

SPHERICAL GEOMETRY EXERCISE

EXERCISE CREATED BY Dr. C.N.Scultes

Objective: To understand why non-Euclidean geometries are plausible and even desirable in some situations. To gain a measure of intuitive knowledge about a non-Euclidean geometry.

Summary of the Exercise: Using a styrofoam ball and pieces of yarn of various lengths, we will explore the geometry of a sphere, the surface of a perfectly round ball, by experimenting with the properties of spherical lines and triangles. **Note:** "Sphere" refers only to the surface of the ball. All figures must lie on the surface.

The Basics

The most fundamental geometric figures are points and lines. A *point* in spherical geometry is a point, a position, on the sphere. Determining what to use for a line is a bit more difficult as there is little "straight" about a sphere.

1. Take a piece of yarn long enough to wrap around the circumference of the sphere and hold down one end with your finger. Holding the yarn taut, place it carefully on the sphere until you come back to where you started. What do you notice about this figure?
2. You should have gotten a *great circle*, a circle which divides the sphere into two identical pieces. Since the yarn was held taut, the shortest distance from one point on this great circle to another point on it must lie along the great circle. For this reason, we will let these great circles be our lines. A *line segment* is a portion of a great circle.
3. Pick two points on the sphere. How many line segments are there which have these two points as endpoints? Does this number depend on which points you choose? How does this compare to Euclidean geometry?

Lines and Angle Measure

4. Construct two lines (great circles) on the sphere. It is important to remember that lines must divide the sphere into two identical pieces. In how many points do these lines intersect?
5. Remove one of these lines and keep the other. Now choose a point not on that line. How many lines are there which pass through this point, but do not intersect the line? Repeat with as many points and lines as you need to. Write a general statement which explains what you have discovered.
6. How could we measure the angle between two lines? What is the smallest possible angle? The largest?

Triangles: A *triangle* is the union of three line segments whose endpoints are three non-collinear points.

7. How many right angles can there be in a triangle in the Euclidean plane? How about on the sphere? How many obtuse angles can there be in a triangle in the Euclidean plane? How about on the sphere? What can you say about the sum of the angles of a triangle in spherical geometry? Why do you think that this is the case?
8. Find and describe a triangle with the largest angle sum that you can.
9. Just by looking at the triangles you made in 7 and 8, or by looking at others in order to test your conjecture, what relationship does there appear to be between of the angle sum of a triangle and the area of that triangle?

Summary

11. Based on the observations you made in this exercise, summarize the similarities and differences between Euclidean and spherical geometry.

For Further Consideration:

Should you wish to explore this geometry further, consider the following questions about circles and quadrilaterals in spherical geometry.

Circles : A circle is the set of all points which are a fixed distance from a given point. The fixed distance is the radius of the circle and the given point is called the center of the circle.

1. Find and describe five examples of circles on the sphere, each with a different radius (estimate distances). Then find and describe a circle on the sphere that is also a line. What is the condition that must hold in order for a circle be a line on the sphere?
2. In Euclidean geometry, about any point, a circle of any size can be constructed. Is this statement true in spherical geometry?
3. Construct any circle on a sphere. Are the center and radius of this circle uniquely determined as they are in Euclidean geometry? If the center and radius are uniquely determined, describe how to find them given an arbitrary circle. If they are not uniquely determined explain why they are not.

Quadrilaterals

4. Is it possible to construct a rectangle on the sphere? How many right angles can a quadrilateral in spherical geometry have? How does this compare with Euclidean geometry?
5. Is it possible to construct a quadrilateral on the sphere for which each pair of opposite sides has the same length?

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