Instructions: Read the “honesty pledge” below and sign where indicated. Complete the exam and scan/photograph the pages. You may either combine them into a multi-page document or separate images. In either case, preserve the question order as best as possible. Your grade will be posted on MyClasses; standard procedures for discussion of grades will apply. Late submissions will not be accepted.

During this exam:

1. ✗ You may not communicate with classmates, peers, faculty, or anybody else about the content on this exam until after the due date and time.

2. ✗ You may not consult any online or digital resource; this includes videos, web-pages, and forums.

3. ✓ You may use your own notes or programs from class lectures or the course textbook (see the course syllabus for text title and edition).

4. ✓ You may prepare your exam submissions digitally (in a word processor or typesetting system).

5. ✓ You may test any code portions of this exam by writing them and compiling on your own system, but only submit the code that is relevant to the question at hand. If a question asks for only a single function, provide only the code for that function.

Honesty Pledge

I, the undersigned, acknowledge that I have read and will adhere to the above instructions and stipulations for the completion of this exam. I understand that violation of any one of these restrictions constitutes academic misconduct in the context of this exam, and will result in 1) a grade of zero on this exam and 2) a report to the Salisbury University Office of Academic Affairs for further action, as appropriate according to the academic policies provided at https://www.salisbury.edu/administration/academic-affairs/misconduct-policy.aspx.

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Name (print)

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Name (sign)
1. (10 pts) What is the difference between the binary-search-tree property and the min-heap property? Can the min-heap property be used to print out the keys of an $n$-node heap in sorted order in $O(n)$ time? Show how, or explain why not.
2. (10 pts) Professor Moriarty decides to write a procedure that produces at random any permutation besides the identity permutation (the permutation that results in no change in the array). He proposes the following procedure:

```plaintext
Permute-Without-Identity(A):
    n = A.length
    for i=1 to n-1
        swap A[i] with A[Random(i+1,n)]
```

where the Random(a,b) subroutine produces a random integer from $a$ to $b$, inclusively.

Does this code do what Professor Moriarty intends?
3. (10 pts) In the context of the HIRE-ASSISTANT problem, suppose there are $n$ candidates.

(a) If the cost of hiring a candidate is $c_H$ and the cost of interviewing is $c_I$, express the best and worst case costs of hiring the best assistant among all candidates.

(b) If the candidates arrive in a uniformly random order, what is the probability of the best and worst-case scenarios?

(c) Prove that if the $n$ candidates arrive in a uniformly random order, the expected number of candidates hired is $\log n$. 

4. (10 pts) Complete the following:

(a) Define and explain the *load factor* of a hash table.

(b) Explain in words and illustrate a hash table that resolves collisions by chaining, along with pseudocode for the procedures to search, insert, and delete from the hash table.

(c) Suppose you modify a Hash Table which resolves collisions by chaining so that insertion and deletion keep each doubly-linked list in some sorted order. How does this affect the cost of inserting, deleting, and searching in the hash table? Give a precise answer and provide justification.
5. (10 pts) Suppose that we build a BST so that each node has only a pointer to its left child, right child, and successor node (if one exists). You may still assume that the tree has a pointer to its root.

(a) Give an implementation of a subroutine, \texttt{Parent}(u), that takes a pointer to a node and returns its parent in the tree (or \texttt{null} for the root). This should complete using \(O(h)\) operations.

(b) Using your subroutine above, show how to implement the \texttt{Search}(k) function to find a key \(k\) in the tree, if it exists, using \(O(h)\) operations.
6. (10 points) Show that any $n$-node BST can be transformed into any other $n$-node BST by using only $O(n)$ rotations.

(a) First show that $n - 1$ right rotations suffice to transform the tree into a right-going chain (i.e. a BST where every node is either the root or a right-child).

(b) Use the above method to give a full transformation algorithm. You can assume the input is two trees, $T_1$ and $T_2$, and that the output should be a sequence of rotation commands.

(c) Demonstrate your algorithm on an example with at least 6 nodes.

Hint: show that the right-going chain on a set of $n$ elements is unique. Then use the fact that rotation is an invertible process.
7. (10 pts) Complete the following:

(a) What are the five properties maintained in a Red-Black tree?

(b) Explain briefly what the primary advantages of Red-Black trees have, compared to standard Binary Search Trees.

(c) Show the red-black trees that result after successively inserting the keys 41, 38, 31, 12, 19, 8 into an initially empty red-black tree.
8. (10 pts) Show, by means of a counterexample, that the following “greedy” strategy does not always determine an optimal way to cut rods. Define the density of a rod of length $i$ to be $p_i/i$, that is, its value per inch. The greedy strategy for a rod of length $n$ cuts off a first piece of length $i$, where $1 \leq i \leq n$, having maximum density. It then continues by applying the greedy strategy to the remaining piece of length $n - i$.

What is the asymptotic running time of the above greedy algorithm?
9. (10 pts) Consider a directed graph $G(V, E)$ where $V = \{1, 2, 3, 4, 5\}$ and the graph has adjacency matrix

$$M = \begin{pmatrix}
0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0
\end{pmatrix}.$$ 

(a) Draw the graph.
(b) Compute the matrix $M^2$ and draw the corresponding graph (where values greater than 1 are taken to be just 1).
(c) What is the logical relationship between the two graphs?
10. (10 points)

(a) Show the execution of the Matrix-Chain-Multiply algorithm for a chain of matrices whose array of dimensions is $[5, 10, 3, 12, 7, 9]$. Include the computation of the solution matrix, $S$, and the cost matrix, $M$. Write the final parenthesization that would result from the algorithm.

(b) Suppose that one tries to solve the matrix chain multiplication problem in the following (greedy) way, to avoid computing every sub-problem: among the chain $A_i A_{i+1} \cdots A_j$, choose to split after matrix $A_k$ by selecting $k$ to minimize $p_{i-1} p_k p_j$ before computing the sub-problems. Give an example input to this algorithm that causes it to fail at finding the optimal solution.