1. All homework, labs, reviews, exams, and lecture notes since exam 1.

2. What is the expected value of the sum of $n$ random 6-sided dice rolls?

3. Let $A[1\cdots n]$ be an $n$-element array. An inversion of $A$ is a pair $(i, j)$ such that $A[i] > A[j]$. Suppose that the elements of $A$ form a uniform random permutation of the numbers $1, 2, \cdots n$ and use indicator random variables to compute the expected number of inversions.

4. Use indicator random variables to solve the following, known as the hat-check problem: Each of $n$ customers at a restaurant gives a hat to a hat-check person at the restaurant. The hat-check person gives the hats back to the customers in a random order. What is the expected number of customers who get their own hat?

5. Consider the following algorithm:

   ```python
   Permute-With-All(Array A[1,2,...,n]):
   n = A.length
   for i = 1 to n
       swap A[i] with A[Random(1,n)]
   
   ```

   Does the above algorithm produce a uniformly random permutation (i.e. any possible order should occur with equal probability). Why or why not?

6. How can you modify a hash table which resolves collisions by chaining so that, within the same linked list, they keys are stored in sorted order? What will the new complexity be for searching for, inserting, and removing entries?

7. Write an algorithm to find the predecessor of a node in a BST.

8. How many weighings of a balance scale are needed to find a lighter counterfeit coin among four coins? Describe an algorithm to find the lighter coin using this number of weighings.

9. What are the five defining properties of a red-black tree?

10. Draw a valid red-black tree with 7 internal nodes that has exactly 3 red nodes. Then give each internal node a value and show the result of inserting 2 more nodes into the tree. Illustrate each “step” that modifies the tree (coloring, rotation, etc.).

11. Show that any $n$-node BST can be transformed into any other $n$-node BST by using only $O(n)$ rotations. Hint: first show that $n - 1$ right rotations suffice to transform the tree into a right-going chain.