Developing Students’ Conceptual Understanding of Place Value and Decimals

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Introduction

According to Ashlock (2010) and Durkin and Rittle-Johnson (2014), students have difficulties learning and understanding decimals. Common misconceptions include:

1. Students often attempt to apply their knowledge of whole numbers to decimals. For example, students may believe that 0.45 is greater than 0.8 because 45 is greater than 8 (Durkin & Rittle-Johnson, 2014).
2. Students often believe if there is a zero at the end of a decimal number, that the value is increases. For example, students might believe that 0.49 has a lesser value than 0.8090 (Durkin & Rittle-Johnson, 2014).

The purpose of this study was to examine teachers’ thinking about whole number place value and the base ten system and how their understanding of these topics influenced their learning of decimals and decimal computations.

Research Question

How can students’ proficiency be improved in the areas of whole number place value, decimal place value, and decimal computation?

Theoretical Framework

The Strands of Mathematical Proficiency model (National Research Council, 2011, p. 156) was used to shape our approach to the research. In order to address these misconceptions, teachers need to focus on curricula, lesson planning, and the development of activities that focus on student growth in all five of the strands of mathematical proficiency. We can support the development of a deeper and more robust understanding of place value and decimals through a focus on conceptual learning, through individual and group work. The results of this study have demonstrated the importance of spending ample time on each standard before moving on to the next one. It is essential that students have multiple experiences within each standard. One shortcoming of this study was moving too quickly through the standards, which did not allow all students to gain a full understanding.

Methodology: Participants and Procedure

Four students participated in the study: Alex, Bethany, Christina, and Daniel. These students were entering fifth grade in fall 2015. During the summer, students participated in an initial assessment interview, seven one-hour tutoring sessions, and a post-assessment interview. All four students attended each session and participated actively in answering questions, adding their thoughts and opinions during class discussion, and helping one another solve problems.

Learning Trajectory for Place Value and Decimals

The standards shown at the left are the points of entry for the study. All students had mastered these points when they entered the study. (Confrey et al., 2012)

PATHWAYS Cycle of Integrated Teaching and Research

Empirical Teaching and Learning Trajectory:

Initial Assessment Results

All students were able to accurately determine the name and value of the base ten blocks, but none of the students could determine the value of the block immediately to the left of the thousands block. This showed that students were not completely sure of the parts formed within the base ten system. After reviewing the initial assessments of all four students, it was clear that the students were assessing the program with a wide range of ability levels. Alex and Bethany demonstrated procedural fluency, by using algorithms to solve the computations. These two students, however, lacked conceptual understanding of place value, which would cause their inability to apply their knowledge to an array of new, unfamiliar situations.

Instructional Cluster 1

During the first week of the study, students demonstrated their knowledge of whole number base ten blocks by discussing what they knew about the blocks and discovering patterns that can be found among the blocks. Each student created a chart that displayed important information about each block.

Instructional Cluster 2

Once students demonstrated their understanding of place value, students were introduced to adding and subtracting with decimals. Students used manipulatives to represent each number and placed the pieces on a place value mat, which helped students line up the place values when they were adding. The students were asked into small groups of two based on ability level.

Instructional Cluster 3

In the third lesson, students were introduced to multiplication of a whole number and a decimal, for example, 3 x 0.8. Christina and Daniel lacked conceptual understanding of multiplication that was needed to use the manipulatives to represent the problem. For 3 x 0.8, Daniel would set up 3 groups of 8 centi pieces. In order to help students gain a better understanding of how to set up the problem, students were given the opportunity to work with a partner and set up problems that were all the same format. Students demonstrated their adaptive reasoning skills by explaining how they set up their manipulatives and how they found their answer.

Reflection and Discussion

The sequence of the learning progression was easy to follow and each standard transitioned well to the next. The most difficult part of this learning progression is developing a conceptual understanding of each standard. In order to address these difficulties, students should experience conceptual learning, through individual and group work. The results of this study have demonstrated the importance of spending ample time on each standard before moving on to the next one. It is essential that students have multiple experiences within each standard. One shortcoming of this study was moving too quickly through the standards, which did not allow all students to gain a full understanding.

At the beginning of this study, each student participated in an initial 30-minute interview that assessed his or her prior knowledge on topics related to the trajectory for place value and decimals (Confrey et al., 2012).

Each student answered the same set of interview questions at the conclusion of the summer. Data was analyzed from the pre and post questionnaires where students’ learning started, where the students made gains, where the students did not make gains, and where the students made the most gains collectively.

Methodology: Data Gathering and Analysis

Sample Interview Questions

1. What would be the value of the next block to the left of the thousands block?
2. Add 2.4 + 10.3
3. Subtract 2.3
4. Multiply 0.2 x 3

Each instructional session and interview was video recorded. After each lesson was taught, we discussed how the lesson went, which students seemed to have a good understanding of what we taught, and preliminary next steps for instruction. We then transcribed the video. After transcribing, we coded the transcripts and student work using the Five Strands of Mathematical Proficiency and made conjectures about tasks and teaching strategies to help advance students’ thinking.

Post-Assessment Results

In the post interview, Christina demonstrated improved understanding of the base ten blocks by accurately identifying the value of the block immediately to the left of the thousands is 10,000.

Alex and Bethany failed to solve the addition, subtraction, and multiplication problems, but were able to explain their answers using adaptive reasoning and three strategies to develop their understanding.

Alex applied his understanding of multiplication to the problem 2.5x0.3 and was able to draw a picture to represent the problem. A significant improvement for Christina was Daniel’s ability to read decimals correctly, demonstrating an understanding of place value. Bethany also demonstrated understanding by accurately writing the number 0.8302 in expanded form and as a mixed number. The initial and post interview data was analyzed by rating student responses on a scale of 1 to 5, 3, 1 of a 1 for how many of the students did not show any understanding of the problem and a score of 5 for a 5 that students showed a complete understanding of the problem. The results of both interviews were compared to see if students performance rating increased or decreased. Alex, Bethany, and Christina’s scores increased, while David’s score decreased.

Initial and Post Interview Results

References