The goal of this lab is to take you through all you need to know about Maple for Calculus I. It is intended for those who are going into Calculus II or III and did not have a course that implemented the Maple computer algebra system.

Since this lab covers all of the commands you would see in a Calculus I class it will take a while to finish and you may want to finish it in portions. We highly recommend typing in and executing all of the commands discussed in the lab and doing all of the exercises. The solutions to the exercises are at the end of the laboratory.
Basic Commands

What is Maple?

Maple is a computer algebra system, which means that it does symbol manipulation in the same way we do when solving a problem by hand. One advantage to using a system like this is that it will keep its answers in exact form whenever possible and in many cases you need to force it to give you an approximation. Maple is a mathematical “worksheet”. It works primarily by what is called command-line interaction. You type in a command, in a way Maple will understand, and Maple will do the calculations and give you an answer. It is not completely menu-driven like many other computer applications. For example, when you start up Maple you will see something like the following

The [>] is your command line prompt. You will type in your command here. When you type a command it will show up in red and every command must end with a semi-colon. After you type in the semi-colon and hit Enter Maple will do its thing. Let’s start out with an easy command. Say we wanted to add 3 and 4 together. The command would be

\[ 3+4; \]

Note that you only type in \( 3+4; \) Also note that Maple output the solution, in exact form, in blue and centered it. Try the following simple commands.

\[ 1/2+1/3; \]

\[ 1/2-1/3; \]
\[
\frac{1}{6}
\]

> \[1/3;\]

\[
\frac{7}{3}
\]

> \[8.3+1/3;\]

8.633333333

In the last command notice that it gave us a decimal output, that is an approximate solution. Maple has two modes, exact and approximate. Maple will stay in exact mode as long as it can or until you force it to go approximate. There are several commands you can use to get approximate solutions and we will look at many of these in this lab. Another way to force Maple to give you an approximate solution is to include a decimal number in your input.

**Getting Started with Maple**

Since Maple is a computer algebra system it can deal with variables as easily as it can deal with numbers. For example,

> \[3*x-2*y;\]

\[3 \times -2 \times y\]

> \[x^2;\]

\[x^2\]

> \[x^{(1/2);}\]

\[\sqrt{x}\]

> \[4^{(1/2);}\]

\[\sqrt{4}\]

> \[simplify(4^{(1/2))};\]

2

The syntax for mathematical expressions in Maple is quite similar to other programs like Excel and to most graphing calculators. The main thing to watch is that you must always use an * to denote multiplication. Otherwise, it is simply + for addition, – for subtraction, * for multiplication, / for division, ^ for powers and we always use parentheses ( ) for grouping. Note that [ ] and { } have other uses in Maple. For example,

> \[3*x^2-2*x+7;\]

\[3 \times x^2 - 2 \times x + 7\]
\[
4^{(x-2)};
\]

Maple will also give you an error if the expression you input is in some way ambiguous. For example,

\[
> x^x^x;
\text{Error, `^` unexpected}
\]

\[
> x^{(x^x)};
\]

\[
> (x^x)^x;
\]

As you know parentheses can make a big difference in the meaning of an expression. Note the difference in the following outputs.

\[
> (x^2-7*x+2)/(x+3);
\]

\[
> x^2-7*x+2/x+3;
\]

\[
> x^2-7*x+2/(x+3);
\]

**Exercises:**

1. Type in the command that will give you the expression 
   \[
   \frac{2}{x + y} + \frac{3}{z} - 5
   \]
2. Type in the command that will give you the expression 
   \[
   \left(x - \frac{y}{2} + 7 z\right)^{(2/3)}
   \]
3. Type in the command that will give you the expression 
   \[
   x^{(y+3)} + \frac{y + 3}{z + 2}
   \]
In Maple, percentage signs are used to refer to previously computed expressions. Specifically, the % operator reevaluates the last expression computed, the %% operator reevaluates the second last expression computed, and the %%% operator reevaluates the third last expression computed. Be careful, the last expression is not always the one directly above the %, it is the last one done in the session. For example, if we execute the following commands in order,

\[
> \frac{1}{2} + \frac{1}{3}; \\
\frac{5}{6}
\]

\[
> \frac{1}{2} - \frac{1}{3}; \\
\frac{1}{6}
\]

\[
> 7 \times \frac{1}{3};
\frac{7}{3}
\]

Then the following ditto commands will return the following values.

\[
> \%;
\frac{7}{3}
\]

\[
> \%\%;
\frac{1}{6}
\]

\[
> \%\%\%;
\frac{5}{6}
\]

If we execute the same commands in reverse order then the following ditto commands will return the following values.

\[
> \%;
\frac{5}{6}
\]

\[
> \%\%;
\frac{1}{6}
\]

\[
> \%\%\%;
\frac{7}{3}
\]
Setting a Variable to a Value

One very useful capability of Maple is the ability to store an expression in a variable name. This is much like storing a value into the memory of a pocket calculator. We simply give the value of expression a name and then simply by using the name we get the entire expression. To set a variable to a value you simply start the command with the variable name followed by := and then the value or expression. For example, to define the variable $x$ to be the number 5 we would use,

\[
> x := 5;
\]

From now on every time $x$ is used the value of 5 replaces it. For example,

\[
> x;
\]

\[
5
\]

\[
> x + 3;
\]

\[
8
\]

\[
> x + 1/2;
\]

\[
\frac{11}{2}
\]

To define the variable $x$ to be the expression $2h - 7$ we would use,

\[
> x := 2*h - 7;
\]

\[
x := 2h - 7
\]

Now when we use $x$ the expression $2h - 7$ replaces it. For example,

\[
> 2*x;
\]

\[
4h - 14
\]

\[
> x^2;
\]

\[
(2h - 7)^2
\]

\[
> \text{expand}(x^2);
\]

\[
4h^2 - 28h + 49
\]

As you can see, this could cause a problem if you wanted to define the expression $x^2$ but you wanted $x$ to be a variable and not $2h - 7$. If you do define a variable to be a particular value or expression you need to reset it if you then want the variable to be a
variable. To reset a variable back to a variable start with the variable followed by the `:=` followed by the variable in single quotes. For example,

```
> x:='x';
```

```
x
```

Now we have the following values for the above expressions.

```
> 2*x;
```

```
2 x
```

```
> x^2;
```

```
x^2
```

```
> expand(x^2);
```

```
x^2
```

Do not use this method to define a function. For example, if we wanted to define to define the function \( f(x) = 2x^2 + 3x - 5 \) the command

```
> f:=2*x^2+3*x-5;
```

```
f := 2 x^2 + 3 x - 5
```

Would not do the trick. It does define \( f \) to be the given expression but it does not view \( f \) as a function. Note the output of the following commands.

```
> f;
```

```
2 x^2 + 3 x - 5
```

```
> f(3);
```

```
2 x(3)^2 + 3 x(3) - 5
```

The correct way to define this function is by,

```
> f:=x->2*x^2+3*x-5;
```

```
f := x \rightarrow 2 x^2 + 3 x - 5
```

```
> f(3);
```

```
22
```

We will talk more about function later in the lab.
Exercises:

4. Type the Maple command that will set the variable \( a \) to the value of 17.
5. Type the Maple command that will set the variable \( b \) to the variable \( a \).
6. Get the value of \( b \). What happened?
7. Set the variable \( c \) to the expression \( \frac{2}{x+y} + \frac{3}{z} = 5 \).
8. Set the variable \( d \) to the expression \( \left( x - \frac{y}{2} + 7z \right)^{2/3} \).
9. Set the variable \( e \) to the expression \( x^{(y+3)} + \frac{y + 3}{z + 2} \).
10. Set the variable \( x \) to the expression \( x^{(y+3)} + \frac{y + 3}{z + 2} \).

What happened and why?

11. Reset the variables \( a, b, c, d \) and \( e \) back to variables.

Mathematical Functions

The following is a list of some of the more useful mathematical functions and their Maple syntax.

Trigonometric and Hyperbolic Functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Maple Syntax</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin(x) )</td>
<td>( \sin(x) )</td>
<td>The sine function.</td>
</tr>
<tr>
<td>( \cos(x) )</td>
<td>( \cos(x) )</td>
<td>The cosine function.</td>
</tr>
<tr>
<td>( \tan(x) )</td>
<td>( \tan(x) )</td>
<td>The tangent function.</td>
</tr>
<tr>
<td>( \cot(x) )</td>
<td>( \cot(x) )</td>
<td>The cotangent function.</td>
</tr>
<tr>
<td>( \sec(x) )</td>
<td>( \sec(x) )</td>
<td>The secant function.</td>
</tr>
<tr>
<td>( \csc(x) )</td>
<td>( \csc(x) )</td>
<td>The cosecant function.</td>
</tr>
<tr>
<td>( \sinh(x) )</td>
<td>( \sinh(x) )</td>
<td>The hyperbolic sine function.</td>
</tr>
<tr>
<td>( \cosh(x) )</td>
<td>( \cosh(x) )</td>
<td>The hyperbolic cosine function.</td>
</tr>
<tr>
<td>( \tanh(x) )</td>
<td>( \tanh(x) )</td>
<td>The hyperbolic tangent function.</td>
</tr>
<tr>
<td>( \coth(x) )</td>
<td>( \coth(x) )</td>
<td>The hyperbolic cotangent function.</td>
</tr>
<tr>
<td>( \sech(x) )</td>
<td>( \sech(x) )</td>
<td>The hyperbolic secant function.</td>
</tr>
<tr>
<td>Function</td>
<td>Maple Syntax</td>
<td>Notes</td>
</tr>
<tr>
<td>----------</td>
<td>--------------</td>
<td>-------</td>
</tr>
<tr>
<td>$\sin^{-1}(x)$</td>
<td>arcsin(x)</td>
<td>The inverse sine function.</td>
</tr>
<tr>
<td>$\cos^{-1}(x)$</td>
<td>arccos(x)</td>
<td>The inverse cosine function.</td>
</tr>
<tr>
<td>$\tan^{-1}(x)$</td>
<td>arctan(x)</td>
<td>The inverse tangent function.</td>
</tr>
<tr>
<td>$\cot^{-1}(x)$</td>
<td>arccot(x)</td>
<td>The inverse cotangent function.</td>
</tr>
<tr>
<td>$\sec^{-1}(x)$</td>
<td>arccsec(x)</td>
<td>The inverse secant function.</td>
</tr>
<tr>
<td>$\csc^{-1}(x)$</td>
<td>arccsc(x)</td>
<td>The inverse cosecant function.</td>
</tr>
<tr>
<td>$\sinh^{-1}(x)$</td>
<td>arcsinh(x)</td>
<td>The inverse hyperbolic sine function.</td>
</tr>
<tr>
<td>$\cosh^{-1}(x)$</td>
<td>arccosh(x)</td>
<td>The inverse hyperbolic cosine function.</td>
</tr>
<tr>
<td>$\tanh^{-1}(x)$</td>
<td>arctanh(x)</td>
<td>The inverse hyperbolic tangent function.</td>
</tr>
<tr>
<td>$\coth^{-1}(x)$</td>
<td>arccoth(x)</td>
<td>The inverse hyperbolic cotangent function.</td>
</tr>
<tr>
<td>$\sech^{-1}(x)$</td>
<td>arcsech(x)</td>
<td>The inverse hyperbolic secant function.</td>
</tr>
<tr>
<td>$\csch^{-1}(x)$</td>
<td>arccsch(x)</td>
<td>The inverse hyperbolic cosecant function.</td>
</tr>
</tbody>
</table>

### Exponential and Logarithmic Functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Maple Syntax</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^x$</td>
<td>exp(x)</td>
<td>The exponential function.</td>
</tr>
<tr>
<td>$\ln(x)$</td>
<td>ln(x)</td>
<td>The natural logarithm function.</td>
</tr>
<tr>
<td>$\log(x)$</td>
<td>log10(x)</td>
<td>The common logarithm function.</td>
</tr>
<tr>
<td>$\log_b(x)$</td>
<td>log<a href="x">b</a></td>
<td>The general logarithm function, $b &gt; 0$.</td>
</tr>
</tbody>
</table>

### Root Functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Maple Syntax</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{x}$</td>
<td>sqrt(x)</td>
<td>The square root function.</td>
</tr>
<tr>
<td>$\sqrt[n]{x}$</td>
<td>surd(x, n)</td>
<td>The $n^{th}$ root function.</td>
</tr>
</tbody>
</table>

### Other Functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Maple Syntax</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>x</td>
<td>$</td>
</tr>
</tbody>
</table>

### Constants

<table>
<thead>
<tr>
<th>Function</th>
<th>Maple Syntax</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>Pi</td>
<td>Pi</td>
</tr>
<tr>
<td>$e$</td>
<td>exp(1)</td>
<td>$e$</td>
</tr>
</tbody>
</table>
The only ones that may seem a little strange are \( \log_{b}(x) \) and \( \text{surd}(x, n) \). We will spend a little extra time on these in a moment. Here are some examples of the other mathematical functions,

\[
\begin{align*}
&> \sin(\pi); \quad 0 \\
&> \cos(\pi); \quad -1 \\
&> \sin(\pi/2); \quad 1 \\
&> \arctan(1); \quad \frac{\pi}{4} \\
&> \sinh(3); \quad \sinh(3) \\
&> \text{evalf}(\%); \quad 10.01787493 \\
&> \sqrt{4}; \quad 2 \\
&> \sqrt{x+y}; \quad \sqrt{x+y} \\
&> \ln(3); \quad \ln(3) \\
&> \log_{10}(1000); \quad \frac{\ln(1000)}{\ln(10)} \\
&> \text{simplify}(\%); \quad 3 \\
&> \exp(x); \quad e^x \\
&> \ln(\exp(1)); \quad 1 \\
&> \exp(\ln(x)); \quad x
\end{align*}
\]

Note the uses of the \text{evalf} and \text{simplify} commands, and the use of \%. We will look at the \text{evalf} and \text{simplify} commands later in the lab but their function is fairly self-explanatory.
The evalf command will approximate an answer and simplify will simplify an answer but keep it in exact form. Recall that the % gets the last result. Now let’s look at the two non-standard functions. First \( \log_b(x) \) will find the logarithm base \( b \) of \( x \). For example,

\[
\begin{align*}
> \log[2](16); & \quad \frac{\ln(16)}{\ln(2)} \\
> \text{simplify}(\%); & \quad 4 \\
> \log[1/2](32); & \quad -\frac{\ln(32)}{\ln(2)} \\
> \text{simplify}(\%); & \quad -5
\end{align*}
\]

Note that the command simply applies the base-change formula to the expression. We further need to use the simplify command to acquire the value. The \text{surd}(x,n)\) command will find the \( n \)th root of \( x \). This is one situation where we need to be very careful about how maple interprets our commands. In mathematics we write \( \sqrt[n]{x} \) and \( x^{1/n} \) freely as the same thing. Most computers and calculators do not see these as the same. If you want \( \sqrt[n]{x} \) in Maple you must use the command \text{surd}(x,n). The following examples will illustrate the difference in output.

\[
\begin{align*}
> \text{surd}(8,3); & \quad 2 \\
> \text{surd}(-8,3); & \quad -2 \\
> 8^{(1/3)}; & \quad 8^{(1/3)} \\
> \text{simplify}(\%); & \quad 2 \\
> (-8)^{(1/3)}; & \quad (-8)^{(1/3)} \\
> \text{simplify}(\%); & \quad 1 + \sqrt{3} \cdot i
\end{align*}
\]

Notice this last result. Mathematically it is correct. If you cube that complex number you will end up with –8. On the other hand, it is not what we would expect. Although
we have not discussed how to do plots yet the plotting of the two expressions \( \text{surd}(x, 3) \) and \( x^{1/3} \) illustrate the differences quite well.

\[
> \text{plot}(\text{surd}(x, 3), x=-3..3, y=-2..2);
\]

\[
> \text{plot}(x^{1/3}, x=-3..3, y=-2..2);
\]

The top graph is the correct graph of \( y = \sqrt[3]{x} \) the bottom one is not.

**Exercises:**

12. Type in the command that will give you the expression \( \sin(2x - 7) + \cos(y - 2) \)

13. Type in the command that will give you the expression \( \sin^{-1}(x) + \cosh(y) \)

14. Type in the command that will give you the expression
\[
\ln(x') + \log_b(x + y)
\]

15. Type in the command that will give you the expression
\[
\sinh^{-1}(t) - \sqrt{t}
\]

16. Type in the command that will give you the expression
\[
\sqrt{x + y} - \ln\left(\sqrt{x - y}\right)
\]

17. Type in the command that will give you the expression
\[
e^{x/y} + 4^{-t}
\]

### Functions

#### Defining a Function of a Single Variable

To define a function of a single variable, start the command with the function
name followed by `:=` followed by the independent variable followed by the “arrow” `->`
followed by the mathematical expression for the function. For example, to define the
function \( f(x) = 2x^2 + 3x - 5 \) we use the command,

\[
> f := x \rightarrow 2x^2 + 3x - 5;
\]

\[
f := x \rightarrow 2x^2 + 3x - 5
\]

Now you can use standard mathematical function notation to evaluate the function. For
example,

\[
> f(0); \\
-5
\]

\[
> f(2); \\
9
\]

\[
> (f(x+h)-f(x))/h;
\]

\[
\frac{2(x+h)^2 + 3h - 2x^2}{h}
\]

### Exercises:

18. Define the function
\( f(x) = \sin(2x - 7) + \cos(x - 2) \)

19. Define the function
\( g(x) = \sin^{-1}(x) + \cosh(x) \)

20. Define the function
\( h(t) = \sinh^{-1}(t) - \sqrt{t} \)
21. Define the function

\[ j(r) = \sqrt[3]{3r} - \ln\left( \frac{r}{3} \right) \]

22. Define the function

\[ k(w) = e^w + 4^{-w} \]

**Defining Piecewise Functions**

Piecewise defined functions can be created with the `piecewise` command. In the `piecewise` command you input the piecewise function in pairs. The first component is the range in which to apply the piece and the second component is the function to apply to that piece. You may put in as many pairs as you would like and the last entry need not be a pair but the function to use otherwise. If there is no default (otherwise) function Maple will assume that it is 0. Note that the output of the `piecewise` command is in a pretty-print mode unless you use it in a function definition. We would suggest that you try the command without a function definition to check your syntax before placing the command in a function definition. For example,

\[
> \text{piecewise}( x=0, 1, \sin(x)/x );
\]

\[
\begin{align*}
1 & \quad x = 0 \\
\frac{\sin(x)}{x} & \quad \text{otherwise}
\end{align*}
\]

\[
> \text{piecewise}( x=0, 1 );
\]

\[
\begin{align*}
1 & \quad x = 0 \\
0 & \quad \text{otherwise}
\end{align*}
\]

\[
> \text{piecewise}( x>0, 1,-1 );
\]

\[
\begin{align*}
1 & \quad 0 < x \\
-1 & \quad \text{otherwise}
\end{align*}
\]

\[
> \text{piecewise}( x<0, 1, x<1, 2, x<2, 3, 4 );
\]

\[
\begin{align*}
1 & \quad x < 0 \\
2 & \quad x < 1 \\
3 & \quad x < 2 \\
4 & \quad \text{otherwise}
\end{align*}
\]

Although we have not discussed plots yet, we included the following plots so you could see the graphs of the functions.

\[
> \text{plot(piecewise( } x<0, 1, x<1, 2, x<2, 3, 4 ),x=-2..5,y=0..5,\text{discont=true});
\]
\[ f(x) = \begin{cases} 1 & x < 0 \\ x & x < 1 \\ 3 - x & x < 2 \\ \sin(x) + 2 & \text{otherwise} \end{cases} \]

\[ g(x) = \begin{cases} 1 & -2 - x \leq 0 \ \text{and} \ x + 1 \leq 0 \\ x & x < 1 \\ 3 - x & x < 2 \\ \sin(x) + 2 & \text{otherwise} \end{cases} \]
Notice the vertical lines at –2 and –1 despite the fact that we used discont=true. This was due to the use of “and” in the defining of one of the ranges. If we rearrange the piecewise command a little we do get a better image of the function.

> g := x -> piecewise(x < -2, x, x <= -1, 1, x < 1, x, x < 2, 3 - x, sin(x) + 2);

> plot(g(x), x = -5 .. 5, y = -3 .. 3, discont = true);

Exercises:

23. Create the following piecewise defined expression

\[
\begin{align*}
&1 & x \leq 0 \\
&\frac{1}{x} & \text{otherwise}
\end{align*}
\]

24. Take the expression above and create a piecewise defined function out of it.

25. Create the following piecewise defined expression

\[
\begin{align*}
&1 & x < 0 \\
&\frac{1}{x} & 0 < x \\
&3 & \text{otherwise}
\end{align*}
\]

26. Take the expression above and create a piecewise defined function out of it.

27. Create the following piecewise defined expression
\[ \begin{cases} 1 & x < 0 \\ \frac{1}{x} & 0 < x \\ 3 & x = 0 \end{cases} \]

28. Take the expression above and create a piecewise defined function out of it.

29. Create the following piecewise defined expression
\[ \begin{cases} 3 & 1 < t \\ 4 & 0 < t \\ 2 & -3 < t \\ 5 & \text{otherwise} \end{cases} \]

30. Take the expression above and create a piecewise defined function out of it.

31. Create the following piecewise defined expression
\[ \begin{cases} t & 1 < t \\ 3 & t - 1 \quad 0 < t \\ \sin(t) & -3 < t \\ \tan(t) & \text{otherwise} \end{cases} \]

32. Take the expression above and create a piecewise defined function out of it.

**Defining a List and a Set**

In many applications it is advantageous or even necessary to create a list or set of items. You need to be aware that there is a major difference between the two even though their syntax is very similar. To define a set we begin with the name of the set followed by := followed by a list of items in curly brackets separated by commas. For example,

\[
> t := \{1, 2, 3, 4, 4, 4, 5, 6\};
> t := \{1, 2, 3, 4, 5, 6\}
\]

\[
> t;
\{1, 2, 3, 4, 5, 6\}
\]

\[
> s := \{x, y, t, w-2, 3, 4\};
> s := \{3, 4, x, y, w-2, t\}
\]

Notice that in a set duplicates are removed. We define a list in the same manner except that we use square brackets instead of curly brackets. Notice here that duplicates are not removed.

\[
> t := [1, 2, 3, 4, 4, 4, 5, 6];
> t := [1, 2, 3, 4, 4, 5, 6]
\]

\[
> t;
[1, 2, 3, 4, 4, 5, 6]
\]
Another difference between lists and sets that you may have noticed above is that a set may rearrange the order of the items whereas a list will not. For example,

```maple
t := {1, 2, 5, 8, 3, 4, 4, 4, 5, 6, 2, 2, 7};
t
```

Exercises:

33. Create a set containing the numbers: 1, 12, 21, 2, 2, 1, 12, 14, 5, 8, 67, 14, 19, 25.
34. Create a list containing the numbers: 1, 12, 21, 2, 2, 1, 12, 14, 5, 8, 67, 14, 19, 25.
35. Create a set containing the letters: a, b, d, e, r, s, t, y.
36. Define a to be 5, b to be 7, d to be a, e to be x, r to be 15, and y to be x – 5. Now create a set containing the letters: a, b, d, e, r, s, t, y.

Using the map Command

The map command is a quick way to evaluate a function at a number of values. Before using the map command you should define the function you wish to use and a list of values you want to use it on. Make sure that you use a list here and not a set.

```maple
f := x -> x^2;
lst := [-2, -1, 0, 2, 5, 10, 104.8];
map(f, lst);
```

Now type in the map command using the function name and the list name. Note that you are to use only the function name.

```maple
map(f, lst);
```

In the same manner we can map an already existing function.

```maple
map(sin, lst);
```
\textbf{Exercises:}

37. Create the function \( f(x) = x^3 \) and create a list containing the numbers: 1, 12, 21, 2, 2, 1, 12, 14, 5, 8, 67, 14, 19, 25. Now use the map command to evaluate the function at each value in the list.

38. Create the function \( f(x) = \sin(x) + \cos(x) \) and create a list containing the numbers: 0, \( \frac{\pi}{6} \), \( \frac{\pi}{4} \), \( \frac{\pi}{3} \), \( \frac{\pi}{2} \), \( \pi \). Now use the map command to evaluate the function at each value in the list.

39. Create a list containing the numbers: 0, \( \frac{\pi}{6} \), \( \frac{\pi}{4} \), \( \frac{\pi}{3} \), \( \frac{\pi}{2} \), \( \pi \). Now use the map command to evaluate the sine function at each value in the list.

40. Create a list containing the numbers: 0, \( \frac{\pi}{6} \), \( \frac{\pi}{4} \), \( \frac{\pi}{3} \), \( \frac{\pi}{2} \), \( \pi \). Now use the map command to evaluate the cosine function at each value in the list.

41. Create a list containing the numbers: 0, \( \frac{\pi}{6} \), \( \frac{\pi}{4} \), \( \frac{\pi}{3} \), \( \frac{\pi}{2} \), \( \pi \). Now use the map command to evaluate the tangent function at each value in the list.

\textbf{The eval Command}

The eval command has many different uses. It is mainly used to evaluate an expression or function at a particular value or to force an evaluation from another command that did not do an evaluation. For example,

\begin{verbatim}
> f:=x->x^2;
   \textit{f} := x \rightarrow x^2

> f(x);
   x^2

> eval(f(x),x=3);
   9

> eval(x^2,x=3);
\end{verbatim}
The main difference between the eval command and the subs command, that we will see later, is that the subs command will simply do the substitution, no matter if it makes sense or not. The eval command, on the other hand, will do the substitution and then the evaluation of the result. For example,

```
> eval(cos(x)/sin(x), x=0);
Error, numeric exception: division by zero

> subs(x=0, cos(x)/sin(x));
\frac{\cos(0)}{\sin(0)}

> eval(%%);
Error, numeric exception: division by zero
```

Exercises:

42. Create the function \( f(x) = \sin(x) + \cos(x) \) and use the eval command to evaluate the function at \( 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2} \) and \( \pi \).

43. Create the expression \( \sin(x) + \cos(x) \) and use the eval command to evaluate the expression at \( 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2} \) and \( \pi \).

44. Create the expression \( \sin(x) + \cos(x) \) and use the subs command to substitute the values \( 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2} \) and \( \pi \) into the expression and then use the eval command to do the evaluation.

The evalf Command

The evalf command will find an approximation to whatever it is given. For example,

```
> evalf(1/3);
0.3333333333

> evalf(Pi);
3.141592654
```

There is an optional value you can include to specify the number of decimal places the approximation will use. This value either goes in square brackets after the evalf name or as a parameter after the expression to be evaluated.
> evalf[100](Pi);
3.1415926535897932384626433832795028841971693993751058209749445923078
   6406286208998628034825342117068

> evalf(Pi,20); 3.1415926535897932385

Another way to force Maple to give you an approximate answer, or at least a decimal answer is to at some point place a decimal number onto the expression. For example,

> f(4.7); 22.09
> f(47/100); 2209  
10000

Exercises:

45. Find an approximation to $e$.
46. Find an approximation to $e$ to 25 decimal places.
47. Find an approximation to $e$ to 1000 decimal places.
48. Find an approximation to $\cos(7)$ to 25 decimal places.
49. Find an approximation to $\cos(7)$ to 1000 decimal places.
50. Find an approximation to $\log(3)$ to 50 decimal places.

The solve Command

The solve command is for finding exact solutions to equations or systems of equations. The syntax is simple

$$\text{solve}(\text{expr}, \text{vars})$$

where expr represents the equation or system of equations and vars is a variable or list of variables. Note that the vars argument may be omitted; in this case the variables will be automatically taken to be all of the variables present in the equation or system of equations. Also, when inputting a system of equations or a list of more than one variable you should place them in a list. We will begin with an easy example. To solve the equation $x^4 - 5x^2 + 6x = 2$ we could use either

> solve(x^4-5*x^2+6*x=2);
$$1, 1, -1 + \sqrt{3}, -1 - \sqrt{3}$$
or

```maple
> solve(x^4-5*x^2+6*x=2, x);
1, 1, -1+\sqrt{3}, -1-\sqrt{3}
```

In the first solve command we let Maple determine that \( x \) was the variable. In the second solve command we explicitly told Maple that our variable was \( x \). To solve the system of linear equations

\[
2x + 3y = 7 \\
5x + 8y = 9
\]

we could use

```maple
> solve({2*x+3*y=7, 5*x+8*y=9});
{y = -17, x = 29}
```

or

```maple
> solve({2*x+3*y=7, 5*x+8*y=9}, {x, y});
{y = -17, x = 29}
```

Note that the list of variables that are input in the solve command will make a difference in the outputs. For example consider the following solve commands.

```maple
> solve(2*x+3*y+z=7);
{ z = -2 x - 3 y + 7, x = x, y = y }
> solve(2*x+3*y+z=7, z);
-2 x - 3 y + 7
> solve({2*x+3*y+z-w=7, 5*x+8*y-3*w+8*z=9}, {x, y});
{ y = -17 - 11 z + w, x = 29 + 16 z - w }
> solve({2*x+3*y+z-w=7, 5*x+8*y-3*w+8*z=9}, {x, y, z});
{ y = -17 - 11 z + w, x = 29 + 16 z - w, z = z }
> solve({2*x+3*y+z-w=7, 5*x+8*y-3*w+8*z=9}, {x, y, z, w});
{ y = y, z = z, x = 12 - y + 5 z, w = 17 + y + 11 z }
> solve({2*x+3*y+z-w=7, 5*x+8*y-3*w+8*z=9}, {x, z, w});
{ y = y, z = z, x = 12 - y + 5 z, w = 17 + y + 11 z }
> solve({2*x+3*y+z-w=7, 5*x+8*y-3*w+8*z=9}, x);
```

Since the solve command finds exact solutions the output can at times be very long. For example.
To obtain a numeric approximation to solutions of this form simply apply the \texttt{evalf} command to the output.

\begin{verbatim}
> evalf(%);
6.630098729 - 0.1 10^9 I, -0.5126801310 - 0.1732050808 10^8 I,
0.8825814030 + 0.1732050808 10^8 I
\end{verbatim}

As will frequently happen when you \texttt{evalf} something like this you will get some round-off error. Note that all of the imaginary parts of the above solution are extremely small. This is an indication that the solutions are in fact real. Also since the \texttt{solve} command gives exact solutions there may be times when it can not give you a nice closed form. When Maple can’t give you an exact solution it will usually give a \texttt{RootOf} statement. Consider the following output.

\begin{verbatim}
> solve(x^4-5*x^2+6*x=3,x);
\texttt{RootOf}(_Z^4 - 5 _Z^2 + 6 _Z - 3, \texttt{index} = 1), \texttt{RootOf}(_Z^4 - 5 _Z^2 + 6 _Z - 3, \texttt{index} = 2),
\texttt{RootOf}(_Z^4 - 5 _Z^2 + 6 _Z - 3, \texttt{index} = 3),
\texttt{RootOf}(_Z^4 - 5 _Z^2 + 6 _Z - 3, \texttt{index} = 4)
\end{verbatim}

As you can see, Maple knows that the equation has four solutions but it can’t find any of them exactly. This is why it has essentially given you the question back in the solution. As above, to obtain an approximate solution to the equation, apply the \texttt{evalf} command to the above output.

\begin{verbatim}
> evalf(%);
\end{verbatim}
Exercises: In each of the following exercises you are asked to find the solution or solutions to an equation or set of equations. If the answer does not come out in exact form that is understandable use the evalf command to get an approximate solution.

51. Find the solution to the following equation.
\[ \sin(x) = \cos(x) \]

52. Find the solution to the following equation.
\[ x^2 - 3x + 2 = 5 \]

53. Find the solution to the following equation.
\[ x^3 - 3x + 2 = 0 \]

54. Find the solution to the following equation.
\[ x^3 - 2x + 2 = 0 \]

55. Find the solution to the following equation.
\[ x^5 - 4x^4 + 3x^2 - 2x + 2 = 0 \]

56. Find the solution to the following system of equations.
\[ 3x^2y - 2xy^3 + 2 = 0 \]
\[ xy - x^2y^2 + 1 = 0 \]

57. Find the solution to the following system of equations.
\[ 3x^2 - y^2 = 2 \]
\[ x + y - xy^2 = -1 \]

58. Find the solution to the following system of equations.
\[ 3x^5 - y^5 + z^5 = 0 \]
\[ xyz - x^2y^2 = -1 \]
\[ x + y + z = 5 \]

59. Find the solution to the following system of equations.
\[ \sin(x) = \cos(y) + \frac{1}{2} \]
\[ \cos(y) = \sin(z) + \frac{1}{2} \]
\[ \sin(z) = \cos(x) + \frac{1}{3} \]

60. Find the solution to the following system of equations.
\[ \sin(x) = \cos(y) + \frac{1}{2} \]
\[
\cos(y) = \sin(z) \\
\sin(z) = \cos(x) + \frac{1}{2}
\]

The \texttt{fsolve} Command

The \texttt{fsolve} command is for finding numeric approximations to the solutions of an equation or system of equations. If the equation to be solved is a polynomial of a single variable the \texttt{fsolve} command will attempt to find all of the real solutions to the equation. If the equation is not a polynomial or if there is a system of equations to be solved the \texttt{fsolve} command will attempt to find a single solution. Note that even for polynomials of a single variable the \texttt{fsolve} command may not find all of the solutions. It is good practice to examine any equation or system of equations to determine approximate values for the solutions and then use the interval option in the \texttt{fsolve} command. The syntactical style of the \texttt{fsolve} command is the same as the \texttt{solve} command.

\[
\texttt{fsolve}(\texttt{expr},\texttt{vars})
\]

where \texttt{expr} represents the equation or system of equations and \texttt{vars} is a variable or list of variables. Note that the \texttt{vars} argument may be omitted, in this case the variables will be automatically taken to be all of the variables present in the equation or system of equations. Also, when inputting a system of equations or a list of more than one variable you should place them in a list. We will start with a couple polynomials. Note that the following \texttt{fsolve} command returns two real solutions for the solution.

\[
> \texttt{fsolve}(x^4-5*x^2+6*x=3,x); \\
\{-2.752432296, 1.538751996\}
\]

Using the \texttt{solve} command and the \texttt{evalf} command we see that the other two solutions are in fact complex (non-real). Hence the \texttt{fsolve} command found all of the real solutions.

\[
> \texttt{solve}(x^4-5*x^2+6*x=3,x); \\
\texttt{RootOf}(\_Z^4 - 5 \_Z^2 + 6 \_Z - 3, \texttt{index} = 1), \texttt{RootOf}(\_Z^4 - 5 \_Z^2 + 6 \_Z - 3, \texttt{index} = 2), \\
\texttt{RootOf}(\_Z^4 - 5 \_Z^2 + 6 \_Z - 3, \texttt{index} = 3), \\
\texttt{RootOf}(\_Z^4 - 5 \_Z^2 + 6 \_Z - 3, \texttt{index} = 4) \\
> \texttt{evalf}(\%); \\
1.538751996, 0.6068401503 + 0.5831600070 \texttt{I}, -2.752432296, \\
0.6068401503 - 0.5831600070 \texttt{I}
\]

If we omit the variable, Maple assumes that it is \texttt{x} and solves the equation.

\[
> \texttt{fsolve}(x^3-7*x^2+2*x+3=0); \\
\]
For equations that are not polynomials we expect a single solution as output from the 
fsolve command, as below.

> \texttt{fsolve(sin(x)=x-2);}
2.554195953

> \texttt{fsolve(sin(x)=x-2,x);}
2.554195953

If we plot the curves we see that there is only one real solution to the equation and hence 
no need to look for any more.

> \texttt{with(plots):}
\texttt{Warning, the name changecoords has been redefined}

> \texttt{implicitplot({y=sin(x),y=x-2},x=-10..10,y=-10..10,}
\texttt{grid=[50,50]);}

On the other hand, say we wanted to solve the equation $\sin(x) = 0$. This clearly has an 
infinite number of solutions and although we know what they are let’s see how the fsolve 
command deals with an equation like this. Note that the simple application of the fsolve 
command returns 0 as a solution.

> \texttt{fsolve(sin(x)=0,x);}
0.

If we were interested in finding a solution between 3 and 5 we would execute,

> \texttt{fsolve(sin(x)=0,x=3..5);}
3.141592654
Even if there is more than one solution in a given interval the fsolve command will return only one. For example,

\begin{verbatim}
> fsolve(sin(x)=0,x=20..30);
25.13274123
> fsolve(sin(x)=0,x=27..30);
28.27433388
\end{verbatim}

Let’s look at a system of nonlinear equations. The following two commands are identical, at least to Maple, and they have the same output.

\begin{verbatim}
> fsolve({4*x^2+sin(y)=2,y/2+cos(x)=1});
\{x = 0.6361393915, y = 0.3912093747\}
> fsolve({4*x^2+sin(y)=2,y/2+cos(x)=1},{x,y});
\{x = 0.6361393915, y = 0.3912093747\}
\end{verbatim}

Note that we get an error if we try to solve for only one of the variables. The fsolve command must find a numeric solution to all of the variables in the equations.

\begin{verbatim}
> fsolve({4*x^2+sin(y)=2,y/2+cos(x)=1},x);
Error, (in fsolve) y is in the equation, and is not solved for
\end{verbatim}

If we graph the system of equations we see that there is another possible solution.

\begin{verbatim}
> implicitplot({4*x^2+sin(y)=2,y/2+cos(x)=1},x=-5..5,y=-5..5,grid=[50,50]);
\end{verbatim}

We can use the fsolve command to approximate this solution by restricting both \( x \) and \( y \) to a rectangle that contains the other solution. For example,

\begin{verbatim}
> fsolve({4*x^2+sin(y)=2,y/2+cos(x)=1},{x=-5..0,y=-2..2});
\{x = -0.6361393915, y = 0.3912093747\}
\end{verbatim}
There is one more thing to aware of when using the fsolve command. As you know from linear algebra, if you have \( n \) variables and you want to find numeric solutions for all of them then you will need at least \( n \) equations. The same is true for nonlinear equations. Maple knows this and hence it does not even try to fsolve a system if there are fewer equations than there are variables.

```maple
> fsolve({4*x^2*z+sin(y)=2,y+z/2+cos(x)=1});
Error, (in fsolve) number of equations, 2, does not match number of variables, 3
```

Exercises:

61. Find an approximate solution to the following equation.
\[
\sin(x) = \cos(x)
\]

62. Find an approximate solution to the following equation that is between 10 and 11.
\[
\sin(x) = \cos(x)
\]

63. Find an approximate solution to the following equation. Did you get all of the solutions? How do you know?
\[
x^2 - 3x + 2 = 5
\]

64. Find an approximate solution to the following equation. Did you get all of the solutions? How do you know?
\[
x^3 - 3x + 2 = 0
\]

65. Find an approximate solution to the following equation. Did you get all of the solutions? How do you know?
\[
x^3 - 2x + 1 = 0
\]

66. Find an approximate solution to the following equation. Did you get all of the solutions? How do you know?
\[
x^5 - 4x^4 + 3x^2 - 2x + 2 = 0
\]

67. Find an approximate solution to the following system of equations.
\[
\begin{align*}
3x^2y - 2xy^3 + 2 &= 0 \\
x - y + 1 &= 0
\end{align*}
\]

68. Find an approximate solution to the following system of equations.
\[
\begin{align*}
3x^2 - y^2 &= 2 \\
x + y - xy^2 &= -1
\end{align*}
\]

69. Find an approximate solution to the following system of equations.
\[
\begin{align*}
3x^2 - y^2 + z^2 &= 0 \\
x - y + z &= -1
\end{align*}
\]

70. Find an approximate solution to the following system of equations.
\[
\begin{align*}
\sin(x) & = \cos(y) + \frac{1}{2} \\
\cos(y) & = \sin(z) + \frac{1}{2} \\
\sin(z) & = \cos(x) + \frac{1}{3}
\end{align*}
\]

71. Find an approximate solution to the following system of equations.
\[
\begin{align*}
\sin(x) & = \cos(y) + \frac{1}{2} \\
\cos(y) & = \sin(z) \\
\sin(z) & = \cos(x) + \frac{1}{2}
\end{align*}
\]

72. Find an approximate solution to the following system of equations. This time, force the \( x \) value to be between 13 and 15, the \( y \) value between –2 and 0 and that the \( z \) value between 0 and 1.
\[
\begin{align*}
\sin(x) & = \cos(y) + \frac{1}{2} \\
\cos(y) & = \sin(z) \\
\sin(z) & = \cos(x) + \frac{1}{2}
\end{align*}
\]

**The subs and algsubs Commands**

The subs command is for substituting expressions into expressions. That is, it will take all occurrences of an expression and replace it with another expression. The syntax for the command is

\[\text{subs(eqn, expr);}\]

where eqn is an equation where the left hand side is the expression to be substituted for and the right hand side is the expression to substituted in. The expr is the expression that the substitution is being done on. For example,

\[> \text{subs(x=2, 3*x^2+2*x-1);}\]

\[\quad 15\]

\[> \text{subs(x=x-h, 3*x^2+2*x-1);}\]

\[\quad 3\,(x-h)^2 + 2\,x - 2\,h - 1\]

\[> \text{subs(cos(x)=y, cos(x)*(sin(x)+cos(x)));}\]

\[\quad y\,(\sin(x) + y)\]
Note that the subs command does not care if the substitution is legitimate or not. Sometimes the expression is simplified, as above, and sometimes it is not. For example,

```maple
> subs(x=0,cos(x)/sin(x));
  cos(0)
  sin(0)
```

```maple
> eval(%);
Error, numeric exception: division by zero
```

Also, there are cases where the substitution does not go through the way we would want it to. This usually happens when there is a simplification step done, usually without our knowledge, before the substitution is done. In this case there is another substitution command called algsubs that is a bit more powerful.

```maple
> subs(x+1=x-a,3*(x+1)^2+2*(x+1)-1);
  3 (x - a)^2 + 2 x + 1
```

```maple
> algsubs(x+1=x-a,3*(x+1)^2+2*(x+1)-1);
  3 (x - a)^2 - 1 + 2 x - 2 a
```

The algsubs command will even go as far as to find the expression by factoring or doing some other manipulation. For example,

```maple
> expand((x+1)^4);
  x^4 + 4 x^3 + 6 x^2 + 4 x + 1
```

```maple
> algsubs(x+1=y,x^4+4*x^3+6*x^2+4*x+1);
  y^4
```

```maple
> expand((x+1)^4+2*x+1);
  x^4 + 4 x^3 + 6 x^2 + 6 x + 2
```

```maple
> algsubs(x+1=y,x^4+4*x^3+6*x^2+6*x+2);
  2 y - 1 + y^4
```

**Exercises:**

73. Substitute the value 5 in for \( x \) in the following expression.

\[
x^3 - 2 x^2 - x + 1
\]

74. Substitute the expression \( x - h \) in for \( x \) in the following expression.

\[
x^3 - 2 x^2 - x + 1
\]

75. Substitute the expression \( 3a^2 - 2a + h \) in for \( x \) in the following expression.

\[
x^3 - 2 x^2 - x + 1
\]

76. Use the subs command to substitute \( y \) in for \( x + 1 \) into the following expression.
77. Use the algsubs command to substitute \( y \) in for \( x+1 \) into the following expression. Is there a difference between the output of this command and the output of the subs command used above.

\[ x^3 - x^2 - 5 \, x - 3 \]

The seq Command

The seq command is for creating a sequence of expressions or objects that are separated by commas. This makes the creation of special lists easy and quick. The seq command has the following syntax,

\[
\text{seq} (\text{expr}, \text{rng})
\]

where expr is the general expression that will change with a change in a variable value and rng is a range for that variable value. For example,

\[
\text{seq} (n^2, n=1..20)
\]

\[ 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400 \]

It is customary for the range to begin and end at integer values but it not necessary. In the following example we begin the sequence at 1.5. Note that this creates a sequence of halves until the point where the next number exceeds the ending value.

\[
\text{seq} (n^2, n=1.5..10)
\]

\[ 2.25, 6.25, 12.25, 20.25, 30.25, 42.25, 56.25, 72.25, 90.25 \]

We can create sequences of functional values as well.

\[
f := x \rightarrow x^2 - 2 \, x + 7
\]

\[
\text{seq} (f(t), t=-5..5)
\]

\[ 42, 31, 22, 15, 10, 7, 6, 7, 10, 15, 22 \]

Another option of the seq command is that one can use a list instead of a range. For example the following command will do the same thing as the map command.

\[
\text{seq} (f(t), t=[1, 3/2, 5.2, 7, 9.11235])
\]

\[ 6, \frac{25}{4}, 23.64, 42, 71.81022252 \]
The seq command is great for defining lists. For example, say we are interested in creating a list of values to numerically examine the limit of a function. To create a list of inputs we would need to create a sequence of numbers that is approaching a particular number. For example, say that we wanted to create a sequence of numbers that approached 1 from above. One way to do it would be as follows.

```maple
> seq(1+1/2^n,n=0..20);
```

We can evalf the list to produce decimal approximations to these values.

```maple
> evalf(seq(1+1/2^n,n=0..20));
```

To create a list we simply need to place the seq command inside square brackets.

```maple
> lst:= [evalf(seq(1+1/2^n,n=0..20))];
```

To analyze the function we then define the function and use the map command on it and the list.

```maple
> f:=x->(x^2-1)/(x-1);
```

```maple
> map(f,lst);
```

Exercises:

78. Create a sequence of 20 values that get close to 5 from below.
79. Create a sequence of 20 values that get close to \( \pi \) from below.
80. Create a list of 20 values that get close to \( \pi \) from below.
81. Use this list in a map command (use evalf on it as well) that evaluates the sine function at each of the list values.
82. Create a sequence of 25 values that increase “without bound”.
83. Create a list of 25 values that increase “without bound”.
84. Use this list in a map command (use evalf on it as well) that evaluates the arctangent function at each of the list values.
Graphing

Maple has many advantages for teaching Calculus but one of its strongest is the plethora of graphing commands. After you create a Maple graph you can click on the image to bring up the image toolbar. This toolbar will allow you to change several of the image options simply by clicking a button. Another option you have is to right-click on the image. This will bring up a pop-up menu of options as well as facilities to save the image in a number of different file formats. As always, you can copy and paste Maple graphs into any application that supports drag and drop graphics.

The plot Command: Plotting Functions

The plot command is for plotting functions or parametrically defined equations. You can use a large number of options to alter the image to suit your needs as well as use a number of different coordinate systems. The syntax for the plot command is very simple,

```
plot(expr, rng, options)
```

where `expr` is the expression or function to plot, `rng` is the range of the independent variable and `options` represents a list of options. The `expr` and `rng` arguments are necessary but you do not need to have any. On the other hand, you may have as many options as you would like. We will not discuss all of the plot options here but we will hit some of the ones you will use most often. For further option listings please see the Maple help system. Let’s look at a few examples.

```maple
> f:=x->sin(x);
f := sin

> plot(f(x), x=-2*Pi..2*Pi);
```

To graph more than one function on the same graph we simply place the set of functions either in a list or a set. For example,
As with all sets and lists, the list retains its order and a set might not. Hence, if you use a set here the colors of the different graphs may differ from the same command that uses a list. When Maple graphs a function it does it so that the graph fits the box. This, in many cases, will distort the image of the graph. To get a true picture of the function you can include the scaling=constrained option. This will graph the function in a 1-1 manner. You can get the same result by clicking the 1-1 button in the toolbar.

```
> plot([f(x), g(x), h(x)], x=-2*Pi..2*Pi);
```

Another way to alter the vertical scale is by inputting a vertical scale. For example, consider the difference between the following two graphs,
This last image brings up another option that is frequently used with rational functions. Note that in the above image the vertical asymptote is shown, to eliminate it we can add the discont=true option.

```
> plot(1/(x-1),x=0..2,y=-10..10,discont=true);
```

The color option sets the color of the function. Possible values or the color option are: aquamarine, black, blue, navy, coral, cyan, brown, gold, green, gray, grey, khaki, magenta, maroon, orange, pink, plum, red, sienna, tan, turquoise, violet, wheat, white, and yellow. For example,

```
> plot(f(x),x=-2*Pi..2*Pi,color=black);
```
When plotting more than one function you can use a list for the color option to control each color independently.

```maple
> plot([f(x), g(x), h(x)], x=-2*Pi..2*Pi, color=[red, black, blue]);
```

Another way to alter the lines appearance is with the linestyle option. The values for the linestyle option are SOLID, DOT, DASH, and DASHDOT. Note that as of Maple 8 these options must be typed in uppercase.

```maple
> plot(f(x), x=-2*Pi..2*Pi, color=black, linestyle=DASHDOT);
```

We can control the thickness of a line as well. Thicknesses can range from 1 to 15, a thickness of 16 results in a thickness of 1, 17 to 2 and so on. For example,
In some cases the number of points Maple uses to graph a function is insufficient. To compensate you can manually set the number of points used in a graph with the numpoints option. For example,

\[
> \text{plot}(\sin(1/x), x=0..1, \text{numpoints}=10000);
\]

Remember that the larger the number of points the longer it will take Maple to graph the function. Another option that is used from time to time is the view option. The view option will display only the portion of the graph you designate, no matter what domain you gave the plot command. For example,
> plot(sin(x), x=-2*Pi..2*Pi, view=[0..5, 0..1]);

Exercises:

85. Plot the function \( \tan(x) \)
on the interval \([-10, 10]\) without using any options.
86. Plot the function \( \tan(x) \)
on the interval \([-10, 10]\) using the discont=true option.
87. Plot the function \( \tan(x) \)
on the interval \([-10, 10]\) using the discont=true option and set the color of the graph to green.
88. Plot the function \( \tan(x) \)
on the interval \([-10, 10]\) using the discont=true option, set the color of the graph to black and restrict the vertical axis to be between \(-5\) and 5.
89. Plot the four functions \( x^2, x^3, x^4, x^5 \)
on the interval \([-5, 5]\), set the color of the graphs to red, green, blue and black respectively. Furthermore restrict the vertical axis to a value that gives you a nice image.
90. Plot the function \( \sin(\cos(\tan(x))) \)
on the interval \([0, \pi]\) without options.
91. Plot the function \( \sin(\cos(\tan(x))) \)
on the interval \([0, \pi]\) using 1000 points for the graph.
92. Plot the function \( \sin(\cos(\tan(x))) \)
on the interval \([0, \pi]\) using 10000 points for the graph.
93. Plot the following piecewise defined function on the interval \([-5, 5]\).
\[
\begin{cases}
1 & x \leq 0 \\
1 & x \\
\frac{1}{x} & \text{otherwise}
\end{cases}
\]

94. Plot the following piecewise defined function on the interval \([-5, 5]\).
\[
\begin{cases}
1 & x < 0 \\
\frac{1}{x} & 0 < x \\
3 & \text{otherwise}
\end{cases}
\]

95. Plot the following piecewise defined function on the interval \([-5, 5]\).
\[
\begin{cases}
3 & 1 < t \\
4 & 0 < t \\
2 & -3 < t \\
5 & \text{otherwise}
\end{cases}
\]

96. Plot the following piecewise defined function on the interval \([-5, 5]\).
\[
\begin{cases}
t & 1 < t \\
3t - 1 & 0 < t \\
\sin(t) & -3 < t \\
\tan(t) & \text{otherwise}
\end{cases}
\]

The implicitplot Command

The implicitplot command is for graphing implicitly defined relations. The syntax for the implicit plot command is similar to the plot command except that it requires ranges for both \(x\) and \(y\). The implicitplot command is not loaded automatically by Maple, it resides in the plots package and hence needs to be loaded with the with command. For example,

\[
> \text{with(plots)}:
\text{Warning, the name changecoords has been redefined}
\]

Don’t worry about the warning. Note that we used a colon at the end of this command instead of the semicolon. The colon will suppress the output of the command. Had we used a semicolon we would see a long blue list of the new commands that were loaded. Many of these commands will be used in Calculus III.

\[
> \text{implicitplot}(x^2-y^2=3,x=-5..5,y=-5..5);
\]
Most of the plot options also work with the implicitplot command, please see the plot command or the Maple help documentation for a more detailed explanation of the available options. For example,

\[ \text{implicitplot}(x^2 - y^2 = 3, x = -5..5, y = -5..5, \text{color} = \text{black}, \text{linestyle} = \text{DASHDOT}); \]

One option that is not available is the numpoints option. When graphing an implicitly defined relation, Maple uses a grid of points and not a list of points. So if you want to increase the number of points used to graph the curve you need to add the grid option to the implicitplot command. For example, look at the following image.

\[ \text{implicitplot}(x^2 - y^2 = 0, x = -2..2, y = -2..2); \]
Notice that there is a square in the center. This square is an error produced by the fact that the curve has a self-intersection at the origin. We can minimize the size of the square, and sometimes remove it, by increasing the grid divisions as in the next command.

> implicitplot(x^2-y^2=0,x=-2..2,y=-2..2,grid=[100,100]);

As with the numpoints option the larger the grid divisions the longer it will take to produce an image. One difference between the plot command and the implicit plot command is when you are graphing several implicit plots together. The relations must be inside a set (curly brackets), you will receive an error if you place them in a list. Otherwise the syntax is the same. For example,

> implicitplot({x^2-x*y^3=3,x=y^3-y^2},x=-5..5,y=-5..5,grid=[50,50],color=black);

Exercises:

97. Use the implicitplot command to graph the equation \( x^2y^2 - xy^3 = 3 \) over the region \([-5, 5] \times [-5, 5]\).

98. Use the implicitplot command to graph the equation \( x^2y^5 - x y^3 = 3 \) over the region \([-5, 5] \times [-5, 5]\) and using a 50 X 50 grid.
99. Use the implicitplot command to graph the equation
\[ x^2 y^2 - x y^3 = 3 \]
over the region \([-5, 5] \times [-5, 5]\), using a 50 X 50 grid and make the color of the graph blue.

100. Use the implicitplot command to graph the equations
\[ \sin(x^2) = \cos(y) \]
\[ y^3 = x^2 + x \]
together over the region \([-3, 3] \times [-3, 3]\), use an appropriate color and grid.

**Infinity Plots**

There are no special commands for infinity plots, in fact you simply use the plot command. Infinity plots are a bit different so we gave them their own small section. Whenever you place infinity as a bound for one of the ranges in a plot Maple will automatically create an infinity plot where the endpoints of the axes are infinite. For example,

\[ \text{plot}(\sin(x), x=0..\text{infinity}); \]

\[ \text{plot}([\exp(x), \ln(x), x^2, \sqrt{x}, \sin(x), (7*x^2-2)/(x^2+5)], x=-\text{infinity}..\text{infinity}); \]

This is a really nice feature when examining end behavior of a function.
Exercises:

101. Plot the four functions $x^2, x^3, x^4, x^5$ on the interval $[-\infty, \infty]$.

102. Plot the arctangent function on the interval $[-\infty, \infty]$.

103. Plot the logarithm functions with bases 2, 3, 5, 10 and 20 on the interval $[-\infty, \infty]$.

104. Plot the root functions with indexes 2, 3, 5, 10 and 20 on the interval $[-\infty, \infty]$.

105. Plot the function $1/x$ on the interval $[-\infty, \infty]$.

106. Plot the function $\frac{2x - 3}{x + 7}$ on the interval $[-\infty, \infty]$.

The logplot and loglogplot Commands

The logplot command produces a semi-log plot of the function or set of functions. The loglogplot produces a log-log plot of the function or set of functions. The syntax for the logplot and loglogplot commands is as follows:

```
logplot(expr, xrng)
loglogplot(expr, xrng)
```

where expr is the function or set of functions and xrng is the range for the $x$ direction. Of course, you may, in addition, have any options you wish. The logplot and loglogplot commands are in the plots package, so you will have to load the plots package into your worksheet.

```
> with(plots):
Warning, the name changecoords has been redefined

> logplot(exp(x), x=-3..10);
```
> loglogplot(x^7, x=1..10);

Exercises:

107. Plot the following four functions using the logplot

\[ x^2 \quad x^3 \quad x^4 \quad x^5 \]

108. Plot the following four functions using the loglogplot

\[ x^2 \quad x^3 \quad x^4 \quad x^5 \]

109. Plot the root functions with indices 2, 3, 5, 10 and 20 using the logplot.

110. Plot the root functions with indices 2, 3, 5, 10 and 20 using the loglogplot.

111. Plot the exponential functions with bases 2, 3, 5, 10 and 20 using the logplot.

112. Plot the exponential functions with bases 2, 3, 5, 10 and 20 using the loglogplot.

The display command

The display command is simply a way to paste several plots together. Its syntax is simple,

\[ \text{display}(\text{expr}) \]

where expr is a sequence of plots separated by commas. Note that the sequence does not have to be in a list or set. For example,

\[ a:=\text{plot}(x^2, x=-2..2, y=-2..2): \]
\[ b:=\text{implicitplot}(x^2-y+2=1, x=-2..2, y=-2..2): \]
\[ c:=\text{plot}(\sin(2*t), t=0..2*\text{Pi}, \text{coords}=\text{polar}): \]
\[ \text{display}(a, b, c); \]
Notice that the three plot commands end with a colon. As we have pointed out before this suppresses the output. If we had used a semicolon the output would be a huge set of blue numbers, not very informative. Also note that we can place the plot commands directly into the display command. For example,

> display(plot(x^2,x=-2..2,y=-2..2),implicitplot(x^2-y+2=1,x=-2..2,y=-2..2),plot(sin(2*t),t=0..2*Pi, coords=polar));

Exercises:

113. Use the display command to join the plots of the implicit plot

\[ x^2y^2 - xy^3 = 3 \]

and the plot of

\[ \sin(\cos(\tan(x))) \]

on the interval \([0, \pi]\)

114. Use the display command to join the plots of the implicit plot

\[ x^2y^2 - xy^3 = 3 \]

and the plot of
on the interval \([-5, 5]\)

115. Use the display command to join the plots of

$$\begin{cases} t & 1 < t \\ 3t - 1 & 0 < t \\ \sin(t) & -3 < t \\ \tan(t) & \text{otherwise} \end{cases}$$

and

$$\begin{cases} 3 & 1 < t \\ 4 & 0 < t \\ 2 & -3 < t \\ 5 & \text{otherwise} \end{cases}$$
Calculus Related Commands

Evaluating Limits of Function of a Single Variable

There are two main commands for limits, limit and Limit. As with most Maple commands that have both a capitalized and lowercase version the capitalized version will display the operation in a pretty-print manner and the lowercase version will perform the operations. If you do use the capitalized form of the command you can then evaluate the expression using the value command. The general syntax for the limit function is

$$\text{limit}(\text{expr}, \text{pos}, \text{dir});$$

where expr represents the expression or function we are taking the limit of, pos represents the limit point, and dir is an optional argument for the direction of the limit, that is, either left or right. For example,

$$\text{limit}(x^2-1, x=2);$$

$$\text{Limit}(x^2-1, x=2);$$

$$\lim_{x \to 2} x^2 - 1$$

$$\text{value}(\%);$$

$$\text{limit}((x^2-1)/(x-1), x=1);$$

$$\text{limit}(\text{abs}(x)/x, x=0);$$

$$\text{limit}(\text{abs}(x)/x, x=0, \text{left});$$

$$\text{limit}(\text{abs}(x)/x, x=0, \text{right});$$

We can also take limits at infinity by placing infinity or –infinity for the position.

$$\text{limit}(	ext{arctan}(t), t=\text{infinity});$$

$$\text{limit}(	ext{arctan}(t), t=\text{-infinity});$$
Exercises:

116. Find the following limit, \( \lim_{x \to \pi} \sin(x) \)

117. Find the following limit, \( \lim_{x \to \left(\frac{2\pi}{3}\right)} \cos(x) - \sin(x) \)

118. Find the following limit, \( \lim_{x \to \infty} \frac{x - 2}{3 - x} \)

119. Find the following limit, \( \lim_{x \to \left(\frac{\pi}{2}\right)} \tan(x) \)

120. Find the following limit, \( \lim_{x \to \left(\frac{\pi}{2}\right)} \tan(x) \)

121. Create the following display, \( \lim_{x \to \pi} \sin(x) \)

122. Create the following display, \( \lim_{x \to \left(\frac{2\pi}{3}\right)} \cos(x) - \sin(x) \)

123. Create the following display, \( \lim_{x \to \infty} \frac{x - 2}{3 - x} \)

124. Create the following display, \( \lim_{x \to \left(\frac{\pi}{2}\right)} \tan(x) \)

125. Create the following display, \( \lim_{x \to \left(\frac{\pi}{2}\right)} \tan(x) \)

Derivatives of Functions of a Single Variable

There are four derivative commands: D, Diff, diff and implicitdiff. The implicitdiff command is for finding derivatives of implicitly defined expressions, that is, for doing implicit differentiation. The other three are for derivatives of explicitly defined functions.
but there are major differences between the D command and the two diff commands. We will start with Diff and diff since these are the ones that will be used most often. Diff and diff are another pair of commands that will either produce a pretty-print version, Diff, or evaluate the derivative, diff. The syntax for either of these commands is

\[ \text{diff(expr, var)}; \]

where expr is the function or expression to be derived and var is the variable that we are differentiating with respect to. For example,

\[
> f := x \rightarrow x^2 - 3x + 2; \\
> \text{diff}(f(x), x); \\
> \text{value}(%);
\]

We can take higher order derivatives simply by adding more variables to the list of arguments. For example,

\[
> \text{diff}(f(x), x, x); \\
> \text{diff}(f(x), x, x, x); \\
> \text{Diff}(f(x), x, x); \\
> \text{Diff}(f(x), x, x, x); \\
> \text{value}(\%);
\]

We can also use the $ repeater to do higher order derivatives. The $ will repeat an expression a given number of times. For example,

\[
> x^2; \\
> x^3;
\]
Using this in conjunction with the diff command allows us to short-cut some of the notation for higher order derivatives. For example,

\begin{verbatim}
> diff(f(x),x$2);
2
> diff(sin(x),x$1023);
-cos(x)
\end{verbatim}

One minor difficulty with the Diff and diff commands is that they return the derivative as an expression and not as a function. Frequently we wish to have the derivative of a function defined as a function. If we use the Diff or diff command we must use the unapply command to turn the expression into a function. The syntax of the unapply command is

\begin{verbatim}
unapply(expr,var)
\end{verbatim}

where expr is the expression to be converted to a function and var is the independent variable. For example,

\begin{verbatim}
> f:=x->x^2-3*x+2;
f := x → x² - 3 x + 2
> diff(f(x),x);
2 x - 3
> df:=unapply(% ,x);
df := x → 2 x - 3
> df(x);
2 x - 3
> df(2);
1
\end{verbatim}

The D command will also find the derivative of a function but its inputs and outputs are quite different than those of Diff and diff. The D command takes as input a function name only, just like the map command. It also outputs a function definition and not an expression. For example,
> \( f := x \rightarrow x^2 - 3x + 2; \)  
\( f := x \rightarrow x^2 - 3x + 2 \)

> \( D(f); \)  
\( x \rightarrow 2x - 3 \)

Notice that the output suggests that what \( D \) is returning is a function that maps \( x \) to \( 2x - 3 \). To create a function that represents the derivative of \( f \) we simply need to assign a name to the output of the \( D \) command, the unapply command is not necessary. For example,

> \( df := D(f); \)  
\( df := x \rightarrow 2x - 3 \)

> \( df(2); \)  
1

In fact we can also use the \( D \) command output as a function itself simply by appending an \( (x) \) or evaluate it at a particular value.

> \( D(f)(x); \)  
\( 2x - 3 \)

> \( D(f)(2); \)  
1

Higher order derivatives can be accomplished with the \( D \) command if we append the \( @@n \) operator, where \( n \) denotes the order of differentiation. For example the second and third derivatives of \( f \) can be found using,

> \( (D@@2)(f); \)  
2

> \( (D@@3)(f); \)  
0

respectively. The \( implicitdiff \) command will find derivatives of implicitly defined relations. The syntax of the \( implicitdiff \) command is as follows,

\[
\text{implicitdiff}(\text{expr}, \text{var1}, \text{var2})
\]

where \( \text{expr} \) is the implicitly defined expression, \( \text{var1} \) and \( \text{var2} \) represent the derivative \( \frac{d \text{var1}}{d \text{var2}} \). For example, to find \( \frac{dy}{dx} \) we use the command,
\[ \text{implicitdiff}(x^2 - y^2 + x \cdot y = \sin(x \cdot y), y, x); \]
\[ \frac{2 \cdot x + y - \cos(x \cdot y) \cdot y}{2 \cdot y - x + \cos(x \cdot y) \cdot x} \]

and to find \( \frac{dx}{dy} \) we use the command,

\[ \text{implicitdiff}(x^2 - y^2 + x \cdot y = \sin(x \cdot y), x, y); \]
\[ \frac{2 \cdot y - x + \cos(x \cdot y) \cdot x}{-2 \cdot x - y + \cos(x \cdot y) \cdot y} \]

Similarly, we can give the implicit expression a name and use it in the implicitdiff command. Note that the following definition is an assignment of an expression to \( t \) and not a function definition.

\[ t := x \cdot y - 2 \cdot y = x^3; \]
\[ t := x \cdot y - 2 \cdot y = x^3 \]

\[ \text{implicitdiff}(t, y, x); \]
\[ \frac{-y + 3 \cdot x^2}{x - 2} \]

Exercises:

126. Find the following derivative using the diff command,
\[ \frac{d}{dx} \cos(x) \]

127. Find the following derivative using the diff command,
\[ \frac{d}{dx} \left( \frac{1}{x^2 - 3 \cdot x + 1} \right) \]

128. Find the following derivative using the diff command,
\[ \frac{d}{dx} \arctan(x) \]

129. Find the following derivative using the diff command,
\[ \frac{d^2}{dx^2} \tan(x) \]

130. Find the following derivative using the diff command,
\[ \frac{d^2}{dx^2} \tan(x) \]

131. Create the following display using the Diff command,
132. Create the following display using the Diff command,
\[ \frac{d}{dx} \cos(x) \]

133. Create the following display using the Diff command,
\[ \frac{d}{dx} \left( \frac{1}{x^2 - 3x + 1} \right) \]

134. Create the following display using the Diff command,
\[ \frac{d}{dx} \arctan(x) \]

135. Create the following display using the Diff command,
\[ \frac{d^2}{dx^2} \tan(x) \]

136. Find the following derivative using the D command,
\[ \frac{d}{dx} \cos(x) \]

137. Find the following derivative using the D command,
\[ \frac{d}{dx} \left( \frac{1}{x^2 - 3x + 1} \right) \]

138. Find the following derivative using the D command,
\[ \frac{d}{dx} \arctan(x) \]

139. Find the following derivative using the D command,
\[ \frac{d^2}{dx^2} \tan(x) \]

140. Find the following derivative using the D command,
\[ \frac{d^2}{dx^2} \tan(x) \]

141. Find \( \frac{dy}{dx} \) where
\[ 4x^2 y - 3y x^3 + \frac{x}{y} = 7xy \]

142. Find \( \frac{dy}{dx} \) where
\[ \frac{\sin(\sin(x) \cos(y))}{\cos(\cos(x) \sin(y))} \]

143. Find \( \frac{dx}{dy} \) where
\[ 4x^2y - 3y^3x^3 + \frac{x}{y} = 7xy \]

144. Find \( \frac{dx}{dy} \) where
\[
\frac{\sin(\sin(x) \cos(y))}{\cos(\cos(x) \sin(y))}
\]

**Integration of Single Variable Functions**

Finding indefinite integrals with Maple is a snap. Although there are several special types of integral commands in Maple you can almost always get by with just two, `Int` and `int`. As with all capitalized and lowercase pairs of commands the capitalized one returns a pretty-print version and the lowercase one does the operation. Also, as with the other capitalized commands in Maple, the `value` command will force Maple to do the operation. To find an indefinite integral we use the syntax,

\[
\text{int} (\text{expr}, \text{var})
\]

where the `expr` is the expression or function to be integrated and the `var` is the variable we are integrating with respect to. For example,

\[
> f := x -> x^2 - 3x + 2;
\]

\[
> \text{int} (f(x), x);
\]

\[
\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x
\]

\[
> \text{Int} (f(x), x);
\]

\[
\int x^2 - 3x + 2 \, dx
\]

\[
> \text{value}(\%);
\]

\[
\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x
\]

**Exercises:**

145. Find the following integral,
\[
\int 7x^3 - 13x^2 + 5x - 6 \, dx
\]

146. Find the following integral,
\[
\int 3^x - x^3 \, dx
\]
147. Find the following integral,
\[ \int \sin(x) \sqrt{\cos(x)} \, dx \]

148. Display the following integral,
\[ \int 7x^3 - 13x^2 + 5x - 6 \, dx \]

149. Display the following integral,
\[ \int 3^x - x^3 \, dx \]

150. Display the following integral,
\[ \int \sin(x) \sqrt{\cos(x)} \, dx \]
Packages

Maple is a very large and versatile mathematical software package. If we were to load all of the Maple commands into memory when we started it up it would take up far too many system resources. Instead, when we load Maple only a small number of commands are loaded. Basically, there is enough to do simple calculations in a number of mathematical areas including Calculus but more advanced commands are omitted. These more advanced commands are in a number of code files called packages. To gain access to these commands you can either reference the package the command is in or more simply load the package into memory with the with command. Specifically, the command

\[
\text{with(pkg):}
\]

loads in the package named pkg. For example to load the plots package we use the command

\[
\text{with(plots):}
\]

Note that we usually use the colon at the end of the with command. If we use the semicolon we will get a listing of all of the functions in that package. The colon suppresses this output. In this section we will discuss some of the Maple packages you may encounter in Calculus I, II and III. There are many other packages that are specific to certain areas of mathematics but we will not discuss them here.

The student Package

The student package has many specialized commands primarily for teaching and learning undergraduate mathematics, specifically Calculus I, II & III. The student package has recently been superseded by the Student package, which is a package with one subpackage, Calculus1. We will discuss the Student package later. The student package can be loaded using the command

\[
\text{with(student):}
\]

The commands that are loaded are: D, Diff, Doubleint, Int, Limit, Lineint, Product, Sum, Tripleint, changevar, completesquare, distance, equate, integrand, intercept, intparts, leftbox, leftsum, makeproc, middlebox, middlesum, midpoint, powsubs, rightbox, rightsum, showtangent, simpson, slope, summand, trapezoid. You will note that some of the commands that are loaded, like D, Diff and Int, already exist in Maple. We will look at a few of these commands.
The changevar Command

The changevar command will do a change of variable for an integral.

The syntax is the same for the indefinite integral, except that we do a substitution at the end. Take the integral,

\[ \int \sqrt{x^3 - x^2 + 3x - 4} \cdot (6x^2 - 4x + 6) \, dx \]

do the substitution,

\[ \text{changevar}(u=x^3-x^2+3x-4, \% , u) \]

find the value,

\[ \text{value}(\%) \]

substitute back,

\[ \text{subs}(u=x^3-x^2+3x-4, \%) \]

and you are done. Below we check our answer.

Exercises:

151. Use the changevar command to do the substitution 

\[ u = \cos(x) \]

in the integral.
\[ \int \sin(x) \sqrt{\cos(x)} \, dx \]

152. Use the changevar command to do the substitution

\[ u = \arctan(x) \]

in the integral.

\[ \int \frac{\arctan(x)^5}{1 + x^2} \, dx \]

**The distance Command**

The distance command simply finds the Euclidean distance between two points.

> `distance(-3, 5);`

\[ 8 \]

> `distance([1,2], [3,5]);`

\[ \sqrt{13} \]

> `distance([a,b], [c,d]);`

\[ \sqrt{(a-c)^2 + (b-d)^2} \]

> `distance([a,b,c,d,e], [f,g,h,i,j]);`

\[ \sqrt{(b-g)^2 + (a-f)^2 + (i-d)^2 + (h-c)^2 + (e-j)^2} \]

> `distance([6,4,-2,1], [8,-4,-3,8]);`

\[ \sqrt{118} \]

**Exercises:**

153. Use the distance command to find the distance between the points \((17,5)\) and \((-3,2)\).

154. Use the distance command to find the distance between the points \((17,5,4)\) and \((-3,2,-5)\).

155. Use the distance command to find the distance between the points \((a,b,c)\) and \((1,2,3)\).
The intercept Command

The intercept command will find the intersection between two curves. If one curve is input the result will be the y intercept and when two curves are input the result will be the intersection of the two curves. For example,

\[ \text{intercept}(y = x^2+5); \]  
\[ \{ y = 5, x = 0 \} \]

\[ \text{intercept}(y = x^2+5, y=0); \]  
\[ \{ y = 0, x = \text{RootOf}(5 + _Z^2, label = _L5) \} \]

\[ \text{evalf}(%); \]  
\[ \{ y = 0., x = 2.236067978 I \} \]

\[ \text{intercept}(y = x^2-5, y=0); \]  
\[ \{ y = 0, x = \text{RootOf}(-5 + _Z^2, label = _L3) \} \]

\[ \text{evalf}(%); \]  
\[ \{ y = 0., x = 2.236067977 \} \]

\[ \text{intercept}(y = \sin(x), y=x-2); \]  
\[ \{ x = 2 + \sin(\text{RootOf}(-_Z + 2 + \sin(_Z))), y = \sin(\text{RootOf}(-_Z + 2 + \sin(_Z))) \} \]

\[ \text{evalf}(%); \]  
\[ \{ x = 2.554195953, y = 0.5541959527 \} \]

Exercises:

156. Use the intercept command to find the roots of the equation

\[ y = x^3 - x^2 \]

157. Use the intercept command to find the intersection of the functions

\[ y = x^3 - x^2 \]

and

\[ y = \frac{1}{100} \]

158. Use the intercept command to find the intersection of the functions

\[ y = x^3 - x^2 \]

and

\[ y = x^2 - 1 \]
The midpoint Command

The midpoint command finds the midpoint between two points. For example,

\[
\text{midpoint}([a,b],[c,d]);
\]

\[
\left[\frac{a+c}{2}, \frac{b+d}{2}\right]
\]

Exercises:

159. Use the midpoint command to find the midpoint between the points (17,5) and (-3,2).
160. Use the midpoint command to find the midpoint between the points (17,5,4) and (-3,2,-5).
161. Use the midpoint command to find the midpoint between the points (a,b,c) and (1,2,3).

The showtangent Command

The showtangent command graphs a curve and its tangent line to a specified point on the curve. All you need to input is the curve and the point of tangency. Other options may be input as well, the most common are \( x \) and \( y \) range restrictions. For example,

\[
\text{showtangent}(x^2+x+5, \ x = 2);
\]

\[
\text{showtangent}(x^2+x+5, \ x = 2, x=-2..3);
\]
Exercises:

162. Use the showtangent command to graph the tangent line to the curve
\[ y = \sin(x) \]
at \( x = \frac{\pi}{4} \).

163. Use the showtangent command to graph the tangent line to the curve
\[ y = \sin(3x) - \cos(2x) \]
at \( x = \frac{\pi}{6} \).

164. Use the showtangent command to graph the tangent line to the curve
\[ y = x^3 - 5x^2 + 3x - 1 \]
at \( x = 2 \). Restrict the \( x \) range to be between –2 and 5 and restrict the \( y \) range to be between –10 and 10.

The Student Package: Student[Calculus1]

Like the student package, the Student package has many specialized commands primarily for teaching and learning undergraduate mathematics, specifically Calculus I, II & III. The Student package is newer and contains commands that can be used in place of the
commands found in the student package. The Student package can be loaded using the command

\[
\text{with(Student[Calculus1])}:
\]

The commands that are loaded are: AntiderivativePlot, ApproximateInt, ArcLength, Asymptotes, Clear, CriticalPoints, DerivativePlot, ExtremePoints, FunctionAverage, FunctionChart, GetMessage, GetNumProblems, GetProblem, Hint, InflectionPoints, Integrand, InversePlot, MeanValueTheorem, NewtonQuotient, NewtonsMethod, PointInterpolation, RiemannSum, RollesTheorem, Roots, Rule, Show, ShowIncomplete, ShowSteps, Summand, SurfaceOfRevolution, Tangent, TaylorApproximation, Understand, Undo, VolumeOfRevolution, WhatProblem. We will look at a few of these commands.

The AntiderivativePlot Command

The AntiderivativePlot command will plot a function along with one of its antiderivatives. The syntax for the command is

\[
\text{AntiderivativePlot(expr, rng, opts)}
\]

where expr is the expression or function to be graphed, rng is the range of the independent variable and opts is a list of options. Two of the main options are value and showclass. The value option allows you to select the \( y \)-value of the left hand endpoint or choose a point for the antiderivative to pass through. The showclass option displays a set of antiderivatives. For example,

\[
> \text{with(Student[Calculus1])}:
> f := x \rightarrow x^2+2*x-7;
> f := x \rightarrow x^2+2 \cdot x - 7
\]

\[
> \text{AntiderivativePlot}(f(x), x=-3..3, \text{value} = 0);
\]

\[
> \text{AntiderivativePlot}(f(x), x=-3..3, \text{value} = 15);
\]
\textbf{AntiderivativePlot}(f(x), x=-3..3, value = [0,0]);

\textbf{AntiderivativePlot}(f(x), x=-3..3, value = [0,5]);

\textbf{AntiderivativePlot}(f(x), -3..3, value = 15, showclass);
Exercises:

165. Use the AntiderivativePlot command to graph the antiderivative of
\[ y = x^3 - 5x^2 + 3x - 1 \]

166. Use the AntiderivativePlot command to graph the antiderivative of
\[ y = \sin(3x) - \cos(2x) \]

167. Use the AntiderivativePlot command to graph the antiderivative of
\[ y = x^3 - 5x^2 + 3x - 1 \]
but restrict the left hand endpoint to be at the height of 7.

168. Use the AntiderivativePlot command to graph the antiderivative of
\[ y = \sin(3x) - \cos(2x) \]
that passes through the point (0,1).

169. Use the AntiderivativePlot command to graph the family of antiderivatives of
\[ y = x^3 - 5x^2 + 3x - 1 \]

170. Use the AntiderivativePlot command to graph the family of antiderivatives of
\[ y = \sin(3x) - \cos(2x) \]

The DerivativePlot Command

The derivativeplot command will graph a function with its derivatives or a set of
derivatives. The syntax for this command is

\[
\text{DerivativePlot}(\text{expr}, \text{rng}, \text{opts})
\]

where expr is the expression or function to be integrated, rng is the range of the
independent variable and opts is a list of options. The main options are derivativeoptions
which takes a list of options for the derivative curve, functionoptions which takes a list of
options for the curve, order which specifies which derivative or set of derivatives to plot,
and view which sets the viewing window. For example,

\[
> \text{with(Student[Calculus1])}:
> f := x \rightarrow \sin(x);
\]

\[
f := \sin
\]

\[
> \text{DerivativePlot}(f(x), x=-5..5);
\]
Exercises:

171. Use the DerivativePlot command to graph the derivative of
\[ y = x^3 - 5x^2 + 3x - 1 \]

172. Use the DerivativePlot command to graph the derivative of
\[ y = \sin(3x) - \cos(2x) \]

173. Use the DerivativePlot command to graph the derivative of order two of
\[ y = x^3 - 5x^2 + 3x - 1 \]
174. Use the DerivativePlot command to graph the derivative of order three of
\[ y = \sin(3x) - \cos(2x) \]

175. Use the DerivativePlot command to graph the derivative of orders one through four of
\[ y = x^3 - 5x^2 + 3x - 1 \]

176. Use the DerivativePlot command to graph the derivative of orders one through six of
\[ y = \sin(3x) - \cos(2x) \]

The InversePlot Command

The InversePlot command will graph a function along with its graphical inverse and the line \( y = x \). For example,

> `InversePlot(exp(x), x=-2..2);`

There are several options for controlling the curve attributes, the most common are functionoptions and inverseoptions. Each of these takes a list of options to be applied to the function or the inverse. For example,

> `InversePlot(exp(x), x=-2..2, functionoptions=[color=black], inverseoptions=[color=red, linestyle=HIDDEN]);`
Exercises:

177. Use the InversePlot command to graph the inverse of
\[ y = x^3 - 5x^2 + 3x - 1 \]

178. Use the InversePlot command to graph the inverse of
\[ y = \sin(3x) - \cos(2x) \]

The MeanValueTheorem Command

The MeanValueTheorem command displays a nice graphical image of the Mean Value Theorem. Simply input the function and an interval and Maple will display the function, a line segment between the endpoints and every place in the interval where the slope of the tangent line matches that of the average. For example,

\[ > \text{MeanValueTheorem}(x^3 - x^2, x=0..2); \]

\[ > \text{MeanValueTheorem}(\sin(x), x=0..10); \]

The MeanValueTheorem command has several options for customizing the display one of which is the view option.

\[ > \text{MeanValueTheorem}(\sin(x), x=0..10, \text{view}=[\text{DEFAULT},-2..2]); \]
The MeanValueTheorem command can also output just the points of tangency with no graph by adding the output=points option. For example,

```maple
> MeanValueTheorem(x^3 - x^2, x=0..2, output=points);
```

\[
\begin{bmatrix}
\frac{1}{3} + \frac{\sqrt{7}}{3}
\end{bmatrix}
\]

```maple
> MeanValueTheorem(sin(x), x=0..10, output=points);
```

\[
\begin{bmatrix}
\arccos\left(\frac{1}{10}\sin(10)\right) - \arccos\left(\frac{1}{10}\sin(10)\right) + 2\pi, \arccos\left(\frac{1}{10}\sin(10)\right) + 2\pi
\end{bmatrix}
\]

By adding the numeric=true option we can convert the output to approximate solutions.

```maple
> MeanValueTheorem(sin(x), x=0..10, output=points, numeric=true);
```

\[
\begin{bmatrix}
1.625225308, 4.657959999, 7.908410616
\end{bmatrix}
\]

**Exercises:**

179. Use the MeanValueTheorem command to get a graphical image of the Mean Value Theorem as it applies to 

\[ y = x^3 - 5x^2 + 3x - 1 \]

on the interval \([0, 5]\). Find the exact values of the tangent points and their numerical approximations.

180. Use the MeanValueTheorem command to get a graphical image of the Mean Value Theorem as it applies to 

\[ y = x^3 - 5x^2 + 3x - 1 \]

on the interval \([-2, 2]\). Find the exact values of the tangent points and their numerical approximations.

181. Use the MeanValueTheorem command to get a graphical image of the Mean Value Theorem as it applies to 

\[ y = x^3 - 5x^2 + 3x - 1 \]
on the interval \([-5, 5]\). Find the exact values of the tangent points and their numerical approximations.

182. Use the \texttt{MeanValueTheorem} command to get a graphical image of the Mean Value Theorem as it applies to 

\[ y = \sin(3x) - \cos(2x) \]

on the interval \([-2, 2]\).

183. Use the \texttt{MeanValueTheorem} command to get a graphical image of the Mean Value Theorem as it applies to 

\[ y = \sin(3x) - \cos(2x) \]

on the interval \([0, 5]\).

\textbf{The \texttt{NewtonsMethod} Command}

The \texttt{NewtonsMethod} command displays a nice graphical image of Newton’s Method given a function and an initial starting point. The main controlling options for this command are the output and iterations. The output option can be either: plot, value or sequence. Plot displays the method graphically, value gives the final approximation and sequence displays the sequence of approximations. For example,

\begin{verbatim}
> NewtonsMethod(x^2+x-1,x=2,output=plot);
\end{verbatim}

\begin{verbatim}
> NewtonsMethod(x^2+x+1,x=2,output=plot);
\end{verbatim}

\begin{verbatim}
> NewtonsMethod(x^2+x+1,x=2,output=plot,iterations=100,view=[-2..2,0..10]);
\end{verbatim}
> NewtonsMethod(x^2+x-1, x=2, output=value);
0.6180339889

> NewtonsMethod(x^2+x-1, x=2, output=value, iterations=10);
0.6180339889

> NewtonsMethod(x^2+x-1, x=2, output=value, iterations=3);
0.6190476191

> NewtonsMethod(x^2+x-1, x=2, output=sequence, iterations=3);
2, 1.0000000000, 0.6666666667, 0.6190476191

> NewtonsMethod(x^2+x-1, x=2, output=sequence, iterations=10);
2, 1.0000000000, 0.6666666667, 0.6190476191, 0.6180344477, 0.6180339889

Note in the last command that there are not 10 values in the sequence. If Maple sees that the successive values are the same it will remove them.

Exercises:

184. Use the NewtonsMethod command to get a graphical image of Newton’s Method for
\[ y = x^3 - 5x^2 + 3x - 1 \]
using an initial value of 4. Find the root approximation. Obtain the sequence of values from Newton’s Method up to 10 iterations.

185. Use the NewtonsMethod command to get a graphical image of Newton’s Method for
\[ y = \sin(3x) - \cos(2x) \]
using an initial value of 3. Find the root approximation. Obtain the sequence of values from Newton’s Method up to 10 iterations.

The Tangent Command

The Tangent command can produce a graph, slope or equation of a tangent line to a curve. The general syntax for this function is
**Tangent**(expr, pos, opts)

where expr is the expression or function to be used, pos is the position of the tangent line and opts is a list of options. The main option for this command is the output option. Output can be either plot, slope or line. Plot returns a graphic image, slope returns just the slope of the line and line returns the equation of the tangent line. For example,

> `Tangent(sin(cos(x)), x=1, output=plot);`

![Tangent Graph]

> `Tangent(sin(cos(x)), x=1, output=slope);`

\[-\cos(\cos(1)) \sin(1)\]

> `evalf(%);`

\[-0.7216061490\]

> `Tangent(sin(cos(x)), x=1, output=line);`

\[-x \cos(\cos(1)) \sin(1) + \sin(\cos(1)) + \cos(\cos(1)) \sin(1)\]

> `evalf(%);`

\[-0.7216061490x + 1.236001408\]

**Exercises:**

186. Use the Tangent command to get a graphical image of the tangent line to
\[y = x^3 - 5x^2 + 3x - 1\]

at \(x = 5\). Find the slope of the line. Find the equation of the line.

187. Use the Tangent command to get a graphical image of the tangent line to
\[y = \sin(3x) - \cos(2x)\]

at \(x = \pi\). Find the slope of the line. Find the equation of the line.

**The Asymptotes Command**

The Asymptotes command will find vertical, horizontal and slanted asymptotes. For example,
Asymptotes\((\frac{2x-7}{x-3},x)\);
\[ y = 2, x = 3 \]

Asymptotes\((\frac{2x^2-7}{x-3},x)\);
\[ y = 2x + 6, x = 3 \]

If we restrict the domain, only asymptotes in that domain are listed.

Asymptotes\((\frac{2x-7}{x-3}, x=-2..2)\);
\[ y = 2 \]

Maple will also try to find all of the asymptotes of a function. If there are an infinite number of them Maple will make note of it.

Asymptotes\((\tan(x),x)\);
Warning, the expression has an infinity of asymptotes, some examples of which are given
\[ \left[ x = -\frac{\pi}{2}, x = \frac{\pi}{2}, x = \frac{3\pi}{2} \right] \]

Asymptotes\((\tan(x), x=0..20)\);
\[ \left[ x = \frac{\pi}{2}, x = \frac{3\pi}{2}, x = \frac{5\pi}{2}, x = \frac{7\pi}{2}, x = \frac{9\pi}{2}, x = \frac{11\pi}{2} \right] \]

Asymptotes\((\arctan(x),x)\);
\[ \left[ y = \frac{\pi}{2}, y = -\frac{\pi}{2} \right] \]

product\((\frac{x-i}{x-i}, i=1..10)\);
\[ (x-1)(x-2)(x-3)(x-4)(x-5)(x-6)(x-7)(x-8)(x-9)(x-10) \]

Asymptotes\(\left(\frac{1}{\text{product}\((x-i), i=1..10\)},x\right)\);
\[ [y = 0, x = 1, x = 2, x = 3, x = 4, x = 5, x = 6, x = 7, x = 8, x = 9, x = 10] \]

Exercises:

188. Find the asymptotes of
\[ \frac{x^2 - 3x - 5}{x - 2} \]

189. Find the asymptotes of
\[ \frac{x^2 - 3x - 5}{x^2 - 2} \]

190. Find the asymptotes of
\[
\frac{1}{\sin(x)}
\]

**The CriticalPoints Command**

The CriticalPoints command will find all of the critical points of a function. For example,

\[
> \text{CriticalPoints}(3x^4-16x^3-66x^2+360x+5, x);
\]

\[
[-3, 2, 5]
\]

If we restrict the domain, critical points outside the domain are ignored.

\[
> \text{CriticalPoints}(3x^4-16x^3-66x^2+360x+5, x=0..10);
\]

\[
[2, 5]
\]

If Maple notices that there are an infinite number of critical points it make note of it.

\[
> \text{CriticalPoints}(\sin(x), x);
\]

Warning, the expression has an infinity of critical points, some examples of which are given

\[
\begin{bmatrix}
-\pi/2 & \pi/2 & 3\pi/2
\end{bmatrix}
\]

**Exercises:**

191. Find the critical points of

\[
x^2 - 3 x - 5
\]

\[
x - 2
\]

192. Find the critical points of

\[
y = x^3 - 5 x^2 + 3 x - 1
\]

193. Find the critical points of

\[
y = \sin(3 x) - \cos(2 x)
\]

**The ExtremePoints Command**

The ExtremePoints command will find all of the points that produce an extreme (local or global) of a function. Note that it is the \( x \) value of the extreme point that is returned. For example,

\[
> \text{ExtremePoints}(3x^4-16x^3-66x^2+360x+5, x);
\]

\[
[-3, 2, 5]
\]
> CriticalPoints(x^3, x);  
\[0\]

> ExtremePoints(x^3, x);  
\[]

> ExtremePoints( sin(x), x=0..5 );
\[0, \frac{\pi}{2}, \frac{3\pi}{2}, 5\]

> ExtremePoints( sin(x), x);
Warning, the expression has an infinity of extreme points, some examples of which are given
\[-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}\]

Exercises:

194. Find the extreme points of
\[
\frac{x^2 - 3x - 5}{x - 2}
\]

195. Find the extreme points of
\[
y = x^3 - 5x^2 + 3x - 1
\]

196. Find the extreme points of
\[
y = \sin(3x) - \cos(2x)
\]

The InflectionPoints Command

The InflectionPoints command will find all of the inflection points of a function. For example,

> InflectionPoints(x^3, x);  
\[0\]

> InflectionPoints(3*x^4-16*x^3-66*x^2+360*x+5, x);
\[-1, \frac{11}{3}\]

> InflectionPoints(3*x^4-16*x^3-66*x^2+360*x+5, x=0..5);
\[\frac{11}{3}\]
Exercises:

197. Find the inflection points of

\[ y = x^3 - 5x^2 + 3x - 1 \]

198. Find the inflection points of

\[ y = \sin(x) \]

The Roots Command

The Roots command will find all of the real roots of a function. As with other commands of this type we can restrict the domain and we can include the numeric option to force Maple to return numeric values. For example,

\[
> \text{Roots}(3x^4 - 16x^3 - 66x^2 + 360x + 5, x);
\]

\[
\left(-8 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/6}\right.
\]

\[+ \sqrt{2} \left(98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818\right)\]

\[+ \sqrt{2} \left(196 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} \right)
\]

\[
\sqrt[6]{98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818 - 3
\]

\[
\sqrt[3]{98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818
\]

\[
\left(15294 + 6 \sqrt[3]{315595}\right)^{2/3}
\]

\[- 1818 \sqrt[6]{98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818
\]

\[+ 572 \sqrt{2} \sqrt[2]{15294 + 6 \sqrt[3]{315595}}\)

\[
\left(6 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/6}\right)
\]

\[
\left(98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818\right)\left(8\right.
\]

\[
\left(15294 + 6 \sqrt[3]{315595}\right)^{1/6}
\]

\[
\left(98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818\right)
\]

\[- \sqrt{2} \left(98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818\right)\]

\[+ \sqrt{2} \left(196 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} \right)
\]

\[
\sqrt[6]{98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818 - 3
\]

\[
\sqrt[3]{98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818
\]

\[
\left(15294 + 6 \sqrt[3]{315595}\right)^{2/3}
\]

\[- 1818 \sqrt[6]{98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818
\]

\[+ 572 \sqrt{2} \sqrt[2]{15294 + 6 \sqrt[3]{315595}}\)

\[
\left(6 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/6}\right)
\]

\[
\left(98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818\right)\left(8\right.
\]

\[
\left(15294 + 6 \sqrt[3]{315595}\right)^{1/6}
\]

\[
\left(98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818\right)
\]

\[- \sqrt{2} \left(98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818\right)\]

\[+ \sqrt{2} \left(196 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} \right)
\]

\[
\sqrt[6]{98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818 - 3
\]

\[
\sqrt[3]{98 \left(15294 + 6 \sqrt[3]{315595}\right)^{1/3} + 3 \left(15294 + 6 \sqrt[3]{315595}\right)^{2/3} + 1818
\]

\[
\left(15294 + 6 \sqrt[3]{315595}\right)^{2/3}
\]
\[
\sqrt{98 (15294 + 6 \sqrt{315595})^{\frac{1}{3}}} + 3 (15294 + 6 \sqrt{315595})^{\frac{2}{3}} + 1818 - 3
\]

\[
\sqrt{98 (15294 + 6 \sqrt{315595})^{\frac{1}{3}}} + 3 (15294 + 6 \sqrt{315595})^{\frac{2}{3}} + 1818
\]

\[
(15294 + 6 \sqrt{315595})^{\frac{2}{3}}
\]

\[
\frac{- 1818 \sqrt{98 (15294 + 6 \sqrt{315595})^{\frac{1}{3}}} + 3 (15294 + 6 \sqrt{315595})^{\frac{2}{3}} + 1818}{6 \sqrt{15294 + 6 \sqrt{315595}}^{\frac{1}{2}}}
\]

\[
\frac{(98 (15294 + 6 \sqrt{315595})^{\frac{1}{3}}} + 3 (15294 + 6 \sqrt{315595})^{\frac{2}{3}} + 1818^{\frac{1}{4}}\right)}
\]

\[
> \text{Roots}(3x^4-16x^3-66x^2+360x+5,x,\text{numeric});
\]

\[
[-4.714892098, -0.01385382051]
\]

\[
> \text{Roots}(3x^4-16x^3-66x^2+360x+5,x=-1..1,\text{numeric});
\]

\[
[-0.01385382051]
\]

\[
> \text{Roots}(x^2+1,x);
\]

\[
[\text{[]}]
\]

**Exercises:**

199. Find the roots of

\[
y = x^3 - 5x^2 + 3x - 1
\]

200. Find the roots of

\[
y = \sin(3x) - \cos(2x)
\]
Using Maple Help

The Maple help system is quite different from other help systems you may have used. This can make it very frustrating at first when you want to find something. In fact, the most frustrating part of the help system is its lack of a search mechanism. We have found that once you know a little about the Maple program the help system becomes easier to use since you have more of an idea of what to look for. There are two main ways to invoke the help system. One is by selecting Help > Introduction from the menu. This will bring up the help system beginning at the Maple introduction, as below.

Notice the 5 panels across the top. These are subtopic panels. If you select an option that has subtopics the subtopic listing will be displayed in the panel directly to the right. So you keep selecting subtopics until you get to a topic that does not have a subtopic listing and then help text will be displayed in the window below the panels. The help text usually starts out with a statement of general syntax, followed by a list of options for the command, followed by examples and ending with links to related topics. For example, the help image below is for the solve command. Note that we went all the way down to the fourth panel before we found the help we needed. This is another reason the Maple help system is frustrating at first, you need to have an idea where the command fits in the scheme of Maple if you have any hope of finding it. This is also why the help system
gets much easier to use after you have a little experience with Maple. At that point you will have a better idea where commands are located.

Another way to invoke the help system is by using the ? command in the worksheet. That is, at any time you can type in ? followed by the name of the command you want to look up and press enter (no semicolon needed). This will automatically search for the command in the help system and send you directly to the page for that command. The unfortunate thing here is that you need to know the name of the command. For example, the above screen was obtained by the command,

```plaintext
>solve
```

You can also change the amount you are shown and the format in which is shown to you by using either ?? or ??? instead of ?. For example,

```plaintext
>?solve
```

or

```plaintext
>??solve
```

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>solve</code></td>
<td>solves equations.</td>
</tr>
<tr>
<td><code>Calling Sequence</code></td>
<td><code>solve(eqns, vars)</code>.</td>
</tr>
<tr>
<td><code>Parameters</code></td>
<td><code>eqns</code> - equation, inequality, or procedure.</td>
</tr>
<tr>
<td></td>
<td><code>vars</code> - (optional) set of names (unknowns to solve for)</td>
</tr>
<tr>
<td></td>
<td><code>eqns</code> - set of equations or inequalities</td>
</tr>
<tr>
<td></td>
<td><code>vars</code> - (optional) unknowns to solve for</td>
</tr>
</tbody>
</table>

The `solve` command attempts to find a solution for the given equations. If it fails to find a solution, it returns an empty set. The `??` and `???` commands are used to show more information, with `???` showing the most comprehensive information.
Solutions:

1. Type in the command that will give you the expression
\[ \frac{2}{x+y} + \frac{3}{z} - 5 \]
\[ > \frac{2}{x+y} + \frac{3}{z} - 5; \]

2. Type in the command that will give you the expression
\[ \left( x - \frac{y}{2} + 7z \right)^{\frac{2}{3}} \]
\[ > \left( x - \frac{y}{2} + 7z \right)^{\frac{2}{3}}; \]

3. Type in the command that will give you the expression
\[ x^{(y+3)} + \frac{y+3}{z+2} \]
\[ > x^{(y+3)} + \frac{y+3}{z+2}; \]

4. Type in the Maple command that will set the variable \( a \) to the value of 17.
\[ > a := 17; \]
\[ a := 17 \]

5. Type in the Maple command that will set the variable \( b \) to the variable \( a \).
\[ > b := a; \]
\[ b := 17 \]

6. Get the value of \( b \). What happened?
\[ > b; \]
\[ 17 \]
\( b \) has the value of \( a \).

7. Set the variable \( c \) to the expression
\[ \frac{2}{x+y} + \frac{3}{z} - 5 \]
\[ > c := \frac{2}{x+y} + \frac{3}{z} - 5; \]
8. Set the variable \( d \) to the expression
\[
\left( x - \frac{y}{2} + 7z \right)^{\frac{2}{3}}
\]

\[ d := (x-y/2+7*z)^(2/3); \]

9. Set the variable \( e \) to the expression
\[
x^{(y+3)} + \frac{y + 3}{z + 2}
\]

\[ e := x^(y+3)+(y+3)/(z+2); \]

10. Set the variable \( x \) to the expression
\[
x^{(y+3)} + \frac{y + 3}{z + 2}
\]

What happened and why?

\[ x := x^(y+3)+(y+3)/(z+2); \]

Error, recursive assignment

We got an error since there was an \( x \) on the left and the right of the assignment statement.

11. Reset the variables \( a, b, c, d \) and \( e \) back to variables.

\[ a := 'a'; b := 'b'; c := 'c'; d := 'd'; e := 'e'; \]

12. Type in the command that will give you the expression
\[
\sin(2x-7) + \cos(y-2)
\]

\[ \sin(2*x-7)-\cos(y-2); \]

13. Type in the command that will give you the expression
\[
\sin^{-1}(x) + \cosh(y)
\]

> \texttt{arcsin(x)+cosh(y);}
> \texttt{arcsin(x) + cosh(y)}

14. Type in the command that will give you the expression 
\[\ln(x^y) + \log_7(x + y)\]

> \texttt{ln(x^y)+log[7](x+y);}
> \texttt{\ln(x^y) + \frac{\ln(x + y)}{\ln(7)}}

15. Type in the command that will give you the expression 
\[\sinh^{-1}(t) - \sqrt{t}\]

> \texttt{arcsinh(t)-sqrt(t);}
> \texttt{arcsinh(t) - \sqrt{t}}

16. Type in the command that will give you the expression 
\[\sqrt{x + y} - \ln(\sqrt{x - y})\]

> \texttt{surd(x+y,5)+ln(surd(x-y,7));}
> \texttt{surd(x + y, 5) + \ln(surd(x - y, 7))}

17. Type in the command that will give you the expression 
\[e^{(x-y)} + 4^{(y-x)}\]

> \texttt{exp(x-y)+4^(y-x);}
> \texttt{e^{(x-y)} + 4^{(y-x)}}

18. Define the function 
\[f(x) = \sin(2x - 7) + \cos(x - 2)\]

> \texttt{f:=x->sin(2*x-7)-cos(x-2);}
> \texttt{f := x \rightarrow sin(2 x - 7) - cos(x - 2)}

19. Define the function 
\[g(x) = \sin^{-1}(x) + \cos(x)\]

> \texttt{g:=x->arcsin(x)+cosh(x);}
> \texttt{g := x \rightarrow arcsin(x) + cosh(x)}

20. Define the function 
\[h(t) = \sinh^{-1}(t) - \sqrt{t}\]

> \texttt{h:=t->arcsinh(t)-sqrt(t);}
> \texttt{h := t \rightarrow arcsinh(t) - \sqrt{t}}

21. Define the function 
\[j(r) = \sqrt{3r} - \ln\left(\frac{1}{\sqrt{r}}\right)\]

> \texttt{j:=r->surd(3*r,5)-ln(surd(1/r,7));}
> \texttt{j := r \rightarrow surd(3 r, 5) - \ln\left(surd\left(\frac{1}{r}, 7\right)\right)}
22. Define the function
\[ k(w) = e^w + 4^{-w} \]
\[ k := w \rightarrow \exp(w) + 4^(-w) \]

23. Create the following piecewise defined expression
\[ \begin{cases} 
1 & x \leq 0 \\
\frac{1}{x} & \text{otherwise}
\end{cases} \]
\[ \text{piecewise}(x<=0,1,1/x); \]

24. Take the expression above and create a piecewise defined function out of it.
\[ f := x \rightarrow \text{piecewise}(x \leq 0, 1, \frac{1}{x}) \]

25. Create the following piecewise defined expression
\[ \begin{cases} 
1 & x < 0 \\
\frac{1}{x} & 0 < x \\
3 & \text{otherwise}
\end{cases} \]
\[ \text{piecewise}(x<0,1,x>0,1/x,3); \]

26. Take the expression above and create a piecewise defined function out of it.
\[ f := x \rightarrow \text{piecewise}(x < 0, 1, 0 < x, \frac{1}{x}, 3) \]

27. Create the following piecewise defined expression
\[ \begin{cases} 
1 & x < 0 \\
\frac{1}{x} & 0 < x \\
3 & x = 0
\end{cases} \]
\[ \text{piecewise}(x<0,1,x>0,1/x,x=0,3); \]
28. Take the expression above and create a piecewise defined function out of it.

\[ f := x \rightarrow \text{piecewise}(x < 0, 1, x > 0, 1/x, x = 0, 3) \]

29. Create the following piecewise defined expression

\[
\begin{cases}
3 & 1 < t \\
4 & 0 < t \\
2 & -3 < t \\
5 & \text{otherwise}
\end{cases}
\]

\[ \text{piecewise}(t > 1, 3, t > 0, 4, t > -3, 2, 5); \]

30. Take the expression above and create a piecewise defined function out of it.

\[ f := t \rightarrow \text{piecewise}(1 < t, 3, 0 < t, 4, -3 < t, 2, 5) \]

31. Create the following piecewise defined expression

\[
\begin{cases}
t & 1 < t \\
3t - 1 & 0 < t \\
\sin(t) & -3 < t \\
\tan(t) & \text{otherwise}
\end{cases}
\]

\[ \text{piecewise}(t > 1, t, t > 0, 3t - 1, t > -3, \sin(t), \tan(t)); \]

32. Take the expression above and create a piecewise defined function out of it.

\[ f := t \rightarrow \text{piecewise}(1 < t, t, 0 < t, 3t - 1, -3 < t, \sin(t), \tan(t)) \]

33. Create a set containing the numbers: 1, 12, 21, 2, 2, 1, 12, 14, 5, 8, 67, 14, 19, 25.
34. Create a list containing the numbers: 1, 12, 21, 2, 2, 1, 12, 14, 5, 8, 67, 14, 19, 25.

> \{1, 12, 21, 2, 2, 1, 12, 14, 5, 8, 67, 14, 19, 25\};

\{1, 2, 5, 8, 12, 14, 19, 21, 25, 67\}

35. Create a set containing the letters: a, b, d, e, r, s, t, y.

> \{a, b, d, e, r, s, t, y\};

\{a, y, b, s, r, d, e, t\}

36. Define a to be 5, b to be 7, d to be a, e to be x, r to be 15, and y to be x – 5. Now create a set containing the letters: a, b, d, e, r, s, t, y.

> a:=5; b:=7; d:=a; e:=x; r:=15; y:=x-5;

\begin{align*}
a & := 5 \\
b & := 7 \\
d & := 5 \\
e & := x \\
r & := 15 \\
y & := x - 5
\end{align*}

> \{a, b, d, e, r, s, t, y\};

\{5, 7, 15, x, s, t, x - 5\}

37. Create the function \(f(x) = x^3\) and create a list containing the numbers: 1, 12, 21, 2, 2, 1, 12, 14, 5, 8, 67, 14, 19, 25. Now use the map command to evaluate the function at each value in the list.

> lst:=[1, 12, 21, 2, 2, 1, 12, 14, 5, 8, 67, 14, 19, 25];

\(lst := [1, 12, 21, 2, 2, 1, 12, 14, 5, 8, 67, 14, 19, 25]\)

> f:=x->x^3;

\(f := x \rightarrow x^3\)

> map(f, lst);

\[1, 1728, 9261, 8, 8, 1, 1728, 2744, 125, 512, 300763, 2744, 6859, 15625\]
38. Create the function \( f(x) = \sin(x) + \cos(x) \) and create a list containing the numbers: \( 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi \). Now use the map command to evaluate the function at each value in the list.

\[
\text{lst} := [0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi]
\]

\[
\text{map}(f, \text{lst});
\]

\[
\left[ 1, \frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}, 1, -1 \right]
\]

39. Create a list containing the numbers: \( 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi \). Now use the map command to evaluate the sine function at each value in the list.

\[
\text{lst} := [0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi]
\]

\[
\text{map}(\sin, \text{lst});
\]

\[
\left[ 0, \frac{1}{2}, \frac{\sqrt{2}}{2}, 1, 0 \right]
\]

40. Create a list containing the numbers: \( 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi \). Now use the map command to evaluate the cosine function at each value in the list.

\[
\text{lst} := [0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi]
\]

\[
\text{map}(\cos, \text{lst});
\]

\[
\left[ 1, \frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}, 0, -1 \right]
\]

41. Create a list containing the numbers: \( 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi \). Now use the map command to evaluate the tangent function at each value in the list.

\[
\text{lst} := [0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi]
\]
\[ lst := \left[ 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi \right] \]

\[ > \text{map(tan, lst)}; \]
\[ \text{Error, (in tan) numeric exception: division by zero} \]

42. Create the function \( f(x) = \sin(x) + \cos(x) \) and use the eval command to evaluate the function at \( 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2} \) and \( \pi \).

\[ > f := x \rightarrow \sin(x) + \cos(x); \]
\[ \quad f := x \rightarrow \sin(x) + \cos(x) \]

\[ > \text{eval}(f(x), x = 0); \quad 1 \]

\[ > \text{eval}(f(x), x = \pi/6); \quad \frac{1}{2} + \frac{\sqrt{3}}{2} \]

\[ > \text{eval}(f(x), x = \pi/4); \quad \sqrt{2} \]

\[ > \text{eval}(f(x), x = \pi/3); \quad \frac{1}{2} + \frac{\sqrt{3}}{2} \]

\[ > \text{eval}(f(x), x = \pi/2); \quad 1 \]

\[ > \text{eval}(f(x), x = \pi); \quad -1 \]

43. Create the expression \( \sin(x) + \cos(x) \) and use the eval command to evaluate the expression at \( 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2} \) and \( \pi \).

\[ > \text{expr} := \sin(x) + \cos(x); \quad \text{expr} := \sin(x) + \cos(x) \]

\[ > \text{eval}(\text{expr}, x = 0); \quad 1 \]

\[ > \text{eval}(\text{expr}, x = \pi/6); \quad \frac{1}{2} + \frac{\sqrt{3}}{2} \]

\[ > \text{eval}(\text{expr}, x = \pi/4); \quad \sqrt{2} \]
> eval(expr, x=Pi/3);
\[
\frac{1}{2} + \frac{\sqrt{3}}{2}
\]

> eval(expr, x=Pi/2);
1

> eval(expr, x=Pi);
-1

44. Create the expression \( \sin(x) + \cos(x) \) and use the subs command to substitute the values 0, \( \frac{\pi}{6} \), \( \frac{\pi}{4} \), \( \frac{\pi}{3} \), \( \frac{\pi}{2} \), and \( \pi \) into the expression and then use the eval command to do the evaluation.

\[
\text{expr} := \sin(x) + \cos(x)
\]

> expr := sin(x) + cos(x);
\[
\text{expr} := \sin(x) + \cos(x)
\]

> subs(x=0, expr);
\[
\sin(0) + \cos(0)
\]

> eval(%);
1

> subs(x=Pi/6, expr);
\[
\sin\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{6}\right)
\]

> eval(%);
\[
\frac{1}{2} + \frac{\sqrt{3}}{2}
\]

> subs(x=Pi/4, expr);
\[
\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right)
\]

> eval(%);
\[
\sqrt{2}
\]

> subs(x=Pi/3, expr);
\[
\sin\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{3}\right)
\]

> eval(%);
\[
\frac{1}{2} + \frac{\sqrt{3}}{2}
\]

> subs(x=Pi/2, expr);
\[
\sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right)
\]

> eval(\%);
1

> subs(x=Pi,expr);
sin(\pi) + \cos(\pi)

> eval(\%);
-1

45. Find an approximation to \(e\).

> evalf(exp(1));
2.718281828

46. Find an approximation to \(e\) to 25 decimal places.

> evalf(exp(1), 25);
2.718281828459045235360287

47. Find an approximation to \(e\) to 1000 decimal places.

> evalf(exp(1), 1000);
2.718281828459045235360287471352662497757275410196624968724209698677722988207531297945251175891578578317632458612881484889586263236712024821087782570416059366564246845828645408689011433270462578277826947530347530552133278985756801582173445872008090906889398290473311478031453008819532866768662797338855801721468199837516321791884678857762654304856156887246870505514993001624584270248108171129633229182758692367245833454168037828397842591233036112070086201711785765728341875295670957759759597895909734079082347866477487815141010034623425807959181608588690622730977586101785119307225676321787731959842029435436564058990653787593112780625427663818397601107471480865708935414520535396227081859782859713595800524719528169466622918035528160132355935787179673794514567132817392351560010183507629532332876279434907632338298807535195251019009048353185309373425368603525909469705320434587535781404522248977693984964651058209392398294887933203625094431173012381970684161403970198376793206832823764648042953118023272882509819455815301756717361332069811250996181881593041690351598888519345807273866738589422879228499892086805825749279610484198444363463244968487560233624827041978623209002160990235304369941849146314093431738143640546253152096183690888707016768396424378140592714563549061303107208510383750510115747704171898610687396965521267154688957035035

48. Find an approximation to \(\cos(7)\) to 25 decimal places.

> evalf(cos(7), 25);
0.7539022543433046381411975

49. Find an approximation to $\cos(7)$ to 1000 decimal places.

```maple
> evalf(cos(7), 1000);
0.75390225434330463814119752171918201221831339146012683954361388081383
602672071740562524839108930248254141743479465336244452436691757600
677773634784094005915398132109044366092742512070335564704755671959
487483862426385597873134029662556671454485695843294811475380694476
511719415901457416475702615588959636521107072880088409165304766539
939468073222281568801250057494520186725901462919920369923958493832
022890197223831750407272336820347486390787391101362471399665801885
68914089674104888560138597900467444921990698488982869752691427066
8253920293629819166381875671933711281197747724214143661428877047516
4424470025510529066125486677154311642573979712987758007443307926382
152257119683060421728939387788016462807588310534108441945163411354
3004261434966212977382016184892199994906190458677352502179353063141
484500998349434024173513785127906367960432067125242798499422454244
588740797484181825454256435361933725284717216614527123211709782156
753409364275313759120360629451754708155442817706076444996752560082
05448065

50. Find an approximation to $\log(3)$ to 50 decimal places.

```maple
> evalf(log10(3), 50);
0.47712125471966243790397411371442151867218834468978375727343  

In each of the following exercises you are asked to find the solution or solutions to an equation or set of equations. If the answer does not come out in exact form that is understandable use the evalf command to get an approximate solution.

51. Find the solution to the following equation.

$$\sin(x) = \cos(x)$$

```maple
> solve(sin(x)=cos(x), x);
\frac{\pi}{4}

52. Find the solution to the following equation.

$$x^2 - 3x + 2 = 5$$

```maple
> solve(x^2-3*x+2=5, x);
\[
\frac{3}{2} + \frac{\sqrt{21}}{2}, \quad \frac{3}{2} - \frac{\sqrt{21}}{2}
\]

53. Find the solution to the following equation.
\[x^3 - 3x + 2 = 0\]

\[
> \text{solve}(x^3-3*x+2=0, x);
\]

-2, 1, 1

54. Find the solution to the following equation.
\[x^3 - 2x + 2 = 0\]

\[
> \text{solve}(x^3-2*x+2=0, x);
\]

\[
\frac{1}{2} \left( -\frac{1}{2} + \frac{\sqrt{3}}{2} \right) + 1, \quad \frac{1}{2} \left( -\frac{1}{2} - \frac{\sqrt{3}}{2} \right) + 1
\]

\[
\text{evalf}();
\]

-1.769292354, 0.8846461773 - 0.5897428050 I, 0.8846461773 + 0.5897428050 I

55. Find the solution to the following equation.
\[x^5 - 4x^4 + 3x^2 - 2x + 2 = 0\]

\[
> \text{solve}(x^5-4*x^4+3*x^2-2*x+2=0, x);
\]

1, RootOf(_Z^4 - 3 _Z^3 - 3 _Z^2 - 2, index = 1),
\text{RootOf}(_Z^4 - 3 _Z^3 - 3 _Z^2 - 2, index = 2),
\text{RootOf}(_Z^4 - 3 _Z^3 - 3 _Z^2 - 2, index = 3),
\text{RootOf}(_Z^4 - 3 _Z^3 - 3 _Z^2 - 2, index = 4)

\[
\text{evalf}();
\]

1., 3.820988314, 0.1482696193 + 0.6681269671 I, -1.117527553,
0.1482696193 - 0.6681269671 I

56. Find the solution to the following system of equations.
\[3x^2y - 2xy^3 + 2 = 0\]
\[xy - x^2y^2 + 1 = 0\]

\[
> \text{solve}({3*x^2*y-2*x*y^3+2=0, x*y-x^2*y^2+1=0}, \{x, y\});
\]
{y = \text{RootOf}(-3 - 4 \cdot Z^3 + 2 \cdot Z^4 - 2 \cdot Z + 4 \cdot Z^2), x = \frac{2}{3} \text{RootOf}(-3 - 4 \cdot Z^3 + 2 \cdot Z^4 - 2 \cdot Z + 4 \cdot Z^2)^2 - \frac{2}{3} \text{RootOf}(-3 - 4 \cdot Z^3 + 2 \cdot Z^4 - 2 \cdot Z + 4 \cdot Z^2) + \frac{2}{3}}

> \text{evalf}(%);
\{ y = 1.535797111, x = 1.215250437 \}

57. Find the solution to the following system of equations.

\[ 3 x^2 - y^2 = 2 \]
\[ x + y - x y^2 = -1 \]

> \text{solve}\{3x^2-y^2=2,x+y-x y^2=-1\}, \{x, y\} ;
\{ x = \text{RootOf}(3 \cdot Z^2 - 1, \text{label = } _{L7}), y = -1 \}, \{ y = 3 \text{RootOf}(3 \cdot Z^3 - 3 \cdot Z - 1, \text{label = } _{L8})^2 - 2, x = \text{RootOf}(3 \cdot Z^3 - 3 \cdot Z - 1, \text{label = } _{L8}) \}

> \text{evalf}(%);
\{ y = -1., x = 0.5773502693 \}, \{ y = 1.879385245, x = 1.137158043 \}

58. Find the solution to the following system of equations.

\[ 3 x^2 - y^2 + z^2 = 0 \]
\[ x y z - x y^2 = -1 \]
\[ x + y + z = 5 \]

> \text{solve}\{3x^2-y^2+z^2=0,x y z-x y^2=-1,x+y+z=5\}, \{x, y, z\} ;
\{ x = \text{RootOf}(12 \cdot Z^5 - 30 \cdot Z^4 + 75 \cdot Z^3 - 2 \cdot Z^2 + 20 \cdot Z - 50), y = \frac{3}{10} \text{RootOf}(12 \cdot Z^5 - 30 \cdot Z^4 + 75 \cdot Z^3 - 2 \cdot Z^2 + 20 \cdot Z - 50)^2 + \frac{377}{150} + \frac{2}{125} \text{RootOf}(12 \cdot Z^5 - 30 \cdot Z^4 + 75 \cdot Z^3 - 2 \cdot Z^2 + 20 \cdot Z - 50)^4 + \frac{1}{25} \text{RootOf}(12 \cdot Z^5 - 30 \cdot Z^4 + 75 \cdot Z^3 - 2 \cdot Z^2 + 20 \cdot Z - 50)^3 - \frac{377}{750} \text{RootOf}(12 \cdot Z^5 - 30 \cdot Z^4 + 75 \cdot Z^3 - 2 \cdot Z^2 + 20 \cdot Z - 50), z = \}

92
\[
\begin{align*}
\frac{373}{750} & \text{RootOf}(12\_Z^5 - 30\_Z^4 + 75\_Z^3 - 2\_Z^2 + 20\_Z - 50) \\
-\frac{3}{10} & \text{RootOf}(12\_Z^5 - 30\_Z^4 + 75\_Z^3 - 2\_Z^2 + 20\_Z - 50)^2 + \frac{373}{150} \\
-\frac{2}{125} & \text{RootOf}(12\_Z^5 - 30\_Z^4 + 75\_Z^3 - 2\_Z^2 + 20\_Z - 50)^4 \\
-\frac{1}{25} & \text{RootOf}(12\_Z^5 - 30\_Z^4 + 75\_Z^3 - 2\_Z^2 + 20\_Z - 50)^5
\end{align*}
\]

> `evalf(%)`;

\[
\{x = 0.8405662072, z = 1.824916034, y = 2.334517760\}
\]

59. Find the solution to the following system of equations.
\[
\begin{align*}
\sin(x) & = \cos(y) + \frac{1}{2} \\
\cos(y) & = \sin(z) + \frac{1}{2} \\
\sin(z) & = \cos(x) + \frac{1}{3}
\end{align*}
\]

> `solve({\sin(x)=\cos(y)+1/2, \cos(y)=\sin(z)+1/2, \sin(z)=\cos(x)+1/3},{x,y,z});`

\[
\begin{align*}
&\{z = \arcsin\left(\frac{1}{3} + \frac{1}{3} \text{RootOf}(7 + 8\_Z + 2\_Z^2, label = _L13)\right) \\
y & = \arccos\left(\frac{5}{6} + \frac{1}{3} \text{RootOf}(7 + 8\_Z + 2\_Z^2, label = _L13)\right) \\
x & = \arctan\left(\frac{4}{3} + \frac{1}{3} \text{RootOf}(7 + 8\_Z + 2\_Z^2, label = _L13), \frac{1}{3} \text{RootOf}(7 + 8\_Z + 2\_Z^2, label = _L13)\right)\}
\end{align*}
\]

> `evalf(%)`;

\[
\{x = 2.016357581, y = 1.156693307, z = -0.09778684248\}
\]

60. Find the solution to the following system of equations.
\[
\begin{align*}
\sin(x) & = \cos(y) + \frac{1}{2} \\
\cos(y) & = \sin(z) \\
\sin(z) & = \cos(x) + \frac{1}{2}
\end{align*}
\]

> `solve({\sin(x)=\cos(y)+1/2, \cos(y)=\sin(z), \sin(z)=\cos(x)+1/2},{x,y,z});`
\{ x = \frac{\pi}{2}, \, z = \frac{\pi}{6}, \, y = \frac{\pi}{3} \}, \{ x = \pi, \, z = -\frac{\pi}{6}, \, y = \frac{2}{3} \pi \} 

61. Find an approximate solution to the following equation.
\[ \sin(x) = \cos(x) \]

\[ \text{fsolve}(\sin(x)=\cos(x),x); \]
\[ 0.7853981634 \]

62. Find an approximate solution to the following equation that is between 10 and 11.
\[ \sin(x) = \cos(x) \]

\[ \text{fsolve}(\sin(x)=\cos(x),x=10..11); \]
\[ 10.21017612 \]

63. Find an approximate solution to the following equation. Did you get all of the solutions? How do you know?
\[ x^2 - 3x + 2 = 5 \]

\[ \text{fsolve}(x^2-3*x+2=5,x); \]
\[ -0.7912878475, 3.791287847 \]

These are all of the solutions since a quadratic has exactly two complex solutions.

64. Find an approximate solution to the following equation. Did you get all of the solutions? How do you know?
\[ x^3 - 3x + 2 = 0 \]

\[ \text{fsolve}(x^3-3*x+2=0,x); \]
\[ -2., 1., 1. \]

These are all of the solutions since a cubic has exactly three complex solutions.

65. Find an approximate solution to the following equation. Did you get all of the solutions? How do you know?
\[ x^3 - 2x + 1 = 0 \]

\[ \text{fsolve}(x^3-2*x+1=0,x); \]
\[ -1.618033989, 0.6180339887, 1. \]

These are all of the solutions since a cubic has exactly three complex solutions.

66. Find an approximate solution to the following equation. Did you get all of the solutions? How do you know?
\[ x^5 - 4x^4 + 3x^2 - 2x + 2 = 0 \]
> \texttt{fsolve(x^5-4*x^4+3*x^2-2*x+2=0,x);}
> \texttt{-1.117527553, 1., 3.820988314}

These are not all of the solutions since we would expect five in total. It is possible that these are all of the real solutions, we would need to investigate the equation further to determine this.

67. Find an approximate solution to the following system of equations.
\begin{align*}
3 \ x^2 \ y - 2 \ x \ y^3 + 2 &= 0 \\
\ x \ y - x \ y^2 + 1 &= 0
\end{align*}

> \texttt{fsolve}\{3*x^2*y-2*x*y^3+2=0, x*y-x*y^2+1=0\}, \{x,y\};
> \{x = 1.215250437, y = 1.535797111\}

68. Find an approximate solution to the following system of equations.
\begin{align*}
3 \ x^2 - y^2 &= 2 \\
\ x + y - x \ y^2 &= -1
\end{align*}

> \texttt{fsolve}\{3*x^2-y^2=2, x+y-x*y^2=-1\}, \{x,y\};
> \{y = -1.000000000, x = 1.000000000\}

69. Find an approximate solution to the following system of equations.
\begin{align*}
3 \ x^2 - y^2 + z^2 &= 0 \\
\ x \ y \ z - x \ y^2 &= -1 \\
\ x + y + z &= 5
\end{align*}

> \texttt{fsolve}\{3*x^2-y^2+z^2=0, x*y*z-x*y^2=-1, x+y+z=5\}, \{x,y,z\};
> \{x = 0.8405662072, y = 2.334517760, z = 1.824916033\}

70. Find an approximate solution to the following system of equations.
\begin{align*}
\sin(x) &= \cos(y) + \frac{1}{2} \\
\cos(y) &= \sin(z) + \frac{1}{2} \\
\sin(z) &= \cos(x) + \frac{1}{3}
\end{align*}

> \texttt{fsolve}\{\sin(x)=\cos(y)+1/2, \cos(y)=\sin(z)+1/2, \sin(z)=\cos(x)+1/3\}, \{x,y,z\};
> \{y = -1.639886875, z = -6.888517889, x = -9.870339215\}
71. Find an approximate solution to the following system of equations.

\[
\begin{align*}
\sin(x) &= \cos(y) + \frac{1}{2} \\
\cos(y) &= \sin(z) \\
\sin(z) &= \cos(x) + \frac{1}{2}
\end{align*}
\]

\[>\text{fsolve}\{\sin(x)=\cos(y)+1/2, \cos(y)=\sin(z), \\
\sin(z)=\cos(x)+1/2\}, \{x, y, z\}; \]
\n\{x = -9.424777961, y = -8.901179185, z = -2.094395102\}

72. Find an approximate solution to the following system of equations. This time, force the \(x\) value to be between 13 and 15, the \(y\) value between -2 and 0 and that the \(z\) value between 0 and 1.

\[
\begin{align*}
\sin(x) &= \cos(y) + \frac{1}{2} \\
\cos(y) &= \sin(z) \\
\sin(z) &= \cos(x) + \frac{1}{2}
\end{align*}
\]

\[>\text{fsolve}\{\sin(x)=\cos(y)+1/2, \cos(y)=\sin(z), \\
\sin(z)=\cos(x)+1/2\}, \{x=13..15, y=-2..0, z=0..1\}; \]
\n\{x = 14.13716694, y = -1.047197551, z = 0.5235987756\}

73. Substitute the value 5 in for \(x\) in the following expression.

\[x^3 - 2x^2 - x + 1\]

\[>\text{subs}(x=5, x^3-2*x^2-x+1);\]

74. Substitute the expression \(x - h\) in for \(x\) in the following expression.

\[x^3 - 2x^2 - x + 1\]

\[>\text{subs}(x=x-h, x^3-2*x^2-x+1);\]

\[(x - h)^3 - 2 (x - h)^2 - x + h + 1\]

75. Substitute the expression \(3a^2 - 2a + h\) in for \(x\) in the following expression.

\[x^3 - 2x^2 - x + 1\]

\[>\text{subs}(x=3*a^2-2*a+h, x^3-2*x^2-x+1);\]

\[(3 a^2 - 2 a + h)^3 - 2 (3 a^2 - 2 a + h)^2 - 3 a^2 + 2 a - h + 1\]
76. Use the subs command to substitute $y$ in for $x + 1$ into the following expression.

\[ x^3 - x^2 - 5 x - 3 \]

\[ \text{subs}(x+1=y,x^3-x^2-5*x-3); \]

\[ x^3 - x^2 - 5 x - 3 \]

77. Use the algsubs command to substitute $y$ in for $x + 1$ into the following expression. Is there a difference between the output of this command and the output of the subs command used above.

\[ x^3 - x^2 - 5 x - 3 \]

\[ \text{algsubs}(x+1=y,x^3-x^2-5*x-3); \]

\[ -4y^2 + y^3 \]

78. Create a sequence of 20 values that get close to 5 from below.

\[ \text{seq}(5-1/2^n,n=1..20); \]

\[ \text{evalf}(\%); \]

79. Create a sequence of 20 values that get close to $\pi$ from below.

\[ \text{seq}(\pi-1/2^n,n=1..20); \]

\[ \text{evalf}(\%); \]

80. Create a list of 20 values that get close to $\pi$ from below.
81. Use this list in a map command (use evalf on it as well) that evaluates the sine function at each of the list values.

```maple
> lst := [seq(Pi-1/2^n, n=1..20)];
```

```maple
> map(sin, lst);
```

82. Create a sequence of 25 values that increase "without bound".

```maple
> seq(n^n, n=1..25);
```
83. Create a list of 25 values that increase “without bound”.

```maple
> lst := [seq(n^n, n=1..25)];
```

84. Use this list in a map command (use evalf on it as well) that evaluates the arctangent function at each of the list values.

```maple
> map(arctan, lst);```
\[ \begin{bmatrix} \frac{\pi}{4}, \arctan(4), \arctan(27), \arctan(256), \arctan(3125), \arctan(46656), \arctan(823543), \\
\arctan(16777216), \arctan(387420489), \arctan(1000000000), \\
\arctan(285311670611), \arctan(8916100448256), \arctan(302875106592253), \\
\arctan(1111200682558016), \arctan(437893890380859375), \\
\arctan(18446744073709551616), \arctan(82724026188636764177), \\
\arctan(39346408075296537575424), \arctan(1978419655660313589123979), \\
\arctan(1048576000000000000000000000000000000), \arctan(5842587018385982521381124421), \\
\arctan(34142787736421955736646723584), \arctan(20880467999847912034355032910567), \\
\arctan(1333735776850284124449081472843776), \\
\arctan(8881784197001252323389053344726525) \end{bmatrix} \]

> `evalf(%)`

\[ [0.7853981635, 1.325817664, 1.533776211, 1.566890097, 1.570476327, 1.570774893, \\
1.570795113, 1.570796267, 1.570796324, 1.570796327, 1.570796327, 1.570796327, \\
1.570796327, 1.570796327, 1.570796327, 1.570796327, 1.570796327, 1.570796327, \\
1.570796327, 1.570796327, 1.570796327, 1.570796327, 1.570796327, 1.570796327, \\
1.570796327, 1.570796327, 1.570796327, 1.570796327, 1.570796327, 1.570796327, \\
1.570796327] \]

85. Plot the function \( \tan(x) \) on the interval \([-10, 10]\) without using any options.

\[ \text{plot}(\tan(x), x=-10..10); \]

86. Plot the function \( \tan(x) \) on the interval \([-10, 10]\) using the discont=true option.

\[ \text{plot}(\tan(x), x=-10..10, \text{discont}=\text{true}); \]
87. Plot the function \( \tan(x) \) on the interval \([-10, 10]\) using the \texttt{discont=true} option and set the color of the graph to green.

\[
\texttt{plot(tan(x),x=-10..10,discont=true,color=green);}
\]

88. Plot the function \( \tan(x) \) on the interval \([-10, 10]\) using the \texttt{discont=true} option, set the color of the graph to black and restrict the vertical axis to be between \(-5\) and \(5\).

\[
\texttt{plot(tan(x),x=-10..10,y=-5..5,discont=true,color=black);}
\]
89. Plot the four functions
\[ x^2 \quad x^3 \quad x^4 \quad x^5 \]
on the interval \([-5, 5]\), set the color of the graphs to red, green, blue and black respectively. Furthermore restrict the vertical axis to a value that gives you a nice image.

\[
> \text{plot}([x^2,x^3,x^4,x^5],x=-5..5,y=-50..50,\text{color}=[\text{red, green, blue, black}]);
\]

90. Plot the function
\[ \sin(\cos(\tan(x))) \]
on the interval \([0, \pi]\) without options.

\[
> \text{plot} (\sin(\cos(\tan(x))),x=0..\pi);
\]
91. Plot the function

\[ \sin(\cos(\tan(x))) \]

on the interval \([0, \pi]\) using 1000 points for the graph.

\[ > \text{plot} (\text{sin}(\text{cos}(\text{tan}(x))), x=0..\text{Pi}, \text{numpoints}=1000); \]

92. Plot the function

\[ \sin(\cos(\tan(x))) \]

on the interval \([0, \pi]\) using 10000 points for the graph.

\[ > \text{plot} (\text{sin}(\text{cos}(\text{tan}(x))), x=0..\text{Pi}, \text{numpoints}=10000); \]
93. Plot the following piecewise defined function on the interval \([-5, 5]\).
\[
\begin{cases}
1 & \quad x \leq 0 \\
\frac{1}{x} & \quad \text{otherwise}
\end{cases}
\]
\[
> \text{plot}(f(x), x=-5..5, y=-1..5, \text{discont=true});
\]

94. Plot the following piecewise defined function on the interval \([-5, 5]\).
\[
\begin{cases}
1 & \quad x < 0 \\
\frac{1}{x} & \quad 0 < x \\
3 & \quad \text{otherwise}
\end{cases}
\]
\[
> \text{plot}(f(x), x=-5..5, y=-1..5, \text{discont=true});
\]

95. Plot the following piecewise defined function on the interval \([-5, 5]\).
\[
\begin{cases}
3 & \quad 1 < t \\
4 & \quad 0 < t \\
2 & \quad -3 < t \\
5 & \quad \text{otherwise}
\end{cases}
\]
\[
> \text{plot}(f(x), x=-5..5, y=-1..6, \text{discont=true});
\]
96. Plot the following piecewise defined function on the interval [-5, 5].
\[
\begin{align*}
\text{if } & t < 0 \\
\text{then } & 3t - 1 \\
\text{if } & 0 < t < 1 \\
\text{then } & \sin(t) \\
\text{if } & t > 1 \\
\text{then } & \tan(t)
\end{align*}
\]

\> plot(f(t), t=-5..5, y=-6..6, discont=true);

97. Use the `implicitplot` command to graph the equation
\[ x^2 y^2 - x y^3 = 3 \]
over the region [-5, 5] X [-5, 5].

\> with(plots):
\> implicitplot(x^2*y^2-x*y^3=3, x=-5..5, y=-5..5);
98. Use the implicitplot command to graph the equation 
\[ x^2 y^2 - x y^3 = 3 \]
over the region \([-5, 5] \times [-5, 5]\) and using a 50 X 50 grid.

```maple
> with(plots):
> implicitplot(x^2*y^2-x*y^3=3,x=-5..5,y=-5..5,
grid=[50,50]);
```

99. Use the implicitplot command to graph the equation 
\[ x^2 y^2 - x y^3 = 3 \]
over the region \([-5, 5] \times [-5, 5]\), using a 50 X 50 grid and make the color of the graph blue.

```maple
> with(plots):
> implicitplot(x^2*y^2-x*y^3=3,x=-5..5,y=-5..5,
grid=[50,50],color=blue);
```
100. Use the implicitplot command to graph the equations
\[ \sin(x^2) = \cos(y) \]
\[ y^3 = x^2 + x \]
together over the region \([-3, 3] \times [-3, 3]\), use an appropriate color and grid.

```
> implicitplot({sin(x^2)=cos(y),y^3=x^2+x},x=-3..3,y=-3..3,grid=[50,50],color=black);
```

101. Plot the four functions
\[ x^2 \quad x^3 \quad x^4 \quad x^5 \]
on the interval \([-\infty, \infty]\).

```
> plot([x^2,x^3,x^4,x^5],x=-infinity..infinity,color=[red,green,blue,black]);
```
102. Plot the arctangent function on the interval $[-\infty, \infty]$.

```maple
> plot(arctan(x), x=-infinity..infinity, y=-3..3);
```

103. Plot the logarithm functions with bases 2, 3, 5, 10 and 20 on the interval $[-\infty, \infty]$.

```maple
> plot([log[2](x), log[3](x), log[5](x), log10(x), log[20](x)], x=-infinity..infinity);
```

104. Plot the root functions with indexes 2, 3, 5, 10 and 20 on the interval $[-\infty, \infty]$.

```maple
> plot([sqrt(x), surd(x,3), surd(x,5), surd(x,10), surd(x,20)], x=-infinity..infinity);
```
105. Plot the function $1/x$ on the interval $[-\infty, \infty]$.

> `plot(1/x, x=-infinity..infinity);`

106. Plot the function $\frac{2x - 3}{x + 7}$ on the interval $[-\infty, \infty]$.

> `plot((2*x-3)/(x+7), x=-infinity..infinity);`

107. Plot the following four functions using the logplot

$x^2$, $x^3$, $x^4$, $x^5$
108. Plot the following four functions using the loglogplot.

\[ \text{loglogplot}([x^2, x^3, x^4, x^5], x=0.001..100, y=0.001..100); \]

109. Plot the root functions with indices 2, 3, 5, 10 and 20 using the logplot.

\[ \text{logplot}([\sqrt{x}, \sqrt[3]{x}, \sqrt[5]{x}, \sqrt[10]{x}, \sqrt[20]{x}], x=0..5, y=0.1..5); \]
110. Plot the root functions with indices 2, 3, 5, 10 and 20 using the loglogplot.

> loglogplot([sqrt(x), surd(x,3), surd(x,5), surd(x,10), surd(x,20)], x=0.01..5, y=0.1..5);

111. Plot the exponential functions with bases 2, 3, 5, 10 and 20 using the logplot.

> logplot([2^x, 3^x, 5^x, 10^x, 20^x], x=-5..5, y=0.1..5);

112. Plot the exponential functions with bases 2, 3, 5, 10 and 20 using the loglogplot.

> loglogplot([2^x, 3^x, 5^x, 10^x, 20^x], x=0.001..5, y=0.1..100);
113. Use the display command to join the plots of the implicit plot
\[ x^2 y^2 - x y^3 = 3 \]
and the plot of
\[ \sin(\cos(\tan(x))) \]
on the interval \([0, \pi]\)

\[ \text{with(plots):} \]
\[ a:=\text{implicitplot}(x^2*y^2-x*y^3=3, x=0..\Pi, y=-3..3, \]
\[ \text{grid=[50,50], color=black}): \]
\[ b:=\text{plot}(\sin(\cos(\tan(x))), x=0..\Pi): \]
\[ \text{display}(a,b); \]

114. Use the display command to join the plots of the implicit plot
\[ x^2 y^2 - x y^3 = 3 \]
and the plot of
\[ \begin{align*}
 t & \quad 1 < t \\
 3t - 1 & \quad 0 < t \\
 \sin(t) & \quad -3 < t \\
 \tan(t) & \quad \text{otherwise}
\end{align*} \]
on the interval \([-5, 5]\)

\[ \text{with(plots):} \]
\[ a:=\text{implicitplot}(x^2*y^2-x*y^3=3, x=-5..5, y=-3..3, \]
\[ \text{grid=[50,50], color=black}): \]
\[ f:=t\rightarrow\text{piecewise}(t>1, t, t>0, 3*t-1, t>-3, \sin(t), \tan(t)); \]
\[ f:=t \rightarrow \text{piecewise}(1 < t, t, 0 < t, 3t - 1, -3 < t, \sin(t), \tan(t)) \]
\[ b:=\text{plot}(f(t), t=-5..5, y=-3..3, \text{discont=true}): \]
\[ \text{display}(a,b); \]
115. Use the display command to join the plots of

\[
\begin{align*}
&\begin{cases}
  t & 1 < t \\
  3t - 1 & 0 < t \\
  \sin(t) & -3 < t \\
  \tan(t) & \text{otherwise}
\end{cases} \\
\end{align*}
\]

and

\[
\begin{align*}
&\begin{cases}
  3 & 1 < t \\
  4 & 0 < t \\
  2 & -3 < t \\
  5 & \text{otherwise}
\end{cases}
\end{align*}
\]

> with(plots):
> f := t -> piecewise(t>1, t, t>0, 3*t-1, t>-3, sin(t), tan(t));
> f := t -> piecewise(1 < t, t, 0 < t, 3*t - 1, -3 < t, sin(t), tan(t))
> g := t -> piecewise(t>1, 3, t>0, 4, t>-3, 2, 5);
> g := t -> piecewise(1 < t, 3, 0 < t, 4, -3 < t, 2, 5)
> a := plot(g(t), t=-5..5, y=-6..6, discont=true, color=black):
> b := plot(f(t), t=-5..5, y=-6..6, discont=true):
> display(a,b);

116. Find the following limit,
\[ \lim_{x \to \pi} \sin(x) \]

> \textbf{limit} (\sin(x), x=\text{Pi});
0

117. Find the following limit,
\[ \lim_{x \to \left(\frac{2\pi}{3}\right)} \cos(x) - \sin(x) \]

> \textbf{limit} (\cos(x)-\sin(x), x=2*\text{Pi}/3);
\[-\frac{1}{2} - \frac{\sqrt{3}}{2}\]

118. Find the following limit,
\[ \lim_{x \to \infty} \frac{x - 2}{3 - x} \]

> \textbf{limit} ((x-2)/(3-x), x=\text{infinity});
-1

119. Find the following limit,
\[ \lim_{x \to \left(\frac{\pi}{2}\right)^+} \tan(x) \]

> \textbf{limit} (\tan(x), x=\text{Pi}/2, \text{right});
-\infty

120. Find the following limit,
\[ \lim_{x \to \left(\frac{\pi}{2}\right)^-} \tan(x) \]

> \textbf{limit} (\tan(x), x=\text{Pi}/2, \text{left});
\infty

121. Create the following display,
\[ \lim_{x \to \pi} \sin(x) \]

> \textbf{Limit} (\sin(x), x=\text{Pi});
\[ \lim_{x \to \pi} \sin(x) \]

122. Create the following display,
\[ \lim_{x \to \left(\frac{2\pi}{3}\right)} \cos(x) - \sin(x) \]
> \texttt{Limit(cos(x)-sin(x),x=2*Pi/3)};
\texttt{lim_{x \to \left(\frac{2\pi}{3}\right)} cos(x) - sin(x)}

123. Create the following display,
\[
\lim_{x \to \infty} \frac{x - 2}{3 - x}
\]

> \texttt{Limit((x-2)/(3-x),x=infinity)};
\texttt{lim_{x \to \infty} \frac{x - 2}{3 - x}}

124. Create the following display,
\[
\lim_{x \to \left(\frac{\pi}{2}\right)^+} \tan(x)
\]

> \texttt{Limit(tan(x),x=Pi/2,right)};
\texttt{lim_{x \to \left(\frac{\pi}{2}\right)^+} \tan(x)}

125. Create the following display,
\[
\lim_{x \to \left(\frac{\pi}{2}\right)^-} \tan(x)
\]

> \texttt{Limit(tan(x),x=Pi/2,left)};
\texttt{lim_{x \to \left(\frac{\pi}{2}\right)^-} \tan(x)}

126. Find the following derivative using the \texttt{diff} command,
\[
\frac{d}{dx} \cos(x)
\]

> \texttt{diff(cos(x),x)};
\texttt{-sin(x)}

127. Find the following derivative using the \texttt{diff} command,
\[
\frac{d}{dx} \left(\frac{1}{x^2 - 3x + 1}\right)
\]

> \texttt{diff(1/(x^2-3*x+1),x)};
\texttt{-\frac{2x - 3}{(x^2 - 3x + 1)^2}}

128. Find the following derivative using the \texttt{diff} command,
\[
\frac{d}{dx} \arctan(x)
\]

> \texttt{diff(arctan(x),x)};
\[
\frac{1}{1 + x^2}
\]

129. Find the following derivative using the \texttt{diff} command,

\[
\frac{d^2}{dx^2} \tan(x)
\]

> \texttt{diff(tan(x),x,x)};
\[
2 \tan(x) (1 + \tan(x)^2)
\]

130. Find the following derivative using the \texttt{diff} command,

\[
\frac{d^7}{dx^7} \tan(x)
\]

> \texttt{diff(tan(x),x$7$)};
\[
1824 (1 + \tan(x)^2)^2 \tan(x)^4 + 2880 (1 + \tan(x)^2)^3 \tan(x)^2 + 272 (1 + \tan(x)^2)^4
\]
\[
+ 64 \tan(x)^6 (1 + \tan(x)^2)
\]

131. Create the following display using the \texttt{Diff} command,

\[
\frac{d}{dx} \cos(x)
\]

> \texttt{Diff(cos(x),x)};
\[
\frac{d}{dx} \cos(x)
\]

132. Create the following display using the \texttt{Diff} command,

\[
\frac{d}{dx} \left( \frac{1}{x^2 - 3x + 1} \right)
\]

> \texttt{Diff(1/(x$^2$-3*x+1),x)};
\[
\frac{d}{dx} \left( \frac{1}{x^2 - 3x + 1} \right)
\]

133. Create the following display using the \texttt{Diff} command,

\[
\frac{d}{dx} \arctan(x)
\]

> \texttt{Diff(arctan(x),x)};
134. Create the following display using the Diff command,
\[
\frac{d}{dx} \tan(x)
\]
\[\text{Diff}(\tan(x),x,x);
\]
\[
\frac{d^2}{dx^2} \tan(x)
\]

135. Create the following display using the Diff command,
\[
\frac{d^7}{dx^7} \tan(x)
\]
\[\text{Diff}(\tan(x),x$7$);
\]
\[
\frac{d^7}{dx^7} \tan(x)
\]

136. Find the following derivative using the D command,
\[
\frac{d}{dx} \cos(x)
\]
\[\text{D}(\cos);
\]
\[
-\sin
\]

137. Find the following derivative using the D command,
\[
\frac{d}{dx} \left( \frac{1}{x^2 - 3x + 1} \right)
\]
\[\text{f:=x->1/(x^2-3*x+1)};
\]
\[f:= x \rightarrow \frac{1}{x^2 - 3x + 1}
\]
\[\text{D}(f);
\]
\[x \rightarrow -\frac{2x - 3}{(x^2 - 3x + 1)^2}
\]

138. Find the following derivative using the D command,
\[
\frac{d}{dx} \arctan(x)
\]
\[\text{D}(\arctan);
\]
\[ a \rightarrow \frac{1}{a^2 + 1} \]

139. Find the following derivative using the D command,
\[ \frac{d^2}{dx^2} \tan(x) \]

\[ > \text{(D@@2)}(\tan); \]
\[ 2 (1 + \tan^2) \tan \]

140. Find the following derivative using the D command,
\[ \frac{d^7}{dx^7} \tan(x) \]

\[ > \text{(D@@7)}(\tan); \]
\[ 64 (1 + \tan^2) \tan^6 + 1824 (1 + \tan^2)^2 \tan^4 + 2880 (1 + \tan^2)^3 \tan^2 + 272 (1 + \tan^2)^4 \]

141. Find \( \frac{dy}{dx} \) where
\[ 4 x^2 y - 3 y x^3 + \frac{x}{y} = 7 x y \]

\[ > \text{implicitdiff}(4*x^2*y-3*x*y^3+x/y=7*x*y,y,x); \]
\[ -y \left( 8 x y^2 - 3 y^4 + 1 - 7 y^2 \right) \]
\[ x \left( 4 x y^2 - 9 y^4 - 1 - 7 y^2 \right) \]

142. Find \( \frac{dy}{dx} \) where
\[ \frac{\sin(\sin(x) \cos(y))}{\cos(\cos(x) \sin(y))} \]

\[ > \text{implicitdiff}((\sin(\sin(x) \cos(y))) / (\cos(\cos(x) \sin(y))),y,x); \]
\[ -\left( -\cos(\sin(x) \cos(y)) \cos(\cos(x) \sin(y)) \cos(x) \cos(y) \right. \]
\[ + \sin(\sin(x) \cos(y)) \sin(\cos(x) \sin(y)) \sin(x) \sin(y) / \left( \cos(\sin(x) \cos(y)) \cos(\cos(x) \sin(y)) \sin(x) \sin(y) \right. \]
\[ - \sin(\sin(x) \cos(y)) \sin(\cos(x) \sin(y)) \cos(x) \cos(y) \left) \right) / \right) \right) \]

143. Find \( \frac{dx}{dy} \) where
\[ 4 x^2 y - 3 y x^3 + \frac{x}{y} = 7 x y \]
\[ \text{implicitdiff}(4x^2y - 3xy^3 + x/y = 7xy, x, y); \]
\[ x(-4xy^2 + 9y^4 + 1 + 7y^2) \]
\[ y(-8xy^2 + 3y^4 - 1 + 7y^2) \]

144. Find \( \frac{dx}{dy} \) where
\[ \frac{\sin(\sin(x) \cos(y))}{\cos(\cos(x) \sin(y))} \]

\[ \text{implicitdiff}((\sin(\sin(x)) \cos(y))/(\cos(\cos(x) \sin(y))), x, y); \]
\[ -\cos(\sin(x) \cos(y)) \cos(\cos(x) \sin(y)) \sin(x) \sin(y) \]
\[ -\sin(\sin(x) \cos(y)) \sin(\cos(x) \sin(y)) \cos(x) \cos(y) \]
\[ + \sin(\sin(x) \cos(y)) \sin(\cos(x) \sin(y)) \sin(x) \sin(y) \]

145. Find the following integral,
\[ \int 7x^3 - 13x^2 + 5x - 6 \, dx \]
\[ \text{int}(7x^3 - 13x^2 + 5x - 6, x); \]
\[ \frac{7}{4}x^4 - \frac{13}{3}x^3 + \frac{5}{2}x^2 - 6x \]

146. Find the following integral,
\[ \int 3^x - x^3 \, dx \]
\[ \text{int}(3^x - x^3, x); \]
\[ \frac{3^x}{\ln(3)} - \frac{x^4}{4} \]

147. Find the following integral,
\[ \int \sin(x) \sqrt{\cos(x)} \, dx \]
\[ \text{int}((\sin(x)) \)*sqrt((\cos(x)), x); \]
\[ \frac{2}{3} \cos(x)^{3/2} \]

148. Display the following integral,
\[ \int 7x^3 - 13x^2 + 5x - 6 \, dx \]
\[ \texttt{> Int}(7\times^3-13\times^2+5\times-6,\times); \]
\[ \int 7x^3 - 13x^2 + 5x - 6 \, dx \]

149. Display the following integral,
\[ \int 3^x - x^3 \, dx \]

\[ \texttt{> Int}(3^x-x^3,\times); \]
\[ \int 3^x - x^3 \, dx \]

150. Display the following integral,
\[ \int \sin(x) \sqrt{\cos(x)} \, dx \]

\[ \texttt{> Int} (\sin(\times)) * \text{sqrt} (\cos(\times)),\times); \]
\[ \int \sin(x) \sqrt{\cos(x)} \, dx \]

151. Use the changevar command to do the substitution
\[ u = \cos(x) \]
in the integral.
\[ \int \sin(x) \sqrt{\cos(x)} \, dx \]

\[ \texttt{> with} (\text{student}); \]
\[ \texttt{> changevar} (u=\cos(\times), \text{Int} (\sin(\times)) * \text{sqrt} (\cos(\times)), \times, u); \]
\[ \int -\sqrt{u} \, du \]

152. Use the changevar command to do the substitution
\[ u = \arctan(x) \]
in the integral.
\[ \int \frac{\arctan(x)^5}{1 + x^2} \, dx \]

\[ \texttt{> with} (\text{student}); \]
\[ \texttt{> changevar} (u=\arctan(\times), \text{Int} (\arctan(\times)^5/(1+\times^2)), \times, u); \]
\[ \int u^5 \, du \]
153. Use the distance command to find the distance between the points \((17,5)\) and \((-3,2)\).

\[
\text{with(student):} \\
\text{distance([17, 5], [-3, 2]);} \\
\sqrt{409}
\]

154. Use the distance command to find the distance between the points \((17,5,4)\) and \((-3,2,-5)\).

\[
\text{with(student):} \\
\text{distance([17, 5, 4], [-3, 2, -5]);} \\
7 \sqrt{10}
\]

155. Use the distance command to find the distance between the points \((a,b,c)\) and \((1,2,3)\).

\[
\text{with(student):} \\
\text{distance([a, b, c], [1, 2, 3]);} \\
\sqrt{(a - 1)^2 + (b - 2)^2 + (c - 3)^2}
\]

156. Use the intercept command to find the roots of the equation \(y = x^3 - x^2\)

\[
\text{with(student):} \\
\text{intercept(y=x^3-x^2,y=0);} \\
\{x = 1, y = 0\}, \{y = 0, x = 0\}, \{y = 0, x = 0\}
\]

157. Use the intercept command to find the intersection of the functions \(y = x^3 - x^2\) and \(y = \frac{1}{100}\)

\[
\text{with(student):} \\
\text{intercept(y=x^3-x^2,y=1/100);} \\
\{y = \frac{1}{100}, x = \text{RootOf(-1 + 100 \_Z^3 - 100 \_Z^2, label = _L1 )}\}
\]

\[
\text{evalf(\%);} \\
\{y = 0.01000000000, x = 1.009806714\}
\]

158. Use the intercept command to find the intersection of the functions \(y = x^3 - x^2\) and \(y = \frac{1}{100}\)

\[
\text{with(student):} \\
\text{intercept(y=x^3-x^2,y=1/100);} \\
\{y = \frac{1}{100}, x = \text{RootOf(-1 + 100 \_Z^3 - 100 \_Z^2, label = _L1 )}\}
\]

\[
\text{evalf(\%);} \\
\{y = 0.01000000000, x = 1.009806714\}
\]
\( y = x^2 - 1 \)

\[ \text{with(student):} \]
\[ \text{intercept}(y=x^3-x^2, y=x^2-1); \]
\[ \{ x = 1, y = 0 \}, \]
\[ \{ x = \text{RootOf}(\_Z^2 - \_Z - 1, \text{label} = \_L3), y = \text{RootOf}(\_Z^2 - \_Z - 1, \text{label} = \_L3) \} \]
\[ \text{evalf}({}); \]
\[ \{ x = 1., y = 0. \}, \{ x = 1.618033989, y = 1.618033989 \} \]

159. Use the midpoint command to find the midpoint between the points \((17,5)\) and \((-3,2)\).

\[ \text{with(student):} \]
\[ \text{midpoint}([17,5],[-3,2]); \]
\[ \left[ \frac{7}{2}, \frac{7}{2} \right] \]

160. Use the midpoint command to find the midpoint between the points \((17,5,4)\) and \((-3,2,-5)\).

\[ \text{with(student):} \]
\[ \text{midpoint}([17,5,4],[-3,2,-5]); \]
\[ \left[ 7, \frac{7}{2}, \frac{-1}{2} \right] \]

161. Use the midpoint command to find the midpoint between the points \((a,b,c)\) and \((1,2,3)\).

\[ \text{with(student):} \]
\[ \text{midpoint}([a,b,c],[1,2,3]); \]
\[ \left[ \frac{a}{2} + \frac{1}{2}, \frac{b}{2} + 1, \frac{c}{2} + \frac{3}{2} \right] \]

162. Use the showtangent command to graph the tangent line to the curve \( y = \sin(x) \) at \( x = \frac{\pi}{4} \).

\[ \text{with(student):} \]
\[ \text{showtangent} (\sin(x), x=\pi/4); \]
163. Use the showtangent command to graph the tangent line to the curve

\[ y = \sin(3x) - \cos(2x) \]

at \( x = \frac{\pi}{6} \).

\[ \text{with(student):} \]
\[ \text{showtangent(sin(3*x)-cos(2*x),x=Pi/6,view=[-2..2,-5..5]);} \]

164. Use the showtangent command to graph the tangent line to the curve

\[ y = x^3 - 5x^2 + 3x - 1 \]

at \( x = 2 \). Restrict the \( x \) range to be between –2 and 5 and restrict the \( y \) range to be between –10 and 10.

\[ \text{with(student):} \]
\[ \text{showtangent(x^3-5*x^2+3*x-1,x=2,view=[-2..5,-10..10],numpoints=500);} \]
165. Use the AntiderivativePlot command to graph the antiderivative of 
\[ y = x^3 - 5x^2 + 3x - 1 \]

\[
\begin{align*}
> & \text{with(Student[Calculus1])}: \\
> & f := x \rightarrow x^3 - 5x^2 + 3x - 1; \\
> & \quad f := x \rightarrow x^3 - 5x^2 + 3x - 1 \\
> & \text{AntiderivativePlot}(f(x), x = -4..4);
\end{align*}
\]

166. Use the AntiderivativePlot command to graph the antiderivative of 
\[ y = \sin(3x) - \cos(2x) \]

\[
\begin{align*}
> & \text{with(Student[Calculus1])}: \\
> & g := x \rightarrow \sin(3x) - \cos(2x); \\
> & \quad g := x \rightarrow \sin(3x) - \cos(2x) \\
> & \text{AntiderivativePlot}(g(x), x = -4..4);
\end{align*}
\]
167. Use the AntiderivativePlot command to graph the antiderivative of 
\[ y = x^3 - 5x^2 + 3x - 1 \]
but restrict the left hand endpoint to be at the height of 7.

\[ \text{with(Student[Calculus1])::} \]
\[ f := x \rightarrow x^3 - 5x^2 + 3x - 1; \]
\[ \text{AntiderivativePlot}(f(x), x=-2..2, \text{value}=7); \]

168. Use the AntiderivativePlot command to graph the antiderivative of 
\[ y = \sin(3x) - \cos(2x) \]
that passes through the point (1,0).

\[ \text{with(Student[Calculus1])::} \]
\[ g := x \rightarrow \sin(3x) - \cos(2x); \]
\[ \text{AntiderivativePlot}(g(x), x=-4..4, \text{value}=[0,1]); \]
169. Use the AntiderivativePlot command to graph the family of antiderivatives of
\[ y = x^3 - 5x^2 + 3x - 1 \]

\[
\begin{align*}
> & \text{with(Student[Calculus1]):} \\
> & f := x \rightarrow x^3 - 5x^2 + 3x - 1; \\
> & \quad f := x \rightarrow x^3 - 5x^2 + 3x - 1 \\
> & \text{AntiderivativePlot}(f(x), x=-2..2, \text{showclass});
\end{align*}
\]

170. Use the AntiderivativePlot command to graph the family of antiderivatives of
\[ y = \sin(3x) - \cos(2x) \]

\[
\begin{align*}
> & \text{with(Student[Calculus1]):} \\
> & g := x \rightarrow \sin(3x) - \cos(2x); \\
> & \quad g := x \rightarrow \sin(3x) - \cos(2x) \\
> & \text{AntiderivativePlot}(g(x), x=-4..4, \text{showclass});
\end{align*}
\]

126
171. Use the DerivativePlot command to graph the derivative of
\[ y = x^3 - 5x^2 + 3x - 1 \]

\[
> \text{with(Student[Calculus1]):}
> f := x -> x^3 - 5x^2 + 3x - 1;
> f := x \mapsto x^3 - 5 x^2 + 3 x - 1
> \text{DerivativePlot}(f(x), x=-5..10);
\]

172. Use the DerivativePlot command to graph the derivative of
\[ y = \sin(3x) - \cos(2x) \]

\[
> \text{with(Student[Calculus1]):}
> g := x -> \sin(3x) - \cos(2x);
> g := x \mapsto \sin(3x) - \cos(2x)
> \text{DerivativePlot}(g(x), x=-4..4);
\]
173. Use the DerivativePlot command to graph the derivative of order two of
\[ y = x^3 - 5x^2 + 3x - 1 \]

\[
> \text{with(Student[Calculus1])}: \\
> f := x -> x^3 - 5x^2 + 3x - 1; \\
> f := x \mapsto x^3 - 5x^2 + 3x - 1 \\
> \text{DerivativePlot}(f(x), x=-5..10, \text{order}=2);
\]

174. Use the DerivativePlot command to graph the derivative of order three of
\[ y = \sin(3x) - \cos(2x) \]

\[
> \text{with(Student[Calculus1])}: \\
> g := x -> \sin(3x) - \cos(2x); \\
> g := x \mapsto \sin(3x) - \cos(2x) \\
> \text{DerivativePlot}(g(x), x=-4..4, \text{order}=3);
\]
175. Use the DerivativePlot command to graph the derivative of orders one through four of
\[ y = x^3 - 5x^2 + 3x - 1 \]

\[
\begin{align*}
> & \text{with(Student[Calculus1])}: \\
> & f := x \rightarrow x^3 - 5x^2 + 3x - 1; \\
& \quad f := x \mapsto x^3 - 5x^2 + 3x - 1 \\
> & \text{DerivativePlot}(f(x), x = -2..6, \text{order}=1..4, \text{view}=[-2..6,-20..20]);
\end{align*}
\]

176. Use the DerivativePlot command to graph the derivative of orders one through six of
\[ y = \sin(3x) - \cos(2x) \]

\[
\begin{align*}
> & \text{with(Student[Calculus1])}: \\
> & g := x \rightarrow \sin(3x) - \cos(2x); \\
& \quad g := x \mapsto \sin(3x) - \cos(2x) \\
> & \text{DerivativePlot}(g(x), x = -4..4, \text{order}=1..6);
\end{align*}
\]
177. Use the InversePlot command to graph the inverse of 
\[ y = x^3 - 5x^2 + 3x - 1 \]

```maple
> with(Student[Calculus1]):
> f:=x->x^3-5*x^2+3*x-1;
f:= x → x^3 - 5x^2 + 3x - 1
> InversePlot(f(x), x=-10..10,view=[-10..10,-10..10]);
```

178. Use the InversePlot command to graph the inverse of 
\[ y = \sin(3x) - \cos(2x) \]

```maple
> with(Student[Calculus1]):
> g:=x->sin(3*x)-cos(2*x);
g := x → sin(3x) - cos(2x)
> InversePlot(g(x), x=-4..4,view=[-4..4,-4..4]);
```
179. Use the MeanValueTheorem command to get a graphical image of the Mean Value Theorem as it applies to 
\[ y = x^3 - 5x^2 + 3x - 1 \]
on the interval \([0, 5]\). Find the exact values of the tangent points and their numerical approximations.

\[
> \text{with(Student[Calculus1])}:
> f := x \rightarrow x^3 - 5x^2 + 3x - 1;
> \text{MeanValueTheorem}(f(x), x=0..5);
\]
\[
> \text{MeanValueTheorem}(f(x), x=0..5, \text{output}=\text{points});
\]
\[
> \text{evalf}(\%);
\]

180. Use the MeanValueTheorem command to get a graphical image of the Mean Value Theorem as it applies to 
\[ y = x^3 - 5x^2 + 3x - 1 \]
on the interval \([-2, 2]\). Find the exact values of the tangent points and their numerical approximations.
\[ \text{MeanValueTheorem}(f(x), x = -2..2); \]

\[
\begin{bmatrix}
5 \\
3 - \sqrt{37}/3
\end{bmatrix}
\]

\[ \text{evalf}(%); \]

\[ [-0.360920843] \]

181. Use the MeanValueTheorem command to get a graphical image of the Mean Value Theorem as it applies to 
\[ y = x^3 - 5x^2 + 3x - 1 \]
on the interval \([-5, 5]\). Find the exact values of the tangent points and their numerical approximations.

\[ \text{MeanValueTheorem}(f(x), x = -5..5); \]
182. Use the MeanValueTheorem command to get a graphical image of the Mean Value Theorem as it applies to
\[ y = \sin(3x) - \cos(2x) \]
on the interval \([-2, 2]\).

\[
> \text{evalf}(%);
\]
\[
[ -1.666666667 ]
\]

183. Use the MeanValueTheorem command to get a graphical image of the Mean Value Theorem as it applies to
\[ y = \sin(3x) - \cos(2x) \]
on the interval \([0, 5]\).
184. Use the NewtonsMethod command to get a graphical image of Newton’s Method for
\[ y = x^3 - 5x^2 + 3x - 1 \]
using an initial value of 4. Find the root approximation. Obtain the sequence of values from Newton’s Method up to 10 iterations.

```maple
> with(Student[Calculus1]):
f:=x->x^3-5*x^2+3*x-1;
of := x → x^3 - 5 x^2 + 3 x - 1
> NewtsMethod(f(x),x=4);
4.365230013
> NewtsMethod(f(x),x=4,output=sequence,iterations=10);
4, 4.454545454, 4.368900401, 4.365236599, 4.365230015, 4.365230014, 4.365230015, 4.365230014, 4.365230015
```

185. Use the NewtonsMethod command to get a graphical image of Newton’s Method for
\[ y = \sin(3x) - \cos(2x) \]
using an initial value of 3. Find the root approximation. Obtain the sequence of values from Newton’s Method up to 10 iterations.

\[
\begin{align*}
> & \text{with(Student[Calculus1])}: \\
> & g := x \rightarrow \sin(3x) - \cos(2x); \\
> & g := x \rightarrow \sin(3x) - \cos(2x) \\
> & \text{NewtonsMethod}(g(x), x=3, output=plot); \\
> & \text{NewtonsMethod}(g(x), x=3); \\
> & 2.827433388 \\
> & \text{NewtonsMethod}(g(x), x=3, output=sequence, iterations=10); \\
> & 3, 2.833531324, 2.827458241, 2.827433389, 2.827433388, 2.827433388, 2.827433388, 2.827433388, 2.827433388, 2.827433388, 2.827433388, 2.827433388
\end{align*}
\]

186. Use the Tangent command to get a graphical image of the tangent line to 
\[ y = x^3 - 5x^2 + 3x - 1 \]
at \( x = 5 \). Find the slope of the line. Find the equation of the line.

\[
\begin{align*}
> & \text{with(Student[Calculus1])}: \\
> & f := x \rightarrow x^3 - 5x^2 + 3x - 1; \\
> & f := x \rightarrow x^3 - 5x^2 + 3x - 1 \\
> & \text{Tangent}(f(x), x=5, output=plot); \\
> & \text{Tangent}(f(x), x=5, output=plot); \\
> & \text{Tangent}(f(x), x=5, output=plot); \\
\end{align*}
\]
> Tangent(f(x), x=5);  
> Tangent(f(x), x=5, output=slope);  

28 x – 126
28

187. Use the Tangent command to get a graphical image of the tangent line to 
\[ y = \sin(3 x) – \cos(2 x) \] 
at \( x = \pi \). Find the slope of the line. Find the equation of the line.

> with(Student[Calculus1]):
> g := x -> sin(3*x) - cos(2*x);  
g := x → sin(3 x) – cos(2 x)

> Tangent(g(x), x=Pi, output=plot);

> Tangent(g(x), x=Pi);
-3 x – 1 + 3\pi

> Tangent(g(x), x=Pi, output=slope);
-3

188. Find the asymptotes of
\[ \frac{x^2 – 3 x – 5}{x – 2} \]

> with(Student[Calculus1]):
> Asymptotes((x^2-3*x-5)/(x-2), x);
\[ y = x – 1, x = 2 \]

189. Find the asymptotes of
\[ \frac{x^2 – 3 x – 5}{x^2 – 2} \]

> with(Student[Calculus1]):
> Asymptotes((x^2-3*x-5)/(x^2-2),x);
> \sin(x)

\[ y = 1, x = -\sqrt{2}, x = \sqrt{2} \]

190. Find the asymptotes of

\[
\frac{1}{\sin(x)}
\]

> with(Student[Calculus1]):
> Asymptotes(1/sin(x),x);

Warning, the expression has an infinity of asymptotes, some examples of which are given

\[ x = -\pi, x = 0, x = \pi \]

191. Find the critical points of

\[
\frac{x^2 - 3x - 5}{x - 2}
\]

> with(Student[Calculus1]):
> CriticalPoints((x^2-3*x-5)/(x-2),x);

\[
[2]
\]

192. Find the critical points of

\[
y = x^3 - 5x^2 + 3x - 1
\]

> with(Student[Calculus1]):
> f:=x->x^3-5*x^2+3*x-1;
> f:= x -> x^3 - 5 x^2 + 3 x - 1
> CriticalPoints(f(x),x);

\[
\left[\begin{array}{c}
\frac{1}{3} \\
3
\end{array}\right]
\]

193. Find the critical points of

\[
y = \sin(3x) - \cos(2x)
\]

> with(Student[Calculus1]):
> g:=x->sin(3*x)-cos(2*x);
> g := x \mapsto \sin(3 x) - \cos(2 x)
> CriticalPoints(g(x),x);

Warning, the expression has an infinity of critical points, some examples of which are given

\[
\left[\begin{array}{c}
\arctan\left(\frac{-1 + \sqrt{10}}{\sqrt{25 + 2 \sqrt{10}}}\right) - 3 \pi, \arctan\left(\frac{-1 + \sqrt{10}}{\sqrt{25 + 2 \sqrt{10}}}\right) - \pi, \arctan\left(\frac{-1 + \sqrt{10}}{\sqrt{25 + 2 \sqrt{10}}}\right) + \pi
\end{array}\right]
\]
194. Find the extreme points of 
\[ \frac{x^2 - 3x - 5}{x - 2} \]

\[
> \text{with(Student[Calculus1])}:
> \text{ExtremePoints((x^2-3*x-5)/(x-2),x);}
> \text{[]} \]

195. Find the extreme points of 
y = x^3 - 5x^2 + 3x - 1

\[
> \text{with(Student[Calculus1])}:
> f:=x->x^3-5*x^2+3*x-1;
> f:= x \rightarrow x^3 - 5x^2 + 3x - 1
> \text{ExtremePoints(f(x),x)};
> \begin{bmatrix} \frac{1}{3} \end{bmatrix} \]

196. Find the extreme points of 
y = \sin(3x) - \cos(2x)

\[
> \text{with(Student[Calculus1])}:
> g:=x->\sin(3*x)-\cos(2*x);
> g := x \rightarrow \sin(3x) - \cos(2x)
> \text{ExtremePoints(g(x),x)};
> \text{Warning, the expression has an infinity of extreme points, some examples of which are given}
> \begin{bmatrix} -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2} \end{bmatrix} \]

197. Find the inflection points of 
y = x^3 - 5x^2 + 3x - 1

\[
> \text{with(Student[Calculus1])}:
> f:=x->x^3-5*x^2+3*x-1;
> f:= x \rightarrow x^3 - 5x^2 + 3x - 1
> \text{InflectionPoints(f(x),x)};
> \begin{bmatrix} \frac{5}{3} \end{bmatrix} \]

198. Find the inflection points of
\[ y = \sin(x) \]

\[ \text{with(Student[Calculus1]):} \]
\[ \text{InflectionPoints(sin(x),x);} \]
Warning, the expression has an infinity of inflection points, some examples of which are given
\[ [-\pi, 0, \pi] \]

199. Find the roots of
\[ y = x^3 - 5x^2 + 3x - 1 \]

\[ \text{with(Student[Calculus1]):} \]
\[ f := x \rightarrow x^3 - 5x^2 + 3x - 1; \]
\[ f := x \rightarrow x^3 - 5x^2 + 3x - 1 \]
\[ \text{Roots(f(x),x);} \]
\[ \begin{bmatrix}
\frac{71}{2} + \frac{3}{2} \cdot \frac{\sqrt{105}}{3}
\end{bmatrix}
\]
\[ \text{evalf}(%); \]
\[ [4.365230013] \]

200. Find the roots of
\[ y = \sin(3x) - \cos(2x) \]

\[ \text{with(Student[Calculus1]):} \]
\[ g := x \rightarrow \sin(3x) - \cos(2x); \]
\[ g := x \rightarrow \sin(3x) - \cos(2x) \]
\[ \text{Roots(g(x),x);} \]
Warning, the expression has an infinity of roots, some examples of which are given
\[ \begin{bmatrix}
-\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}
\end{bmatrix}
\]
\[ \text{evalf}(%); \]
\[ [-4.712388981, 1.570796327, 7.853981635] \]