Every natural number can be written as the sum of Distinct powers of 2.
Proof (by complete, aka strong, induction):

I1: $1 = 2^0$ which is a sum (albeit with only one term) of distinct powers of 2.

IH: Suppose that every natural number $j \leq k$ can be written as the sum of distinct powers of 2.

IS: We will show that k+1 can be written as the sum of distinct powers of two.
k+1 is either even or odd. (by the proof done in boardwork)

Case 1: If k+1 is even then (k+1)/2 is a natural number less than or equal to k. By our induction hypothesis this means there exist distinct powers p1, p2, ... pn such that $0 \leq p1 <...< pn$ and $(k+1)/2 = 2^{p1} + 2^{p2} +...+ 2^{pn}$ multiplying both sides by 2 gives:
$k+1 = 2(2^{p1} + 2^{p2} +...+ 2^{pn}) = 2^{p1+1} + 2^{p2+1} +...+ 2^{pn+1}$ which is the sum of distinct powers of 2.

Case 2 If k+1 is odd then k is even and thus when k is expressed as the sum of distinct powers of 2, each power is at least 1, so that they are all divisible by 2. That is
$k = 2^{p1} + 2^{p2} +...+ 2^{pn}$ where $0 < p1 <...< pn$ Thus
$k+1 = 2^{p1} + 2^{p2} +...+ 2^{pn} +1 = 2^{p1} + 2^{p2} +...+ 2^{pn} +2^0$ is the sum of distinct powers of 2.

Since the statement is true for k+1 in both cases, it is true by the principle of mathematical induction for all natural numbers, n.