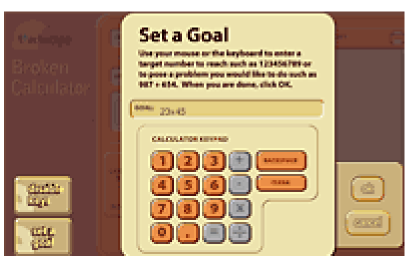

Figure 2 The "Set a Goal" Window

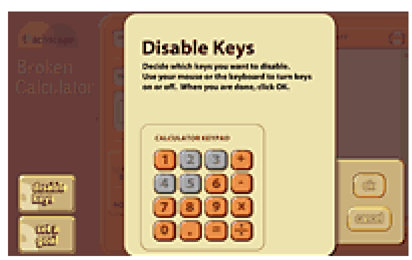

Figure 3 The "Disable Keys" Window

Broken Calculator offers two modes for the user. In "Normal" mode, the keys function much like a traditional calculator, except that some keys will not function. In "Leading Digit" mode, once the first non-zero digit is entered, all the digits from 1 to 9 are disabled. Thus the only numbers that can be entered directly in this mode are single digits multiplied by powers of 10. This mode underscores that all our computational algorithms are based on the single-digit "facts" and a nimble understanding of place value.


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