The Banneker Banner

Maryland Council of Teachers of Mathematics

The Official Journal of the Maryland Council of Teachers of Mathematics

Vol. 28, No. 2 Fall 2013

CONTENTS

Articles

EDITOR
Tricia K. Strickland, Ph.D.
Hood College
401 Rosemont Avenue
Frederick, MD 21701
301-696-3553
strickland@hood.edu

REVIEWERS

The individuals below have given their time and expertise to read and review manuscripts submitted for this edition of the *Banneker Banner*. We are very grateful for their help.

Banneker Banner Submission Guidelines

The Banner welcomes submissions from all members of the mathematics education community, not just MCTM members. To submit an article, please attach a Microsoft Word document to an email addressed to strickland@hood.edu with "Banneker Banner Article Submission" in the subject line. Manuscripts should be original and may not be previously published or under review with other publications. However, published manuscripts may be submitted with written permission from the previous publisher. Manuscripts should be double-spaced, 12 point Times New Roman font, and a maximum of 8 pages. APA format should be used throughout the manuscript with references listed at the end. Figures, tables, and graphs should be embedded in the manuscript. As the Banner uses a blind review process, no author identification should appear on manuscripts. Please include a cover letter containing author(s) name(s) and contact information as well as a statement regarding the originality of the work and that the manuscript is not currently under review elsewhere (unless accompanied by permission from previous publisher). If electronic submission is not possible, please contact the editor to make other arrangements. You will receive confirmation of receipt of your article within a few days, and will hear about the status of your article as soon as possible. Articles are sent out to other mathematics educators for anonymous review, and this process often takes several months. If you have questions about the status of your article during this time, please feel free to contact the editor. Please note that photographs of students require signed releases to be published; if your article is accepted, a copy of the release will be sent to you and it will be your responsibility to get the appropriate signatures. If you would like a copy of this form at an earlier time, please contact the editor.

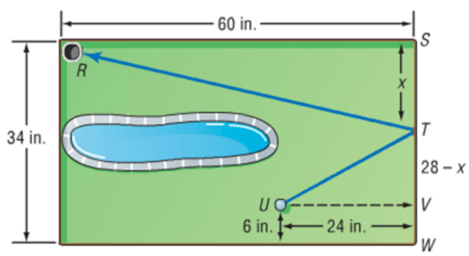
The Gavel:

I am currently taking an online course with Dr. Jo Boaler from Stanford University. The course itself is excellent, of course, but it is an article that Jo had us read that caught my attention. The article chronicles how the teacher took a textbook problem and made it an incredible math task.

"When I Let Them Own the Problem"

From our textbook:

24. GOLF Jessica is playing miniature golf on a hole like the one shown at the right. She wants to putt her ball *U* so that it will bank at *T* and travel into the hole at *R*. Use similar triangles to find where Jessica's ball should strike the wall.



There is essentially nothing left in this problem for students to explore and figure out on their own. If anything, all those labels with numbers and variables conspire to turn kids *off* to math. Ironically even when the problem tells kids what to do (*use similar triangles*), the first thing kids say when they see a problem like this is, "I don't get it."

They say they don't get it because they never got to own the problem.

The author then had the students follow a few directions that set up a picture of a hole in a miniature golf game and then challenged the class to get the golf ball in the hole. What amazing conversations ensued! What struggles took place! Awesome thinking became evident! And not once did a student say he/she did not "get it". Collaborative problem solving took place as all students were engaged in finding a way to make the shot.

As we enter the era of the Common Core State Standards, why not let students own the problem? Take a static problem and turn it in to a dynamic one that students can own. Through the learning process, the author had to work at not jumping right in to assist the students, which admittedly was difficult. Letting the students grapple with the problem gave her the chance to provide guiding questions but not answers. This approach encouraged the students to think deeply in order to creatively solve the problem. And they did. Not only that, some humor became evident as the students began to sing new lyrics to the Rolling Stones hit, *(I Can't Get No) Satisfaction:*

I can't get no similar triangles
I can't get no similar triangles
'Cause I try and I try and I try and I try
I can't get no, I can't get no
When I'm drawing in my lines

I enjoyed reading this article as it challenged me to think about how I could change problems that are gleaned from texts to something, well, more innovative. It will not be the easiest process, but definitely more rewarding. (For access to the article, go to http://fawnnguyen.com/2013/05/07/20130506.aspx)

Where can we get the inspiration for this type of innovative thinking? Make plans to join us at the National Council of Teachers of Mathematics Regional Conference in Baltimore on October 16-18. Many outstanding educators will be there to inspire, encourage, motivate, and enthuse you! MCTM is the host organization, and can offer opportunities to volunteer at the conference. See http://www.nctm.org/conferences/forms.aspx?ekfrm=35276 to volunteer.

On behalf of the Board of Directors of MCTM, I would like to congratulate the 2013 Outstanding Teachers and Educators:

Sarah Voskuhl - Aberdeen High School - Mathematics Educator
Jennifer Novak - Howard County Public Schools - Mathematics Educator
Christy Graybeal - Hood College - College Mathematics Educator
Emily Roberts - Marriotts Ridge High School - Beginning Mathematics Educator

These amazing educators provide rich and challenging environments for math that promotes sense making and thinking—and may already be letting their students own the problems! As the new school year begins, think about an outstanding educator that you know and nominate them for the award.

It has been an amazing and busy two years as President of MCTM, which had many wonderful opportunities! As I pass the gavel to our new President, Andrew Bleichfeld, I know that he will bring a new and exciting perspective to MCTM.

Have a wonderful year!

Sue Vohrer – MCTM President

Common Core State Standards and Teacher Preparation

Darrin Drum

Research reported in this paper was completed in a capstone project for the master's degree in mathematics education in the Graduate School of Hood College.

Common Core State Standards and Teacher Preparation

The Common Core State Standards (CCSS) have prompted changes in mathematics curricula nationwide. These new standards affect not just the public school curriculum that teachers have to teach; they are also affecting the courses and philosophies that colleges and universities use to educate pre-service teachers.

Teacher preparation is a critical component in the development of successful students. Without teachers who are prepared to interpret curriculum and effectively communicate mathematical ideas to their students, more students will graduate from high school unprepared for the demands of the real world, particularly in the use of mathematics.

How do the Common Core State Standards Address Teacher Preparation?

The mission of the CCSS initiative is to "provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

One of the ways that the Common Core State Standards address teacher preparation is in the development of eight specific Standards for Mathematical Practice. These eight standards of student behaviors describe the type of expertise that mathematics teachers at all levels should develop in their students. Thus teachers themselves must be given the opportunity to develop teaching styles that will bring these eight standards out in their students. The eight standards are (http://www.corestandards.org/Math/Practice):

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

While the content standards are the major focus of curriculum writers and curriculum specialists, the need for teachers to be exposed to and understand these practice standards should not be thought of as unnecessary or unimportant. The development of these behaviors will not only benefit students in their math classes but will also allow students to develop the type of thinking that will help them be more successful in school and after their school lives.

The new standards will also affect how mathematics teachers are prepared to teach by colleges and universities. By providing a consistent, well researched and received set of standards (adopted by 45 of the 50 states) (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) teacher education should become more consistent across the country. Instead of learning how to read a general curriculum which could be different from state to state and in some cases from county to county within a state, new teachers can now be taught how to look for specific understandings from their students which will be at a consistent level nationwide. This will enable postsecondary schools to help new teachers focus on how to deliver the curriculum effectively.

Implementation of the CCSS over the next several years will be a critical step as new teachers are educated and help develop "best practices" for teaching the new curriculum. While the CCSS do not specifically dictate how prospective teachers are to be educated at any level, the level of mathematical understanding and connection that will be required of teachers at all levels indicate that a change in teacher education will be needed.

In 2001 the Conference Board of the Mathematical Standards (CBMS) published a book titled *The Mathematical Education of Teachers*. This book made several recommendations on how teachers of mathematics should be taught. In 2012, CBMS published a follow up book titled *The Mathematical Education of Teachers II.* The purpose for the second book was to suggest ways to integrate the CCSS into the education of mathematics teachers as well as to update the recommendations made in the 2001 book.

The overarching recommendations made in the CBMS reports were as follows.

1. Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach.
2. Although the quality of mathematical preparation is more important than the quantity, the following amount of mathematics coursework for prospective teachers is recommended.
    a. Prospective elementary grade teachers should be required to take at least 9 semester-hours on fundamental ideas of elementary school mathematics.
    b. Prospective middle grades teachers of mathematics should be required to take at least 21 semester-hours of mathematics, that includes at least 12 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers.
    c. Prospective high school teachers of mathematics should be required to complete the equivalent of an undergraduate major in mathematics, that includes a 6-hour capstone course connecting their college mathematics

courses with high school mathematics.

3. Courses on fundamental ideas of school mathematics should focus on a thorough development of basic mathematical ideas. All courses designed for prospective teachers should develop careful reasoning and mathematical "common sense" in analyzing conceptual relationships and in solving problems.

4. Along with building mathematical knowledge, mathematics courses for prospective teachers should develop the habits of mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching.

5. Teacher education must be recognized as an important part of mathematics departments' mission at institutions that educate teachers. More mathematicians should consider becoming deeply involved in K-12 mathematics education.

6. The mathematical education of teachers should be seen as a partnership between mathematics faculty and mathematics education faculty.

7. There needs to be greater cooperation between two-year and four-year colleges in the mathematical education of teachers.

8. There needs to be more collaboration between mathematics faculty and school mathematics teachers.

9. Efforts to improve standards for school mathematics instruction, as well as for teacher preparation accreditation and teacher certification, will be strengthened by the full-fledged participation of the academic mathematics community.

10. Teachers need the opportunity to develop their understanding of mathematics and its teaching throughout their careers, through both self-directed and collegial study, and through formal coursework.

11. Mathematics in middle grades (grades 5-8) should be taught by mathematics specialists.

How Are Teachers Prepared to Teach High School Mathematics in Maryland?

The first policy that most colleges and universities in the state of Maryland follow is that students who wish to teach math at the secondary (grades 7-12) level must major in mathematics and have an education minor. Some schools do have a double major in mathematics and education but most do not. Based on the programs at each institution of higher learning, the number of credit hours required by Maryland colleges and universities varies from a low of 70 combined (education and mathematics) hours to a high of 96 combined hours. The following chart, compiled from each college and university website, shows the number of hours of education and mathematics courses required by universities and colleges in the state of Maryland (as specified by the Maryland State Department of Education) that a student must complete in order to become a certified secondary mathematics teacher. (This chart does not include courses designed to help students pass required assessments for certification.)

As one can see, the number of credit hours required by various Maryland colleges and universities varies. The fewest number of education credit hours required are the 32 required by both Loyola University and McDaniel College. The fewest number of mathematics credit hours are the 36 hours required by Mt. St. Mary's University. As noted in the chart the 21 hours required at Washington Adventist University is for a math minor; Washington Adventist does not offer a mathematics major but does offer secondary Mathematics Education certification and is approved by the MSDE. A comparison of the minimum number of hours suggested by the CBMS report to the course requirements at several Maryland universities show that schools in Maryland do a good job requiring secondary level teachers to obtain a degree in mathematics thus meeting the first and second CBMS recommendation from 2001. The second recommendation has not been well met in one particular area. At the time of my research only one school in Maryland had a specific course track for teachers of middle school students. As recommended by CBMS in 2001, schools need to differentiate between middle and high school students. There is a reason students now spend three years in middle school, they are not all developmentally ready for the same rigor expected of a high school student, thus these students need to be taught differently. Maryland colleges need to recognize these differences better and make a clear path for teachers of middle school students to better develop their skills.

Table 1

Education and Mathematics Credit Hours

Institution of Higher Learning	Education Credit hours	Mathematics Credit hours
Bowie State University	34	39
Coppin State University	36	46
Frostburg State University	42.5	39
Goucher College	39	37
Hood College	34	49
Loyola University Maryland	32	42
McDaniel College	32	38
Morgan State University	45	51
Mount St. Mary's University	37	36
Notre Dame of Maryland University	40	41
Salisbury University	37	40
Towson University	45	42
University of Maryland- College Park	34	39
University of Maryland- Eastern Shore	42	43
University of Maryland- Baltimore County	33	43
Washington Adventist University	34	21 Math Minor Only
Averages (Not including Washington Adventist University)	37.5	41.667

Suggestions for implementation

How can a Mathematics teacher in Maryland effectively implement the Common Core State Standards? The most important understanding a Mathematics teacher can have; is to understand the eight standards for mathematical practice outlined in the CCSS. As teachers become more familiar with how their students are expected to think about mathematics, the easier it will be for teachers to implement changes in their teaching styles and lessons. Specifically teachers need to know what it looks like for a student to make sense of problems and persevere. As teachers, we need to set an atmosphere in our rooms where it is ok to be incorrect but it is not ok to give up. How often do we, in order to 'get through' an overwhelming amount of curriculum, just give one or two students an opportunity to solve a difficult problem before we solve it ourselves to keep moving? That type of classroom environment needs to disappear. All of the eight standards are critical for teachers to understand and be prepared to implement. Schools in Maryland and nationwide need to make sure that classes for both pre service and current teachers address each of these critical areas. The CCSS do not change the curriculum as much as they are calling for a change in the way the curriculum is delivered. The same math will still be taught, it just can't be taught in the same old way anymore. College courses that model teaching methods that reflect the standards of the CCSS and help teachers explore like their students are expected to explore will be the single most useful tool to help educators successfully implement the CCSS.

Secondary Mathematics Education is constantly evolving. The education of mathematics teachers is constantly changing to reflect changes in education philosophy and national mandates. The Common Core State Standards are the most recent, well supported, policy to cause universities and colleges to reflect on how teachers are taught. The CCSS are not perfect, no set of standards and curriculum will ever be considered perfect by such a diverse and opinionated group as teachers. They are however, a much better alternative to help teachers enable the students of today to become better mathematical thinkers, not just on a local or state level, but nationwide.

References

Conference Board of the Mathematical Sciences. (2001). *The mathematical education of teachers.* Providence, RI: American Mathematical Society.

Conference Board of the Mathematical Sciences. (2012). *The mathematical education of teachers II.* Providence, RI: American Mathematical Society.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Mathematics Standards.* Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers. Retrieved June 18, 2012 from http://www.corestandards.com

Puzzler

Stew Saphier

A five volume set of books is on a bookshelf in usual fashion. Each book is precisely 1¼ inches thick including the two covers which are each 1/8 inch thick. A bookworm eats through the front cover of volume one all the way through the back cover of volume five always traveling perpendicular to the pages. How far did the bookworm travel?

Answer and explanation on p. 42.

Middle School Mathematics Teachers' Interpretation of Curricular Messages

Christy D. Graybeal, Ph.D.
Hood College
Frederick, MD
This study was funded by a Hood College Board of Associates McCardell Professional Development Grant.

Despite the many constraints placed on them, teachers have significant influence over what and how they teach (Cohen & Hill, 2000; Cuban, 1995; Lipsky, 1980; Wills & Sandholtz, 2009). Mathematics teachers who are using the same curricular materials can enact them in dramatically different ways and afford their students very different experiences (Chávez-López, 2003; Chval, Grouws, Smith, & Ziebarth, 2006; Kilpatrick, 2003; Remillard, 1996; Schwille et al., 1982). Curricular resources (such as textbooks, curriculum guides, and state and national documents) contain messages about mathematics and mathematics teaching. Teachers may not interpret these messages as they are intended by the authors. Furthermore, competing and conflicting messages within and across resources may be interpreted by teachers. One possible reason for *why* teachers can use the same resources in such a wide variety of ways is that individual teachers focus on different messages and understand messages differently.

Because teachers play such a significant role in the mathematics education of students, the larger education community needs to support teachers in their efforts to reform mathematics education and ensure that all children succeed in learning mathematics. In order to reduce the variance between the intended and taught curriculum, those responsible for writing and selecting curricular materials need to learn how teachers are currently interpreting curricular resources. They should then use this information to create or select resources that better support teachers in their enactment of the intended messages.

In order to address these issues, this study[1] explored the messages that middle school mathematics teachers in Maryland interpret from curricular resources and the ways in which these messages relate to their beliefs and practices. Specifically, the research questions were:

- What messages do teachers interpret from available resources?
- How consistent are these messages across resources?
- How do these messages relate to the teachers' expressed beliefs and reported classroom practices?

Survey Design

This study builds off of a previous research study (Graybeal, 2008) which included interviews of five experienced middle school mathematics teachers in Maryland. These interviews explored the messages that the teachers interpreted from student textbooks, school district documents, state documents, and National Council of Teachers of Mathematics (NCTM) documents and the ways in which those interpretations related to their beliefs and practices. The teachers in this previous study interpreted messages in eleven themes. Three themes—*Concepts and Procedures*, *Question Types*, and *Source of Solution Methods*—created the most tension for the teachers. Thus, these three themes are the focus of the current study. Additionally, the teachers' wordings from the initial study were used to form the statements in the survey used in the current research study. Several middle school mathematics teachers piloted drafts of the survey and feedback from them was used to shape the final version of the survey.

The survey was administered online via Survey Monkey. The survey asked teachers for information about education, teaching experience and certifications, and contextual information. Since middle school mathematics teachers often teach a variety of mathematics courses, and may have different thoughts about each of these courses, the teachers were directed to think about the last math class period/block that they taught and to refer to this course when answering all questions in the survey.

Most of the questions asked teachers to respond to sets of similar statements with regard to their own personal beliefs, their own teaching practices, and how they imagine the authors of the textbooks, school district documents, state documents, and NCTM documents would respond to these statements. These statements focused on the three themes identified earlier – the balance of concepts and procedures (*Concepts and Procedures*), the types of questions to ask of students (*Question Types*), and the ways in which students should become acquainted with solution methods (*Source of Solution Methods*). Select survey questions and results of the survey are presented in the following sections.

Participants

An email invitation to participate in the online survey was sent to each of the 1330 middle school mathematics teachers in Maryland. Additionally, members of the Maryland Council of Teachers of Mathematics received encouragement to participate via an email from the president of that organization. Of the 150 teachers who accessed the survey, 114 (76%) viewed the entire survey. Not all participants answered all questions, thus the number of respondents varies from question to question.

Autonomy

One of the first questions on the survey asked teachers about how free they feel to decide what mathematics content to teach and how to teach it. These results are shown in Table 1. The teachers feel little freedom to decide *what* mathematics content to teach, but feel free to decide *how* to teach.

Table 1. Freedom to Decide What and How to Teach

Statement: Think about the last math class period/block that you taught. On a scale of 1 to 5 (with 1 being "strongly disagree" and 5 being "strongly agree") indicate your level of agreement with the following statements.	*Number*	*Mean*	*Standard Deviation*
I feel free to decide what mathematics content I will teach in this course.	143	2.03	1.100
I feel free to decide how I will teach this course.	141	3.52	1.217

Influences of Resources on Instruction

Another question on the survey asked teachers to indicate how much influence the textbook, school district documents, state documents, and NCTM documents have on their teaching. These results are shown in Table 2. Of the four resources listed, the teachers report that the NCTM and the textbooks have the least influence on the teachers' practices while the state and school district materials have the most influence on their practices.

Table 2. Influences of Resources on Instruction

	Statement: On a scale of 1 to 5 (with 1 being "strongly disagree" and 5 being "strongly agree") indicate your level of agreement with the following statements.	*Number*	*Mean*	*Standard Deviation*
School District	The school or district's curriculum guide and/or assessments have a significant influence on my instruction of this course.	121	4.38	.878
State	The Maryland State Curriculum and Assessments have a significant influence on my instruction of this course.	119	4.31	.890
Textbook	The textbook series (and its supplementary resources, if applicable) has a significant influence on my instruction of this course.	119	3.34	1.304
NCTM	The National Council of Teachers of Mathematics has a significant influence on my instruction of this course.	121	2.98	1.190

Concepts and Procedures

The teachers involved in the previous study (Graybeal, 2008) indicated that the balance between concepts and procedures is one with which they struggle. Thus, this was an area of focus for this study. Over the past century, the emphasis on concepts and procedures in mathematics instruction in the United States of America has swung back and forth repeatedly. For the first half of the twentieth century, mathematics instruction primarily focused on mathematical procedures. In the 1950s and 1960s the "new math movement" attempted to redirect attention to mathematical concepts. This was followed by the "back to basics" movement which again focused on procedures. In the 1980s and 1990s the "reform" movement again focused on concepts. In response to this, there have been more recent "back to basics" movements (NRC, 2001, p. 115). Although conceptual understanding and procedural fluency are not mutually exclusive instructional goals, many teachers feel that they are and that they must choose between the two. Despite recent efforts to reform mathematics instruction, most students in the United States of America receive instruction which "continues to emphasize the execution of paper-and-pencil skills in arithmetic through demonstrations of procedures followed by repeated practice" (NRC, 2001, p. 4).

The participants in this survey responded to sets of statements regarding the balance of concepts and procedures. The results for one of these sets of statements are listed in Table 3. Note that the participants imagine that the authors of NCTM documents would more strongly agree with an emphasis of concepts over procedures than the authors of their students' textbooks would.

Question Types

The teachers in the previous study (Graybeal, 2008) also felt that different resources were encouraging them to ask different types of questions of students. One prominent feature of any curricular resource is the types of questions that it asks. Do the questions tell what operations to use or is that left to the students to determine? Are the questions contextualized? If so, are the contexts realistic and/or meaningful to the students? Traditionally textbooks in the United States of America have emphasized computational questions devoid of contexts. As a result, students have had great difficulty in applying what they learn in a particular lesson to other situations both inside and outside of school. In order to help students succeed in applying their knowledge to new situations, some have argued that students should learn mathematics in contexts similar to the situations in which the knowledge will be needed (Barab & Plucker, 2002; Greeno & Moore, 1993). Others have argued that mathematics questions need to not only be contextual, but that these contexts need to be relevant to the students' lives (Gutstein, 2003; Secada, 1992; Tate, 1994).

The participants in the present study responded to sets of statements regarding the types of questions asked of students. The results for one of these sets of statements are listed in Table 4. The participants believe that it is important to ask more contextual word problems than purely computational questions and they feel that the authors of school district documents, state documents, and NCTM documents would agree. They do not think that the authors of their textbooks would agree as strongly. The teachers' reported practices align more closely with their interpretations of their students' textbooks than with their beliefs or interpretations of school district, state or NCTM documents.

Table 3. Concepts and Procedures

	Statement: On a scale of 1 to 5 (with 1 being "strongly disagree" and 5 being "strongly agree" indicate your level of agreement with the following statements.	*Number*	*Mean*	*Standard Deviation*
NCTM	The authors of NCTM documents seem to believe that it is more important for students to understand mathematical concepts than it is for them to be fluent with mathematical procedures.	89	3.74	1.017
School District	The authors of the school or district's curriculum guide and/or assessments seem to believe that it is more important for students to understand mathematical concepts than it is for them to be fluent with mathematical procedures.	116	3.51	1.000
Practices	When teaching this course I emphasize the understanding of mathematical concepts more than fluency with mathematical procedures.	137	3.35	.904
Beliefs	I believe that it is more important for students to understand mathematical concepts than it is for them to be fluent with mathematical procedures.	140	3.29	1.147
State	The authors of the Maryland State Curriculum and the Maryland Assessments seem to believe that it is more important for students to understand mathematical concepts than it is for them to be fluent with mathematical procedures.	108	3.28	1.003
Textbook	The authors of my students' textbooks seem to believe that it is more important for students to understand mathematical concepts than it is for them to be fluent with mathematical procedures.	86	3.13	.968

Table 4. Question Types

	Statement: On a scale of 1 to 5 (with 1 being “strongly disagree” and 5 being “strongly agree” indicate your level of agreement with the following statements.	*Number*	*Mean*	*Standard Deviation*
NCTM	The authors of NCTM documents seem to believe that it is important to ask more contextual word problems than purely computational questions.	96	4.03	1.020
Beliefs	I believe that it is important to ask more contextual word problems than purely computational questions.	139	3.77	1.017
State	The authors of the Maryland State Curriculum and the Maryland Assessments seem to believe that it is important to ask more contextual word problems than purely computational questions.	115	3.76	1.081
School District	The authors of the school or district’s curriculum guide and/or assessments seem to believe that it is important to ask more contextual word problems than purely computational questions.	118	3.75	1.014
Textbook	The authors of my students’ textbooks seem to believe that it is important to ask more contextual word problems than purely computational questions.	90	3.30	.999
Practices	When teaching this course I ask more contextual word problems than purely computational questions.	137	3.28	.939

Source of Solution Methods

One of the most contentious issues in modern mathematics education involves the question of how students should become acquainted with solution methods. Some argue that “students learn by creating mathematics through their own investigations of problematic situations, and that teachers should set up situations and then step aside so that students can learn” (NRC, 2001, p. xiv). Others claim that “students learn by absorbing clearly presented ideas and remembering them, and that teachers should offer careful explanations followed by organized opportunities for students to connect, rehearse, and review what they have learned” (NRC, 2001, p. xiv). Still others contend that these views should not be thought of as opposite ends of a single dimension, but rather as separate dimensions of teaching (Stecher et al., 2006). Nevertheless, these two ways of acquainting students with solution methods have historically been seen as opposites and the teachers in the previous study (Graybeal, 2008) agonized over which approach to use.

The participants in the present study responded to sets of statements regarding the participants’ beliefs, practices, and perceptions of the resources with regard to source of solution methods. The results for one of these sets of statements are listed in Table 5. Note that on average the participants indicate that they do not think that the authors of their students’ textbooks believe that students can and should develop their own solution strategies. In contrast, the participants indicate that they think authors of NCTM documents believe that students can and should develop their own solution strategies.

Table 5. Source of Solution Methods

	Statement: On a scale of 1 to 5 (with 1 being "strongly disagree" and 5 being "strongly agree" indicate your level of agreement with the following statements.	*Number*	*Mean*	*Standard Deviation*
NCTM	The authors of NCTM documents seem to believe that students can and should develop their own solution methods.	95	4.14	.952
Beliefs	I believe that students can and should develop their own solution methods.	139	3.63	.927
School District	The authors of the school or district's curriculum guide and/or assessments seem to believe that students can and should develop their own solution methods.	116	3.44	1.090
Practices	When teaching this course I have students develop their own solution methods.	137	3.36	.913
State	The authors of the Maryland State Curriculum and the Maryland Assessments seem to believe that students can and should develop their own solution methods.	109	3.28	1.178
Textbook	The authors of my students' textbooks seem to believe that students can and should develop their own solution methods.	89	2.98	1.234

Summary and Discussion

The teachers who participated in this survey indicated that they feel that they have autonomy over how they will teach, but little control over what content they teach. They also indicated that while the school district and state documents have significant influence on their instruction, their students' textbooks and NCTM documents have less influence. These results are important for mathematics education leaders to consider as they work to improve the mathematics education of students and they are particularly important to consider as Maryland adopts the Common Core State Standards (CCSS).

Additionally, the teachers in this study interpreted many discrepancies in messages across resources. In particular, the teachers tended to interpret different messages from their students' textbooks and NCTM documents. While most textbooks claim to be aligned with NCTM principles, the teachers saw discrepancies between these resources. The NCTM Process Standards are quite similar to the CCSS Standards for Mathematical Practice. Thus, although many textbooks now claim to be aligned with CCSS, it is likely that teachers will not see alignment. Attention should be paid to how consistently and accurately messages (especially messages about mathematical practices) are portrayed in the textbooks provided to teachers and students. Tools such as those presented in the *Mathematics Curriculum Materials Analysis Project* (2011) will be useful when examining these messages. As Maryland phases in the Common Core State Standards and new curricular resources are being developed and adopted, we have the opportunity to create a cohesive message; let's make wise choices.

References

Barab, S. A., & Plucker, J. A. (2002). Smart people or smart contexts? Cognition, ability, and talent development in an age of situated approaches to knowing and learning. *Educational Psychologist, 37*(3), 165-182.

Chávez-López, O. (2003). *From the textbook to the enacted curriculum: Textbook use in the middle school mathematics classroom.* (Doctoral dissertation, UMI No. 3099613)

Chval, K. B., Grouws, D. A., Smith, M., & Ziebarth, S. (2006, April). *A cross-site study of local factors affecting mathematics curriculum enactment.* Paper presented at the meeting of the American Educational Research Association, San Francisco, CA.

Cohen, D. K., & Hill, H. C. (2000). Instructional policy and classroom performance: The mathematics reform in California. *Teachers College Record, 102*(2), 294-343.

Cuban, L. (1995). How organizations influence teacher responses to secondary science curriculum reform. *Theory into Practice, 34*(1), 4-11.

Graybeal, C.D. (2008). *Mathematics teachers' interpretations of messages in curricular resources and the relations of these interpretations to their beliefs and practices.* (Doctoral dissertation, University of Maryland, 2008). *Dissertation Abstracts International, 69, 09.*

Greeno, J. G., & Moore, J. L. (1993). Situativity and symbols: Response to Vera and Simon. *Cognitive Science, 17*(1), 49-59.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education*, *34*(1), 37-73.

Kilpatrick, J. (2003). What works? In S. S. Senk & D. R. Thompson (Eds.), *Standards-based school mathematics curricula: What are they? What do students learn?* (pp. 471-488). Mahwah, NJ: Erlbaum.

Lipsky, M. (1980). *Street-Level Bureaucracy*. New York: Russell Sage Foundation.

Mathematics Curriculum Materials Analysis Project. (2011). Retrieved March 14, 2013 from http://www.mathedleadership.org/ccss/materials.html

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Remillard, J. T. (1996). *Changing texts, teachers, and teaching: The role of curriculum materials in mathematics education reform.* (Doctoral dissertation, UMI No. 9734181)

Schwille, J., Porter, A., Belli, G., Floden, R., Freeman, D., Knappen, L., et al. (1982). *Teachers as policy brokers in the content of elementary school mathematics* (Report No. ED 225 854 SE 040 213). East Lansing, MI: Institute for Research on Teaching.

Secada, W.G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 623-660). Reston, VA: NCTM.

Stecher, B., Le, V., Hamilton, L., Ryan, G., Robyn, A., & Lockwood, J. R. (2006). Using structured classroom vignettes to measure instructional practices in mathematics. *Educational Evaluation and Policy Analysis, 28*(2), 101-130.

Tate, W.F. (1994). Race, retrenchment, and the reform of school mathematics [Electronic version]. *Phi Delta Kappan, 75*(6), 477-484.

Wills, J. S., & Sandholtz, J. H. (2009). Constrained professionalism: Dilemmas of teaching in the face of test-based accountability. *Teachers College Record, 111*(4), 1065-1114.

The Use of Technology in the Geometry Classroom

Ryan Oben
Albert Einstein High School
Montgomery County Public School

Research reported in this paper was completed in a capstone project for the master's degree in mathematics education in the Graduate School of Hood College.

The Use of Technology in the Geometry Classroom

"What is a circumcenter?" Today this question can be easily answered, not by a Geometry teacher, or even a person for that matter, but rather by technology. Type this question into the answer engine at www.wolframalpha.com and not only do you get a great illustration, but also a full definition accompanied by calculations that require a trigonometric background to understand. Perhaps more impressively, this is only one example of the innumerable technologies that are available to us today and can be used in classrooms. As a Geometry teacher I use many of these technologies, which help me to keep kids focused and fascinated by the material; and as a result, they are better able to learn.

My positive experience with technology is hardly an isolated event; school systems are now pushing to incorporate more technology into the classroom, as evidenced by recent changes in multiple state curricula, as well as national standards, in the hopes that this will improve student performance and learning. Aside from improving performance, incorporating technology will also better prepare students for the future which will undoubtedly be heavily dependent on technology of all sorts.

There are various forms of technologies that are available to today's high school Geometry teacher; those that I most prefer to use in my own Geometry classrooms, include a software program called "Geometer's Sketchpad," graphing calculators, and a relatively new innovation, the Interactive White Board (IWB). There are many benefits associated with the use of technology in the Geometry classroom, as well as countless examples of research literature to support this claim, and it is my goal to make clear that technology truly is a powerful and positive tool that can enrich the understanding of today's students. So while there may be some mixed reviews regarding the infusion of technology into

Geometry classrooms, based on my research, the majority of these counter-arguments are rooted in problems having nothing to do with the technology itself, but rather the misuse of it. Throughout my research I have discovered study after study which supports the claim that technology truly is a wonderful addition to the Geometry classroom, which is something my own teaching experiences have taught me as well.

The one example of technology that I use on a daily basis is the IWB. Some popular brands are the Smart Board (www.smarttech.com) and the Promethean Board (www.prometheanworld.com). The IWB is a large projection screen (approximately 80'' diagonally) which is connected to a computer and has a projector that displays the computer screen on an electronic white screen. The primary teacher-interface is a hand-held pen that allows the teacher to manipulate the screen as you would with a mouse, and the primary student-interfaces are personal input devices referred to as Activotes or Eggs (due to their shape) and the Activslate which is a wireless device that allows students to write on the board from their seats.

The benefits of the use of such technology in the classroom are best summed up in the following statement from the National Council of Teachers of Mathematics (NCTM): "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (2012). One palpable benefit is that the use of technology naturally piques students' curiosity and fosters an inclination to investigate beyond the surface level of problems thereby increasing the scope of their exploration. Second, and perhaps the most assessable benefit of technology, is the deeper levels of understanding technology can help generate and the higher level of thinking it requires. Finally, technology can help foster student interest and motivation and thus improve learning as well as commitment to learning. While many of the benefits that stem from the use of technology are intangible, others are obvious and significant as shown by research literature and studies which provides ample evidence of technology's many benefits.

Whether it is because illustrations are more accurate or are easier to create, or simply because students enjoy interacting with technology, it is clear that through the use of technology students are more willing to go beyond the question at hand and investigate what comes next without being prompted. Alan Brown (1999) detailed a study involving the use of graphing calculators (more specifically the TI-92). The students were allowed to pick a topic (subject to teacher approval) geared towards the exploration of geometric proofs involving polygons. During this study Brown noticed that "dynamic-geometry software allows for the same constructions [as by-hand methods] but ventures far beyond the initial construction by permitting students to investigate multiple what-if scenarios" (p. 818). Brown noted that because of the use of the TI-92 graphing calculator students were able to "discover which properties vary and which remain invariant" (p. 818) which helped them form accurate conclusions. Because the process of exploration was streamlined the students went beyond their initial conjectures and discovered patterns and properties that they were not even looking for. For this reason Brown stated that "interactive technology can enhance the Geometry curriculum" (p. 819) and that the use of technology extended the students' exploration. This makes him only one of many researchers that have come to believe that claim.

Brad Glass (2001) came to a similar conclusion; he described a classroom experiment involving a software program called Logo Geometry MOTIONS (http://el.media.mit.edu/logo-foundation/logo/index.html). In this study students were asked to analyze rigid transformations such as translations and rotations. Students were broken into two separate groups, the first of which was given access to technological support in the form of the Logo software, and the other was not. Glass noticed that the group that was not allowed to use the Logo Geometry MOTIONS application had far more difficulty explaining the transformations given to them and therefore had trouble venturing past the initial activity. One concern Glass voiced regarding the lack of access to the software was that "students may focus on a single and often overly specific diagram as being representative of all cases. For example, students who are searching for patterns while working with a diagram of a square may believe that their findings hold for all rectangles" (p. 225). This concern demonstrates a lack of desire or ability (on the students' part) to extend past the initial example or illustration. Glass noted that through the incorporation of technology teachers would be better equipped to fight against this common pitfall because technology allows students to quickly explore multiple examples in order to test their understanding and thereby avoid the issue of overgeneralization of a single construction or drawing.

Technology also helps fight the common tendency of students to develop only a temporary or surface level understanding of what they study; an understanding that is so shallow and fleeting that it only stays with them until the day after the dreaded assessment. Students who get to use technology not only expand their exploration, but that exploration goes to, and results in, a much deeper level of understanding. Rose Zbiek (1996) detailed an activity titled the "Pentagon Problem" which assumed the use of Geometer's Sketchpad to assist students in their investigation.

By the end of the activity Zbiek credited the use of technology with the ability of most of the class to solve the problem. Zbiek claimed that without the use of technology, very few of the students would have gained sufficient geometric reasoning to master "the Pentagon Problem." However, with the use of technology, most of the students were able to find a solution (in this case a counter-example) for the conjecture. In this example the teacher presented the students with a problem that without technology may have been too challenging, but with the use of Geometer's Sketchpad the majority of students were able to reach a solution thus establishing a more thorough understanding.

Gloriana Gonzalez and Patricio Herbst (2009) provided more evidence for the benefits of technology in the classroom through the use of graphing calculators. They described a four-day study in which students spent two days using "static" diagrams and two days using "dynamic" (or interactive) diagrams using Cabri on graphing calculators. In looking back at the two days spent with the graphing calculators they noted that the technology allowed teachers to draw the students' attention to geometric configurations and patterns that would be difficult to notice or duplicate by hand, thus this extension was made possible through the use of technology.

Despite the numerous benefits of technology that have been well documented, the rapid development and availability of technology represents a change to the modern day classroom that some are reluctant to adjust to. In my research I have encountered several pseudo-problems people have described with technology such as the interactive white board, graphing calculators, and software programs. I refer to these as "pseudo-problems" primarily because their existence does not prove that there is an issue with technology, but rather it serves as an indicator that many teachers are misusing the technological resources brought to their classrooms.

One common misuse that leads to the criticism of technology is underuse, which is an issue that dates back to the television and even the radio as detailed by Larry Cuban (1986). Cuban pointed out that resistance and doubt surrounding new forms of technology is very common, even as early as 1920. However he also pointed out that in most cases this doubt (which usually centered on the effectiveness of the technology) stemmed from the fact that it was not being used often enough to make a difference.

David Coffland and Albert Strickland (2004) conducted research to help guide future research surrounding the proper use of technology. Their study exposed multiple correlations, most surprising of which was the inverse relationship between the number of sections of Geometry a teacher teaches and the amount of time spent using the computer in class. It seemed that teacher underuse was indirectly related to how comfortable the teacher was in his or her ways. As a result of this misuse (or again underuse), the reputation of technology is often tainted in the eyes of some because it seems to be ineffective.

Another common complaint I encountered was the claim that technology is too flashy, too entertainment-driven, and is simply too distracting, but once again the responsibility to prevent this issue falls in the hands of the teachers. Gonzalez and Herbst (2009) noted that students have a tendency to want to play with a new-found technology, while the teacher is simply trying to teach a concept; it therefore falls within the realm of the teacher's responsibility to keep the students focused on the task at hand rather than the tool their using and keep them from being distracted or merely entertained by it. Clearly these researchers recognized the possible distracting features of technology, however they made it perfectly clear that it is the responsibility of the teachers to combat this; it is their job to question students and challenge them to go beyond the technology and use it to derive conclusions (Gonzalez & Herbst, 2009). The conclusion of Gonzalez and Herbst reflect that the issue of distracting technology can be solved by a properly prepared teacher, much like the other issues commonly brought against the infusion of technology in the Geometry classroom.

Interestingly, Robert Marzano (2009) who summarized research that he had conducted with Mark

Haystead involving 85 teachers and 170 classrooms, found that in nearly a quarter of the cases, better results were achieved when interactive white boards were not used. In order to explain what may have caused some of the non-IWB classrooms to perform at a higher level, Marzano stated that "Some teachers speed through the material on these [interactive white] boards without explaining them in detail or at all" (p. 81). This again exemplifies the pattern that it is the misuse of the teacher, rather than an inherent problem with the technology itself, which renders the technology ineffective; this point brings us to the final common pseudo-problem.

Many claim that technology causes teachers to rush too quickly through the material however it is not the fault of the technology but rather the teacher if the pace is too fast. Many researchers agree that the ability of technology to speed things up is actually a strength; it simply must be wielded appropriately. For example Gonzalez and Herbst (2009) noted that by integrating technology into the classroom, teachers are better able to challenge students with a wider range of problems which helps expand their thinking and get them thinking about mathematical concepts in a new light. Again to take advantage of this tool, teachers must be instructed appropriately as well as students. In fact, when Brown (1999) conducted his research, which involved the use of TI-92 graphing calculators in five ninth-grade Geometry classes, there was an initial fifty-minute training period in which the students were walked through the graphing calculator program and instructed on how to use it properly. Without this initial training, it is reasonable to expect the entire interaction with technology to be rendered moot. With proper training and proper pace, the speed of technology is actually an asset rather than a hindrance.

The theme here is clearly that the naysayers regarding the infusion of technology in the Geometry classroom are simply targeting the wrong source. Almost all pseudo-problems I have encountered in my research are unjustly pointing a finger at technology when it is really up to the one wielding that power to use to it correctly. To say that technology is inherently flawed and therefore should not enter the Geometry classroom is similar to saying that microwaves are unsafe because people occasionally set their food on fire, so we should all go back to rubbing sticks together to warm our food.

Technologies such as the Interactive White Board, applications for the graphing calculator, and the software programs available for computers clearly have the power to reshape the Geometry classroom of today. These technologies represent the next wave of the future, and anyone who completes an unbiased analysis will surely determine that the infusion of technology in the Geometry classroom is a beneficial shift that teachers, students and parents alike should embrace. Despite the obvious benefits, the idea of introducing technology into the Geometry classroom has often been received with pessimism and doubt regarding whether or not it is truly worth its inclusion. The counter-arguments against this progress range from claiming that technology is ineffective, or that it causes teachers to rush through material too quickly for students to comprehend, to the claim that many times technology is a distraction which boasts nothing more than entertainment – rather than educational – value. However these pseudo-problems are all pointing the finger in the wrong place. In most cases if technology is ineffective, rushed, or distracting, this is simply due to the misuse of technology on the part of the user – in this case the teacher. When technology is used properly, its benefits in the Geometry classroom are undeniable. Students are able to analyze more, to think more deeply, and are motivated to do so because of technology. Rather than fighting against the clearly beneficial use of technology, it is the duty of our teachers to evolve and adapt their teaching styles to best utilize it and in order to help ensure the best use of technology, it is our duty to support teachers in gaining the skills and knowledge they need to cope with this revolutionary change in teaching Geometry today.

References

Brown, A. (1999) Geometry's Giant Leap. *Mathematics Teacher, 92(9), 816-819*. Retrieved July 7, 2012, from ProQuest Education Journals. (Document ID: 204665347).

Cabri – CabriLog SAS (2009). Cabri II Plus (Version 1.4.5) [Software]. Retrieved on August 25, 2012 from http://www.cabri.com/download-cabri-2-plus.html.

Coffland, D., & Strickland, A.. (2004). Factors Related to Teacher Use of Technology in Secondary Geometry Instruction. *Journal of Computers in Mathematics and Science Teaching, 23*(4), 347-365. Retrieved June 26, 2012 from Education Resource Information Center (ERIC #EJ724756).

Cuban, L.. (1986). Teachers and Machines. The Classroom Use of Technology Since 1920. New York: Teachers College Press.

GeoGebra (2012). GeoGebra 5 (Version 4.2) [Software]. Retrieved on October 27, 2012 from http://www.geogebra.org

Glass, B. (2001) Making Better Use of Computer Tools in Geometry. *Mathematics Teacher, 94(3), 224-229.* Retrieved July 7, 2012, from ProQuest Education Journals. (Document ID: 204632941).

Gonzalez, G., & Herbst, P.. (2009). Students' Conceptions of Congruency through the Use of Dynamic Geometry Software. *International Journal of Computers for Mathematical Learning, 14(2), 153-182.* Retrieved July 12, 2012, from Education Resource Information Center (ERIC #EJ861494).

Hirsch, C..(1981). Those Amazing Triangles. *Mathematics Teacher, 74(6), 444-448*. Retrieved June 27, 2012, from ERIC Educational Resources Information Center. (ERIC # EJ251481)

Marzano, R.. (2009) Teaching with Interactive Whiteboards. *Educational Leadership,* 67(3), 80-82. Retrieved July 3, 2012, from http://www.ascd.org/publications/educational-leadership/nov09/vol67/num03/Teaching-with-Interactive-Whiteboards.aspx

NCTM – National Council of Teachers of Mathematics (2012). The Technology Principle. Retrieved on July 13, 2012 from http://www.nctm.org/standards/content.aspx?id=26809.

Wolfram – Mathematica 8 (2012). Wolfram Mathematica 8 [Software]. Retrieved on August 25, 2012 from http://www.wolfram.com/mathematica/.

Zbiek, R. (1996). The Pentagon Problem: Geometric Reasoning with Technology. *Mathematics Teacher, 89(2), 86-90.* Retrieved July 10, 2012 from Education Resource Information Center (ERIC #EJ518927).

NCTM offers Free Mobile Apps

Pick-A-Path: Help Okta the Octopus reach the target by choosing a path from the top of the maze to the bottom. Seven levels with seven puzzles will test students' skills with powers of ten, negative numbers, fractions, decimals, and more.

Deep Sea Duel: Students may collect cards with a specified sum to win a duel before Okta does! Choose how many cards (9 or 16), what types of numbers (whole numbers or decimals), and Okta's level of strategy (easy to hard).

Equivalent Fractions: Create equivalent fractions by dividing and shading squares or circles, and match each fraction to its location on the number line. Check your work, and use the table feature to capture results and look for patterns.

Math Concentration: Match whole numbers, shapes, fractions, or multiplication facts to equivalent representations.

For more information, visit the NCTM website: http://www.nctm.org/mobileapps/

Dynamic Representations and Flexible Trigonometric Thinking

Randall E. Groth, Ph.D.
Salisbury University
Salisbury, Maryland

Right-triangle diagrams can be used to depict and analyze situations where an angle measurement or side length needs to be determined. Using right triangles in conjunction with the unit circle allows us to determine the values for sine, cosine, and other trigonometric functions in different quadrants of the coordinate plane. The values that trigonometric functions take on, as shown by the unit circle, can be visualized with Cartesian graphs. These graphs help reveal characteristics such as periodic behavior. Much of the beauty of trigonometry lies in how these representations complement and build upon one another.

Unfortunately, our students often miss the beauty of the subject because they have not developed the flexibility of thinking needed to recognize the relationships among different representations. Part of this problem can be attributed to the manner in which trigonometry is conventionally taught. Thompson (2008) noted that the trigonometry of right angles and the trigonometry of periodic functions are often treated in isolation. Weber (2008) agreed, noting that the calculation of ratios in static triangles is often over-emphasized in comparison to time spent on building functional understanding. To avoid these instructional pitfalls, it is important to help students interact with several trigonometric representations in tandem. If students are to develop flexible trigonometric thinking, they need to understand how representations build upon one another rather than to develop isolated bits of knowledge about various representations.

This article describes an activity involving multiple, linked trigonometric representations in a dynamic geometry software environment. The overarching goal of the activity was to illustrate links among trigonometric functions, the unit circle, Cartesian graphs, and tables of values. The *Geometer's Sketchpad* (GSP) (Jackiw 2009) document created to support the activity can be downloaded from http://faculty.salisbury.edu/~regroth/GSP%20files.html. A free demo version of GSP to view the file can be downloaded from www.keypress.com. To begin, the structure of the GSP document will be described below. Then, an instance of the implementation of the activity with a group of prospective secondary school teachers will be discussed.

Dynamic Trigonometry Representations

The GSP document for the activity consists of six pages. Each page contains multiple dynamic representations of trigonometric functions. Students are asked to manipulate different aspects of each representation and observe the results. Items for discussion are included on each page to help students organize their thoughts and reflections about the representations. The dynamic representations in the document and accompanying items for discussion are designed to help students develop in-depth, flexible understanding of sine, cosine, and tangent.

Page 1: Dynagraphs and tables

On the first page of the GSP document (Figure 1), students interact with a non-conventional representation called a dynagraph (Goldenberg, Lewis, and O'Keefe 1992) and an accompanying table of values. The marker on the "output" axis of the dynagraph traces out the range of a function in red as the marker on the "input" axis is dragged left and right. Dynagraphs illustrate the role of domain and range values in defining a function. Students are not immediately given the symbolic representation for the function, but are asked to conjecture what it might be after observing the oscillating behavior of the dynagraph while dragging and recording several input/output pairs in a table. The values from the dynagraph automatically appear in the table, and students can record important values for the function by double-clicking it. After students make their conjectures, the "Show Function f" button can be used to reveal that the function portrayed is $f(x) = \sin x$.

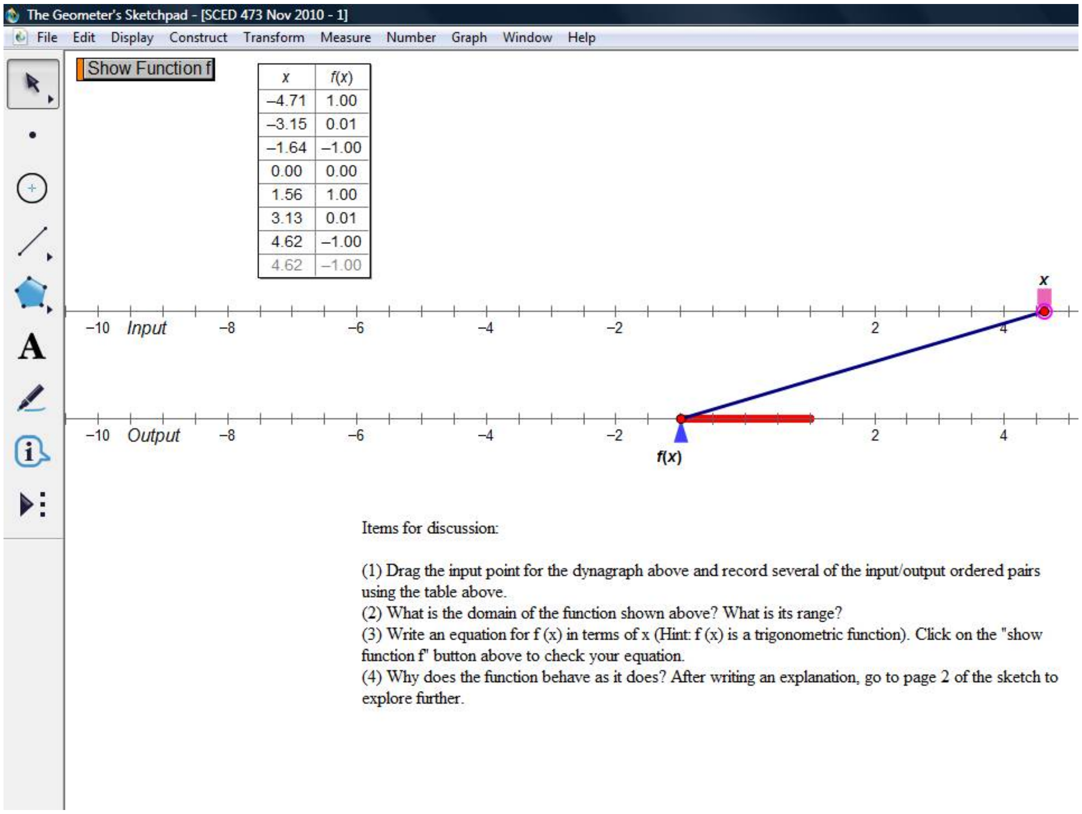


Figure 1. A dynagraph representation of $f(x) = \sin(x)$

Page 2: Unit circles, Cartesian graphs, and tables

Page 2 of the GSP document (Figure 2) contains three linked conventional representations: the unit circle, a table of values, and a Cartesian graph. Students are asked to drag a point around the unit circle and note the characteristics of the Cartesian graph that is traced out as they do so. The table in the bottom left corner of the page can be used to record important values for the function, such as locations of maxima, minima, and roots. In working with page 2, students should begin to associate the *y*-coordinate of the point on the unit circle with the value of the sine function. The animation button in the top left corner can be used to automate the linked processes of dragging the point around the unit circle and observing the effects on the Cartesian graph. The page is designed to enhance students' understanding of the conventional representations by showing, in real time, how a change in one appears in the others.

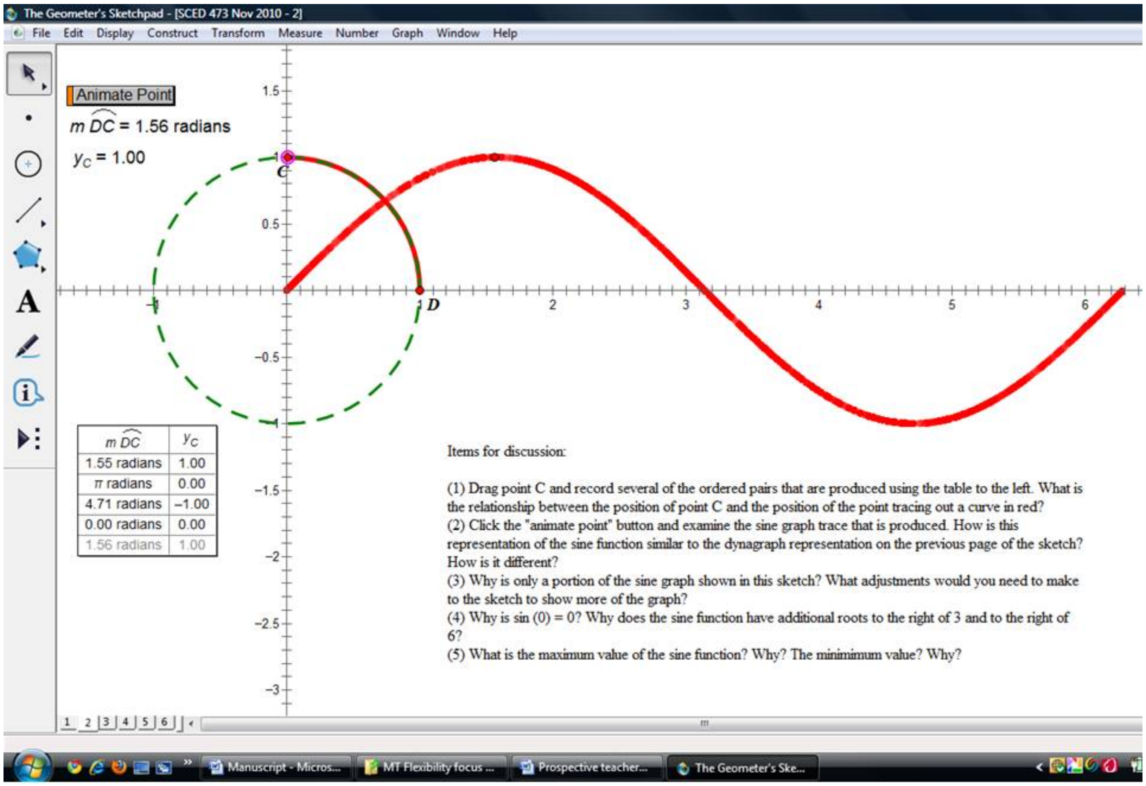


Figure 2. Dynamically linked Cartesian graph, table, and unit circle for $f(x) = \sin(x)$

Pages 3-6: Dynamic representations for cosine and tangent

The rest of the pages in the GSP document focus upon the cosine and tangent functions by using the same techniques illustrated above. On page 3, a dynagraph representation for cosine is given, and page 4 contains a table, unit circle, and Cartesian graph for cosine that are linked to one another. These pages help illustrate the oscillating nature of cosine, its roots, maxima, and minima, and how its Cartesian graph relates to the *x*-coordinates of the unit circle. Comparing pages 2 and 4 to one another helps show that cosine can be thought of as a horizontal shift of the sine function. Several other common characteristics of sine and cosine also become apparent in examining pages 1-4, such as their oscillating natures, their ranges, and their periods.

At first glance, pages 5 and 6 look very similar to pages 3 and 4. The same pattern of examining a dynagraph on one page and then examining three linked conventional representations on the other page is repeated. However, the dynagraph on page 5 exhibits strange behavior. As the input marker approaches several different values, the line connecting the input and output shoots quickly from one direction to the other (Figure 3). Students can use the table linked to the dynagraph to record several values at which this behavior occurs. Using the "Show Function f" button" to reveal that the function illustrated on page 5 is *f* (*x*) = tan (*x*) helps to explain the odd behavior of the dynagraph. Since $\tan(x) = \frac{\sin(x)}{\cos(x)}$, it is undefined for any values of *x* that make cos (*x*) = 0. The dynagraph represents these values of *x* by showing a rapid change in the output value as the input nears one of them.

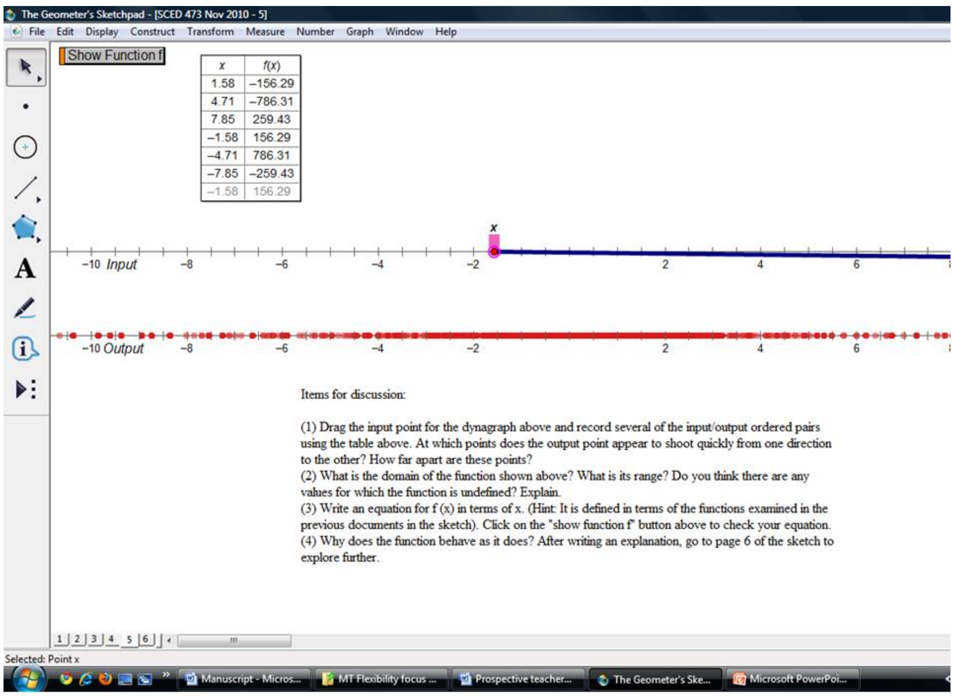


Figure 3. Linked dynagraph and table representations for *f* (*x*) = tan (*x*).

The unusual behavior of the tangent function is further illustrated on page 6, which contains the linked unit circle, table, and Cartesian graph for *f* (*x*) = tan (*x*) (Figure 4). The points at which the dynagraph rapidly changed correspond to vertical asymptotes in the Cartesian graph. Students can also see that the *y*-coordinate of the Cartesian graph of *f* (*x*) = tan (*x*) is determined by a quotient involving the *x* and *y*-coordinates of the movable point on the unit circle. The discussion items on page 6 invite students to predict the behavior of other trigonometric functions defined as quotients, such as secant, cosecant, and cotangent. Therefore, although the GSP document only explicitly provides representations for sine, cosine, and tangent, it provides as a launching pad for analyzing the behavior of all six common trigonometric functions.

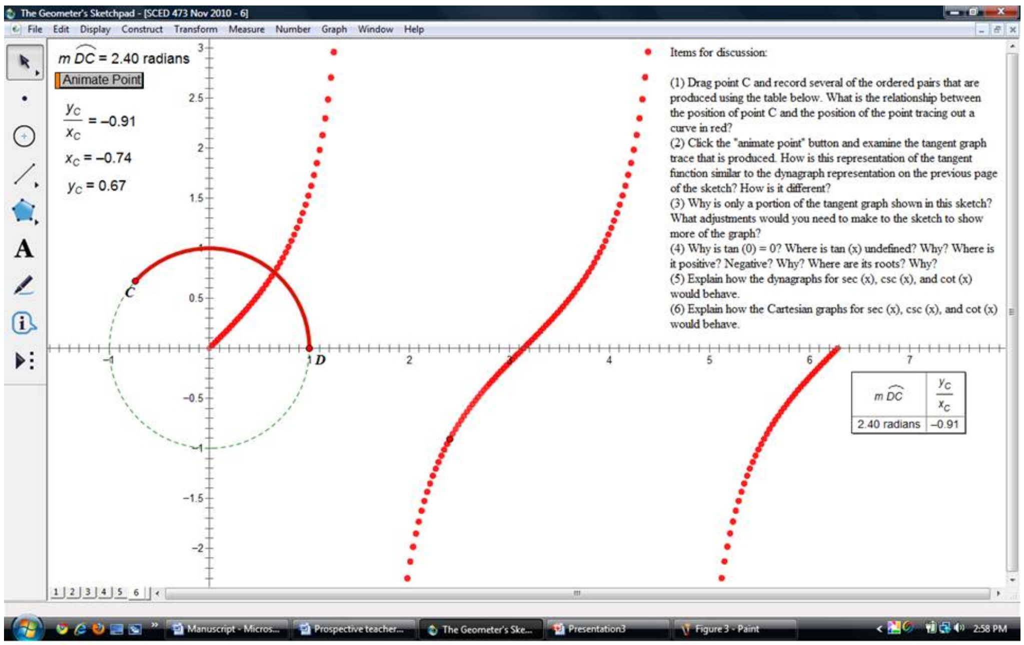


Figure 4. Dynamically linked Cartesian graph, table, and unit circle for *f* (*x*) = tan (*x*)

Implementation with Prospective Teachers

Although the activity described above fits well within most high school trigonometry curricula, it is also useful for the purpose of teacher education. High school teachers need to develop deep understanding of trigonometry, functions, and function representations to help foster students' conceptual understanding. Because of this, the GSP activity described above was used with a group of nine prospective secondary teachers to help enhance their understanding of these ideas. The development of such understanding is a valuable goal because it can help teachers see how concepts build upon one another across grade levels, answer unanticipated student questions, and evaluate students' mathematical claims (Cooney, Beckman, and Lloyd 2010). The group was asked to download the GSP file, work through the questions contained in it, and then participate in an online discussion based upon the following prompts:

- What mathematics did you learn from interacting with the sketch?
- Did you have any difficulties completing the activity? If so, describe them.
- What was your favorite part of the activity? Your least favorite part? Why?
- How might you use or adapt portions of this activity for use in your own classroom in the future?
- How might dynagraphs help students develop better understanding of functions?
- How might the unit circle help students develop better understanding of trigonometry?

As the prospective teachers discussed their work on the activity with one another, it was apparent that they were beginning to develop some of the types of flexible knowledge for teaching functions described in NCTM's *Developing Essential Understanding of Functions*: *Grades 9-12* (Cooney, Beckman, and Lloyd 2010).

Developing Essential Understanding of Functions

One component of a rich understanding of functions involves understanding function definitions conceptually. Teachers should know that "Functions are single-valued mappings from one set – the *domain* of a function – to another – its *range*" (Cooney, Beckman, and Lloyd 2010, p. 8). Some of the prospective teachers completing the activity noted that the dynagraphs helped them understand the idea of function in ways not possible through conventional representations. Joan (a pseudonym, as are the rest of the prospective teachers' names), for example, remarked,

> The dynagraph was very intriguing. It was a great way to visualize the range and domain of the function. It also allowed you to see many values of *x* being plugged into the function and the output. Using this in the classroom could potentially allow students to see what a function does. Normally students just see a few values plugged into the function and the graph. Actually seeing what the function is doing on the number line re-emphasizes the definition of a function.

Joan's comments emphasize some of the power of a dynamic representation to illustrate the idea of a function. Instead of seeing only a limited number of inputs and outputs, as in a conventional puddle diagram or table, dynagraphs can trace out many input/output pairs for a function in just a few seconds. By examining the pairs and the behavior of the dynagraph in motion, students can gain a sense of the relationship between domain and range.

Developing Essential Understanding of Function Representations

Another essential understanding for prospective secondary teachers to develop is that, "Changing the way that a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations highlight different characteristics, and some may show only part of the function" (Cooney, Beckman, and Lloyd 2010, p. 10). The activity sparked some of the prospective teachers' thinking about how dynagraph representations could be used to support the teaching of function representations. James suggested, "Students could come to the board and manipulate the dynagraph and the other students could record the results and make conjectures about the range and domain." Tina added, "Students could work in groups to develop the sine, cosine, and tangent curves and then use the animation to check their answers." These comments suggest that some prospective teachers saw the dynagraphs as not just another representation to learn, but one that can potentially help students understand conventional function representations, such as Cartesian graphs, that are already central to the existing curriculum.

Nonetheless, agreement about the use of dynagraph representations was not universal. Maria conjectured that students would not do well with dynagraphs if left to explore them on their own, and suggested she may just use them as demonstration tools. George echoed Maria's concern and expanded upon it, stating,

> Maria, I understand your concern about having your students use dynagraphs. I definitely think that it would be overwhelming for students to be assigned this lab if they had just recently learned about trigonometric functions. I like your idea of going over the dynagraphs with the whole class.

> This way, the class can put their heads together to figure out what function they are looking at. Also, the teacher can provide hints as needed without giving away the answer.

Although George agreed with Maria's idea of not putting the dynagraphs directly in students' hands, he did still suggest using them as catalysts for student discourse about their corresponding symbolic representations.

Developing Essential Understanding of Trigonometric Functions

The essential understanding most commonly discussed during the GSP activity was one related to trigonometric functions:

> Trigonometric functions are natural and fundamental examples of periodic functions. For angles between 0 and 90 degrees, the trigonometric functions can be defined as the ratios of side lengths in right triangles; these functions are well defined because the ratios of side lengths are equivalent in similar triangles. For general angles, the sine and cosine functions can be viewed as the y- and x-coordinates of points on circles or as the projection of circular motion onto the y- and x-axes" (Cooney, Beckman, and Lloyd 2010, p. 9).

This essential understanding speaks to the central purpose of the activity: to deepen understanding of trigonometric functions by depicting them with linked representations.

Prospective teachers' comments indicated that the activity succeeded in prompting in-depth, flexible thinking about relationships among representations. After doing the activity, George commented, "I thought it was interesting to compare the vertical position of the point on the unit circle to the vertical position of the point on the Cartesian graph. I have never compared these two aspects before." His comment prompted Maria to go back to the GSP file and examine it again. After doing so, she remarked,

> I never thought of it like that, George. Now looking back at the lab it is interesting to see that we are able to compare the vertical position of the point on the unit circle to the vertical position in the Cartesian plane. I too have never compared those two aspects and I think it would be a great idea to show to students in order for them to make that connection.

Others in the discussion agreed that the GSP file helped show the relationship between the unit circle and Cartesian graphs in ways they had not previously considered. This newfound understanding expanded the range of strategies in their possession for representing trigonometric ideas during instruction.

Even though trigonometry is often considered a high school subject, the activity and its results suggest that prospective teachers need opportunities to make connections among trigonometric representations in college before being asked to teach them. The reason for this need was expressed well in an exchange between Brian and Maria:

> Brian: When I first learned trigonometry, I was taught triangle trig only. I did not retain this at all. When I didn't have my calculator, I could not perform trig functions because I really had no knowledge as to how you would find such a thing. In college, I really started to understand circle trig and this sort of approach. If I were introduced to this in high school, I feel like I would have understood trig a lot better in general.
>
> Maria: That is so true, Brian. I too memorized values for trig functions, however I was given a table 0, 30, 60, 90 to memorize the different values for sine, cosine, and tangent at each, then in a later class I was showed the unit circle and it never really stayed with me because I had already memorized the values – therefore I didn't see the point of learning the circle. In addition, I never really understood how trig functions worked and it took me a while to figure it out.

Although Brian and Maria studied trigonometry in high school, they relied upon memorization of discrete facts in order to succeed. The GSP activity helped prompt them to make connections among ideas and representations they previously considered to be unrelated to one another.

Conclusion

Although the activity described above was used with prospective teachers, readers are encouraged to use it with high school students as well. The GSP document for the activity helps users see relationships among multiple trigonometric representations by showing how a change in one representation corresponds to changes in others. Understanding relationships among representations of concepts is a hallmark of flexible mathematical thinking. By taking advantage of dynamic geometry software environments, teachers can better illustrate the beauty of trigonometry by providing a window on how representations work together and complement one another to form a coherent whole.

References

Cooney, Thomas J., Sybilla Beckman, and Gwendolyn M. Lloyd. *Developing Essential Understanding of Functions*: *Grades 9-12*. Reston, VA: NCTM, 2010.

Goldenberg, Paul, Philip Lewis, and James O'Keefe. "Dynamic Representation and the Development of a Process Understanding of Functions." *In The Concept of Functions: Aspects of Epistemology and Pedagogy, MAA Notes, Volume 25,* edited by Guershon Harel and Ed Dubinsky, pp. 235-260. Washington, DC: Mathematical Association of America, 1992.

Jackiw, Nick. (2009). *The Geometer's Sketchpad, v5.0*. Berkeley, CA: Key Curriculum Press.

Thompson, Patrick W. "Conceptual Analysis of Mathematical Ideas: Some Spadework at the Foundations of Mathematics Education." In *Proceedings of the 32nd Annual Meeting of the International Group for the Psychology of Mathematics Education, Volume 1,* edited by Olimpia Figueras, José Luis Cortina, Silvia Alatorre, Teresa Rojano and Armando Sépulveda, pp. 45-64. Morélia, Mexico: PME, 2008.

Weber, Keith. "Teaching Trigonometric Functions: Lessons Learned from Research." *Mathematics Teacher* 102, no. 2 (September 2008): 144-150.

Puzzler - *Stew Saphier*

Answer: 4 inches

Explanation: The front cover of volume one is immediately adjacent to volume two. The back cover of volume five is immediately adjacent to volume four. Therefore the bookworm eats through the front cover of volume one, volumes two, three, and four, and the back cover of volume five. That's 1/8 + 3(1¼) + 1/8 = 4 inches.

Sonia Kovalesvsky: The Patron of Math Days

Betty Mayfield, Ph.D.
Jill Bigley Dunham, Ph.D.
Hood College

Sonia Kovalevsky, Sofia Kovalevskaya, Sophie Kowalevski: however you choose to spell it, this name has been buzzing in the high school mathematics scene in recent years. What does a 19th century Russian have to do with your female students?

A Sonia Kovalevsky High School Mathematics Day (SK Day) is a day-long program for female middle or high school students and their teachers. An SK Day might include talks or workshops on mathematical topics, problem-solving competitions, or information on careers in mathematics (Association for Women in Mathematics, 2011.) SK Days are organized at the local college or university level, so no two are alike. They take place throughout the country and throughout the year. The one commonality among all SK Days is the goal of encouraging young women to consider a future in the area of mathematics.

Who was Sonia Kovalevsky?

The "patron saint" of SK Day, Sonia Kovalevsky was a remarkable woman who overcame a number of societal obstacles in order to pursue mathematics. She was born Sofia Krukovsky in 1850 to an aristocratic family in Moscow, Russia (Osen, 1974.) Her authoritarian father was a general in the Russian army, but her grandfather and uncles were mathematicians. Kovalevsky was raised largely by governesses and nannies at her family's estate at Palobino (Perl, 1978.)

As legend has it, Kovalevsky was first fascinated by mathematics when she saw pages from a calculus textbook used to cover a wall in her bedroom when her family ran out of wallpaper. She was encouraged by her family to some degree; they hired a private tutor to teach her mathematics.

Kovalevsky was interested in both writing and mathematics, but opportunities for her to study at a university were limited.

By 1860, St. Petersburg University was allowing women to attend lectures as non-degree students. However, just as academic life was becoming more accepted for women, a series of student protests in 1862 ended with a shutdown of the universities in Russia. Upon reopening in 1863, women were no longer allowed even to attend lectures, as part of a crackdown on the universities' perceived liberality. After a brief period of increased opportunity, women were again barred from all Russian universities (Perl, 1978.)

Since her own country offered no opportunity for study, Kovalevsky hoped to study in another country. However, foreign travel was frowned-upon for single women. The solution chosen by many women at the time was to find a man to marry platonically; then foreign travel would be allowed, and the woman's sisters could join her as well without scandal. She found such an arrangement with Vladimir Kovalevsky, a paleontology student at Moscow University.

Sonia studied for a time in Heidelberg and then Berlin. She still experienced discrimination: Berlin's university would not accept her, but the famous Karl Weierstrass recognized her abilities and gave her private tutoring. She ultimately earned her Ph.D. from the university at Göttingen, Germany, making her the first woman to earn a Ph.D. in mathematics.

Kovalevsky's research included her dissertation "On the Theory of Partial Differential Equations," papers "On the Reduction of a Definite Class of Abelian Integrals of the Third Range," "Supplementary Research and Observations on Laplace's Research on the Form of the Saturn Ring," "On the Property of a System of Equations," and the Prix Bordin-winning "On the Problem of the Rotation of a Solid Body about a Fixed Point."[2] (These are, of course, the English translations of her work; they were originally published in German, Russian, or French.)

[2] In French: "Mémoire sur un cas particulier du problème de le rotation d'un corps pesant autour d'un point fixe, où l'intégration s'effectue à l'aide des fonctions ultraelliptiques du temps"

Despite being the first woman to be a member of the Russian Academy of Sciences, she was still offered no teaching position in Russia. After moving around Europe for most of her life, she took a position as a professor at the University of Stockholm in Sweden in 1889, the only real professorship available to her as a woman at the time.

In 1891, Kovalevsky contracted influenza while traveling. She died at age 41. In addition to her contributions to mathematics and trailblazing for women in the sciences, she also raised a daughter, was politically active, and wrote several novels.

How Did SK Days Get Started?

In 1985, the Association for Women in Mathematics held a symposium on "The Legacy of Sonya Kovalevskaya" at Radcliffe College (Kovalevskaya, 1985.) One of the current authors was in attendance. It was a truly exciting experience, as mathematicians and historians from across the country spent three days exploring the life and work of Sonia Kovalevsky and the impact her work has had on mathematics. The same year, the first Sonia Kovalevsky High School Mathematics Day was held at the Bunting Institute, near Boston. The focus of this first SK Day was the development of tools for mathematics education.

Over time, the number of SK Days being held across the country has grown. The Association for Women in Mathematics sponsors between twelve and twenty such events every year; other SK Days are held with local or other funding.

Because the funding comes from many sources and Kovalevsky's name is freely associated with these programs, it is almost impossible to estimate how many SK Days are held annually around the United States. A Google search for "Sonia Kovalevsky day" yields over 20,000 results, many of which are sites for individual SK Day programs (Google.com, 2011.)

SK Days at Hood College

Hood College held its first Sonia Kovalevsky Day in 2002. In that first year, we hosted a few hesitant teachers who each brought one or two students. But our county mathematics curriculum specialist also attended, and the next day she raved in a memo to her boss, "One of the neatest things was that it was all women. It was interesting to see the inspiration and love for mathematics come out in all of us for a day – as you know it is so easy to get bogged down with the daily needs within our jobs that to have the freedom to just enjoy learning math for a day was refreshing!!" With her support, our program grew to include more schools, more teachers, and more students. In 2010, Hood held its largest SK Day ever, where more than 40 students from all 10 of Frederick County's high schools participated.

What Goes On at SK Day?

Every event is different, but at Hood we have settled on a program of workshops, a career panel, and some informal time (over lunch, for example) for the high school students to visit with college math students. Some popular math workshops we have offered include:

- The Wacky World of Square Geometry (taxicab geometry)
- To Be or Knot to Be (knot theory)
- Crack the Code! (cryptography)
- The Junk-Mail Mystery (barcodes on mail)
- Coloring a Map (graph theory)
- The Sierpinski Gasket Fractal (created in Geometer's Sketchpad)
- Buffon's Hot Dog (estimating pi by throwing frozen hot dogs onto a grid -- really)
- Listening to Rational and Irrational Numbers (using a monochord to play musical notes)
- Barbie Bungee-Jumping (predicting and estimating).

The idea is to introduce the students to a topic that they have never heard of -- and never thought of as mathematics. The workshops are hands-on, active, and fun. The participants meet and work with students from other high schools.

Our students already know that they can become math teachers if they study math; they have evidence of that in front of them every day. But what else can they do with math? That is where the career panel comes in. We try to find women with interesting careers who use math in their jobs. Some people who have served on our career panel:

- Mathematician at the National Security Agency
- Economist for the Bureau of Economic Analysis
- Patent examiner in cryptography for the U.S. Patent Office
- Scientist at the Johns Hopkins Applied Physics Lab
- Scientific Programming Manager at the National Cancer Institute
- Sociology professor.

We have also invited a representative from our career center to talk about the job prospects for mathematics majors.

The members of our student chapter of the Association for Women in Mathematics give up a day of their spring break to volunteer at this event. They help at registration, play games and work puzzles with early arrivals, administer temporary tattoos, give directions, assist with the workshops, put up signs and balloons, and sit with the high school participants at lunch. One student gives a presentation on the life of Sonia Kovalevsky. Some comments from those Hood students:

> My interest in math, in general, was renewed and new interests were sparked for future studies. The day provided me with a great perspective on the possibilities I can look forward to after graduation.
>
> This was a fun and informative event for the high school girls and for myself. We were introduced to different kinds of mathematics, as well as different careers using mathematics.
>
> I think this is a great program, because as we saw at lunch, many of the students had never been to a college before. This event gave them the opportunity to meet college students, professors, and staff. The students were able to get a feel for what it is like at a college. Also, the career panel gave the students a great idea of how important it is for them to have a college degree. Overall, I think that this is a great program for high school girls in this area and I would love for the program to continue for many years in the future.

What are the Expenses Involved in Hosting an SK Day?

The cost depends on many things: Is the event on a school day? Do you need to pay for substitute teachers? School buses? Some colleges hold SK Day on a Saturday, in order to avoid those costs.

Will you serve lunch? Donuts and coffee? Many schools also supply the participants with T-shirts. Will you give the attendees folders, brochures, pencils, pens?

The Association for Women in Mathematics gives grants to colleges and universities who want to host a Sonia Kovalevsky Day. From their web site (Association for Women in Mathematics, 2011):

> *AWM awards grants ranging on average from $1500 to $2200 each ($3000 maximum) to universities and colleges. Historically Black Colleges and Universities are particularly encouraged to apply. Programs targeted toward inner city or rural high schools are especially welcome.*
>
> *The deadline is in early August each year.*

The Mathematical Association of America, working with the Tensor Foundation, gives awards under its Women and Mathematics Grants program, “for projects designed to encourage college and university women or high school and middle school girls to study mathematics.” One possible project listed on their web site (Mathematical Association of America, 2011) is to “bring high school women onto a college campus for a Math Day with follow-up.” The deadline for these grants is February 12th each year.

Is It Worth It?

A quick perusal of the student feedback forms from previous years gives us plenty of evidence that hosting an SK Day is worth the effort many times over. For one recent year, the responses from the student participants were as follows:

The student comments were equally encouraging:

“Of course! I will definitely take math classes in college!”

“Yes, it opened my eyes to a whole new world of possibilities in math!!”

“I love it [math] even more!”

“I do plan on going to college. Hood is a place I want to go.”

“I realized there are a lot more career options involving math than I thought there were”

“This was really cool- thanks!”

“Cracking codes is freakin fun!”

“I really found topology to be exciting and interesting”

“I am much more interested in persuing [sic] a career in math”

“It has enlightened me about all the other jobs available that I have never considered to be a math profession.”

Table 1

Student evaluations of Sonia Kovalevsky Day at Hood College

(N = 36)

	Strongly Agree (%)	Agree (%)	Neutral (%)	Disagree (%)	Strongly Disagree (%)
I enjoyed SK Day	69	31	0	0	0
I learned some new mathematics today	23	57	17	3	0
Participating in SK Day has given me a greater appreciation for mathematics.	39	50	11	0	0
Because of SK Day, I am more likely to take math classes in college.	35	32	29	3	0

Conclusion

Hosting a Sonia Kovalevsky Day for young women is exhausting, stimulating, and lots of fun. It provides a place for secondary and higher-ed students and faculty to spend a day together, celebrating mathematics and the life of a remarkable mathematician. Most importantly, it encourages students to continue taking math classes in high school and beyond, by showing them how useful and exciting mathematics can be.

Hood students administer Pi Tattoos as part of SK Day

References

Association for Women in Mathematics. (2012). Sonia Kovalevsky High School Mathematics Days. Retrieved from http://sites.google.com/site/awmmath/programs/kovalevsky-days.

Google.com. "Sonia Kovalevsky Day". Accessed August 18, 2011.

Kovalevskaya, S., & Keen, L. (1985). *The Legacy of Sonya Kovalevskaya: Proceedings of a symposium*. Providence, RI: American Mathematical Society.

Mathematical Association of America. (2013). Tensor Women and Mathematics Grants. Retrieved from http://www.maa.org/wam/tensor.html

Osen, L. M. (1974). *Women in Mathematics*. Cambridge, MA: MIT Press.

Perl, T. (1978). *Math equals: Biographies of women mathematicians and related activities.* Parsippany, NJ: Addison-Wesley/Dale Seymour Publications.

The Maryland Council of Teachers of Mathematics (MCTM) is the professional organization for Maryland's teachers of mathematics. Our members represent all levels of mathematics educators, from preschool through college. We are an affiliate of the National Council of Teachers of Mathematics (NCTM). Our goal is to support teachers in their professional endeavors and help them to become agents of change in mathematics education.

First Name: ______________________ Last Name: _________________________

Home Address: ____

City: ________________________ State: ________ Zip: _____________________

Home Phone: _______________ E-mail address: __________________________

MCTM does not share email addresses

School System or Affiliation: ____

Level: Check applicable categories:

__ Early Childhood (Pre-Kindergarten to Grade 2)
__ Elementary (Grades 3 – 5)
__ Middle School (Grades 6 – 8)
__ High School (Grades 9 – 12)
__ Higher Education (Grade 13+)

Signature: _______________________________________ Date:____________

Join on-line at http://www.marylandmath.org or mail a $15 check, made out to MCTM, and this completed form to:

Ed Nolan
10299 Huron Trail
New Market, MD 21774

MCTM Board of Directors

The MCTM Board of Directors and Committee Chairs are here to serve you. To contact a member of the board or committee, click on the e-mail link provided.

President	Susan Vohrer	svohrer@aacps.org
President-Elect	Andrew Bleichfeld	andrew.bleichfeld@hcps.org
Executive Director	Ed Nolan	Edward_C_Nolan@mcpsmd.org
Recording Secretary	Melissa Waggoner	melissa_waggoner@hcpss.org
Treasurer	Dolores Bonincontri	dbonincontri@bcps.org
Central Region Rep.	Jennifer Henschen	jhenschen@bcps.org
Western Region Rep.	Kevin Giffhorn	KLGIFFH@carrollk12.org
Southern Region Rep.	Marcie Sun	Marcia.Frank@pgcps.org
Eastern Shore Rep.	Jeanine Brizendine	jeanine.brizendine@qacps.org
Early ChildhoodLevel Rep.	John SanGiovanni	john_sangiovanni@hcpss.org
Intermediate Level Rep.	Verna Washington	Verna_L_Washington@mcpsmd.org
Middle Level Rep.	Babette Margolies	babsmar@gmail.com
High School Level Rep.	Crystal McCubbin (acting)	cmccubbin@bcps.org
Pre-service Teacher Rep.	Emily Graf	egraf1@students.towson.edu
College Level Rep.	Ming Tomayko	mtomayko@towson.edu
Member-at-Large	Luis Lima	LLima@bcps.k12.md.us
Membership Chairperson	Jenny Novak	jennifer_novak@hcpss.org
Nominations Chairperson	Camille Tomlin	camtom47@hotmail.com
New Teacher Resources	Karen Vaden	karen_vaden@hcpss.org
Awards Chairperson	Christine Oberdorf	christineoberdorf@comcast.net
Math Month Chairperson	Bonnie Yost	byost@bcps.org
Mathalon Coordinator	Stewart Saphier	StewSaphier@aol.com
Webmaster	Bethany Kidd	bkidd@bcps.org
Newsletter Editor-in-Chief	Christy Graybeal	graybeal@hood.edu
Banner Editor-in-Chief	Tricia Strickland	stickland@hood.edu
Historian	Babette Margolies	babette_e_margolies@mcpsmd.org
NCTM Liaison	Bill Barnes	william_barnes@hcpss.org
Government Relations Rep.	Nicole Paris	Nicole.Paris@fcps.org
MSDE Representative	Marci Frye	mfrye@msde.state.md.us

The Maryland Council of Teachers of Mathematics is an affiliate of the National Council of Teachers of Mathematics. Membership in the MCTM is open to all persons with an interest in mathematics education in the state of Maryland. To become an MCTM member, please visit our website: https://www.marylandmath.org/membership/join .

Furthermore, the MCTM Board invites all members to become actively involved in our organization. To become involved, please contact one of the officers listed above. We would love to hear from you!

Maryland Council of Teachers of Mathematics
c/o Ed Nolan
10299 Huron Trail
New Market, MD 21774

MCTM Mission Statement: The MCTM is a public voice of mathematics education, inspiring vision, providing leadership, offering professional development, and supporting equitable mathematics learning of the highest quality for all students.