1 Objectives

In this lab you will focus on the following objectives:

1. Review basic graph representations and operations
2. Develop familiarity with the c++ standard library tools.
3. Implement depth-first search
4. Work with directed acyclic graphs (DAG)
5. Implement method to find strongly-connected graph components and topological sort

2 Tasks

1. Put your code in a folder called “Lab-11”. This folder will be zipped and turned in at the end.

2. Re-use your Graph class from Lab 10 with the following modifications (keep all other methods, such as print):
   
   (a) Modify the class to decide at instantiation time whether the graph is directed or undirected. This should be specified as a parameter to the constructor, and be immutable otherwise. Include a public method to report which type of graph it is.

   (b) The addEdge should add an edge between two vertices (if it doesn’t already exist), and take into account whether the graph needs to be directed or undirected (specifically, if it is undirected, both directions of the edge should be added!)

   (c) Implement Depth-First-Search (DFS) for the graph, as discussed during lecture.

   (d) Add a method to report whether the graph is both directed and acyclic (contains no cycles). Use a modified DFS to determine whether it is acyclic: during the search, if any gray nodes (other than the “parent”) are encountered during a single DFS-visit, then the algorithm has found a cycle.

   (e) Implement the topological sort algorithm on the graph, reporting a valid topological ordering, if one exists (i.e. the graph must be a DAG). A topological ordering (as will be discussed in lecture) is an ordering of the vertices \(\{v_1, v_2, \ldots, v_n\}\) so that if \(i > j\), then there cannot be a path in the graph from \(v_i\) to \(v_j\) (there may or may not be a path from \(v_j\) to \(v_i\)). You can determine a valid topological sort by first performing DFS, then ordering the vertices by decreasing finish times. If there is no such ordering (because it is not a DAG), an error should be reported.
(f) Write a method to report each fully connected component of the graph using DFS. The high-
level approach (discussed in lecture) is to first DFS the graph to compute the finish times, then
reverse all the edges (using a copy of the graph with a copy constructor will be useful here!) and
perform DFS on the reversed graph (aka the transpose graph) where the nodes are visited in
*order of descending finish time*. Each *DFS tree* in the resulting forest computed by the second
DFS contains exactly one fully connected component of the graph. The graph does not have to
be acyclic for this procedure.

3. Write a test program to demonstrate (clearly) the correctness of each of the above functions.

4. Write your *main* function to read the graph adjacency list from a file for testing purposes. Be sure to
document the format your program expects, and remember to submit the files you use to test your
code.

5. Include a *Makefile* to build your code.

6. Include a *README* file to document your code, any interesting design choices you made, and *answer
the following questions completely and thoroughly*:
   (a) Summarize your approach to the problem, and how your code addresses the abstractions needed.
   (b) What is the theoretical time complexity of your algorithms (best and worst case), in terms of the
   size of the graph? Be sure to vary the parameters enough to use the observations to answer the
   next questions!
   (c) How could the code be improved in terms of usability, efficiency, and robustness?

3 Submission

All submitted labs must compile with your provided *Makefile* and run on the COSC Linux environment.

Upload your project files to MyClasses in a single *`.zip`* file.

Turn in (stapled) printouts of your source code, properly commented and formatted with your name,
course number, and complete description of the code.

Also turn in printouts reflecting several different runs of your program (you can copy/past from the
terminal output window). Be sure to test different situations, show how the program handles erroneous
input and different edge cases.

4 Bonus

(Up to 20 pts)

Modify your *Graph* class to use *c++* templates. To accomplish this, you can either store the data directly
inside the graph nodes, but pay a performance/memory penalty in the methods that require moving whole
nodes around by value (instead of *int* variables). The refactoring to accommodate this would also be quite
cumbersome.

A better way is to copy the data into a *vector* or *map* and use the index of each one in the vector as
an integer “key” of a graph node. For instance, if the user wants a graph on the strings "dog", "cat", and
"computer", you can store them in a *std::vector<std::string>*. and alias them in the graph with keys
0,1, and 2, respectively. This will also only require minimal modification of the graph algorithms themselves,
because they will still operate on *int* valued nodes!