INTEGRATING SONIFICATION INTO CALCULUS INSTRUCTION

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ABSTRACT

The primary goal of this proof-of-concept project is to improve student learning and success rate in calculus by integrating sonification throughout a calculus course. Between February 2005 and February 2008, we will:

- Design laboratory activities in which students use sonification to explore and apply calculus.
- Design classroom activities that build on the laboratory activities, so that sound is a ubiquitous representation of mathematics, just as graphics, numerics, and symbolics are now.
- Measure the impact of these laboratory and classroom sonification activities on student learning and success rate in calculus.

In this paper, we present an overview of our project and model activities to the sonification community. We present this information early in our project, hoping to generate discussion that will help us refine our goals and methods for making sonification an effective calculus learning tool.

1. INTRODUCTION

A significant number of students in the United States, including many at Salisbury University, are not successful in their attempt to learn Calculus. This creates a stumbling block to the education of future engineers, scientists, and mathematicians. The Calculus Reform Movement, which began in the late 1980's, was an attempt to increase the number of students succeeding in calculus.

One of its most successful reform efforts has been support for multiple representations of mathematics, such as graphs, abstract symbols, and words. Use of multiple simultaneous representations is a widely accepted method for improving students' learning and success in calculus, facilitating construction of a conceptual basis for the content [1] and the development of problem-solving strategies [2]. The Calculus Reform Movement initially emphasized representing mathematics numerically, graphically, and analytically, and referred to this approach as the Rule of Three. Soon, this Rule expanded to a Rule of Four which added oral or written representations of mathematics [1]. Sonification as a representation of mathematics has not yet been integrated into calculus instruction, yet Gardner's theory of multiple intelligences suggests that students learn better when multiple senses are engaged in the learning process. One example of this approach is to use vision and audition to reinforce each other in the classroom.

We expect that dovetailing sonification with the Rule of Four to create a Rule of Five [3] will be a successful application of the theory of multiple intelligences. This paper outlines our three-year project, *Sound-Calculus*, to integrate sonification into calculus through biweekly laboratory activities and frequent classroom activities and measure the impact these activities have on student learning of calculus. Byproducts of this project include teaching students how to interpret and use sonifications, identifying design principles for mathematical sonifications, and engaging mathematics, mathematics education, and computer science students in undergraduate research in sonification.

2. PROJECT DESCRIPTION

2.1. Overview

To integrate sonification into a calculus course, we will develop, with advice from an ear-training consultant and a psychoacoustic consultant, five laboratory activities employing sonification of mathematics, primarily using pitch and location as our sound variables [4]. These activities will teach students how to interpret sonifications and connect them to mathematical ideas. More importantly, we will develop corresponding and reinforcing classroom materials to integrate sonification into daily activities. Nationally recognized experts in undergraduate mathematics education will review these materials to increase potential to improve learning, to remove distractions in materials, and to improve usability by other college mathematics faculty. As we implement the materials, we will continually assess the impact of sonification on student learning and student retention and success rates. We will publish an instructor's manual on CD and on the internet. This manual will contain the laboratory activities, the classroom materials, sample sonifications, and instructions and software for students to use to create sonifications with tools such as Microsoft Excel.

[1] Partial support for this work was provided by the Course, Curriculum, and Laboratory Improvement program under grant 0442450 and by Salisbury University.

2.2. Sonification Laboratories

Graphic representation of mathematics is ubiquitous in mathematics classrooms, even for the youngest students. To use sonification effectively in the same way, we need activities in which students learn how to create and interpret sonifications in mathematical contexts. In our plan, these activities occur in biweekly hour-long laboratories using standard PCs and headphones. We begin by developing and implementing the following five laboratory activities.

1. Training exercises: The purpose of this activity is to teach students how to listen to, and begin interpreting, sonifications of mathematical objects. The sound variables we will use for our sonifications are pitch and left-right position.
2. Qualitative properties of functions: It is important for students to understand and be able to recognize (from graphs, formulas, and descriptions) qualitative properties of functions. These properties include where the function is constant, increasing, decreasing, concave up, concave down, positive, and negative. Students will listen to and create sonifications that exhibit these properties, and test their ability to recognize and interpret them. They will produce corresponding graphical and verbal representations of what they hear.
3. Limits: In this activity, students will use sonification to study limits and asymptotes of functions. Students will listen to samples and create their own sonifications representing these concepts. This activity will reinforce their understanding of the difference between convergence and divergence and the meaning of vertical and horizontal asymptotes.
4. Sonifying the Derivative: Certain qualitative properties of a function and certain qualitative properties of its derivative are related. “An increasing differentiable function must have a positive derivative” is an example of such a relationship. Internalizing these relationships is central to understanding calculus, and failing to do so in a timely fashion is often one of the biggest impediments to success in a calculus course. Since the derivative is a function in its own right, Lab 4 is an extension of Lab 2. In lab 2, students learned how to hear and interpret qualitative properties of one function. In lab 4, they will analyze the properties of two functions simultaneously, and learn which properties of each function are related.
5. Extreme values of functions: *Extreme values* are the largest and smallest values of a function and occur at endpoints of its domain, or at *critical points*, values for which the derivative is zero or non-existent. This relationship between a function and its derivative is an important calculus concept, but often difficult for students to internalize. There are three common mistakes in looking for extreme values of a function: forgetting to examine endpoints, forgetting to examine critical points, or assuming that all critical points must be locations of extreme values. Students will listen to and create sonifications to hear the location of extreme values of a function. Then, they will use simultaneous sonifications of a function (perhaps in pitch) and its derivative (perhaps in location) to reinforce and internalize the meaning of this relationship.

Eventually, we would also like to include more activities, such as:

1. Area and Integrals: An area function for *f*, $\int_0^t f(x)dx$, which is the accumulated net area of the region bounded by the graph of *f* and the lines $x = 0$, $x = t$, and $y = 0$, is just as important to calculus as the derivative is. An area function for *f* is related to *f*, qualitatively, as *f* is related to its derivative. Some students, especially those who understand lab 4 well, might draw this connection themselves. In some sense, these students hear The Fundamental Theorem of Calculus: *f* is the derivative of its area function. In any case, we will make the connection for all students in the class immediately following the lab. This is an example of sonification lab activities that are naturally integrated directly into classroom instruction.
2. Gravity wells and function extrema [5]: We are intrigued by [5] and the physical model that motivates this method. This approach for exploring the extreme values of a function of several variables has not been used in introductory calculus.
3. Applications of sonification: Sonification is becoming an important tool in quantitative research. By the end of the semester, students will be equipped to create and interpret elementary sonifications. This lab will give them the opportunity to apply sonification and calculus to a discipline other than mathematics.

2.3. Assessment of Sonification’s Impact

Our materials will be in pilot phase from August 2005 through January 2007. For three consecutive semesters we will be developing laboratory and classroom activities and integrating them into calculus. Students and consultants will be queried for guidance in refining, deleting, or adding features for the next semester.

In the spring semester of 2007 we will assess the effectiveness of our sonification activities in improving student learning in calculus. To do this two sections of calculus, Section A and Section B, will be taught by the same instructor. Students in both sections will complete Laboratory 1. Students in Section A will complete Laboratories 2 and 3 with sonification and Laboratories 4 and 5 without sonification, while students in Section B will complete Laboratories 2 and 3 without sonification and Laboratories 4 and 5 with sonification.

Student performance will then be assessed in two ways. First, we will analyze individual students’ performance on homework, exams, and laboratory exercises with and without sonification enhancement (i.e., a within-subject measure). Second, we will analyze sonification vs. traditional methods by activity, using performance from both sections (i.e., a between-subjects measure).

This design is more complex than one where one section would use our materials and the other would not. However, the added complexity allows us to study the improvements in student learning due to sonification, and factor out improvements due to the use of technology and laboratory activities.

3. CONCLUSION

In addition to the primary intended consequence of this project, improving student learning in calculus, there are three other potential benefits. First, for students who cannot effectively use graphics; i.e., those who are not visual learners or are visually impaired, sonification offers a new channel for communication and discovery. Next, since properties of functions are central to mathematics, our instructional materials will be useful across the mathematics curriculum; e.g., pre-calculus, algebra, statistics, mathematics for liberal arts majors, and mathematics for pre-service teachers. Finally, this project offers mathematics and computer science students significant and accessible problems for undergraduate research.

In conclusion, much needs to be done. We will develop laboratory activities for the students. We will develop an instructor's guide and daily sonification materials, including Excel worksheets that students can use to create and analyze sonifications, and activities for the classroom and homework assignments. In Spring 2007, we will collect data on the impact of sonification on student learning in calculus. This data will be analyzed and disseminated in a variety of venues during the following year. Our goal is to determine how sonification can be an effective means for improving student learning, especially when viewed as part of a Rule of Five. We hope to engage a wide-ranging interdisciplinary group in studying sonification for learning mathematics, using our materials, developing other materials, and assessing the impact of sonification.

4. REFERENCES

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