The Safe Distance Between Airplanes and the Complexity of an Airspace Sector

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Summary

We determine the minimum safe spacing between aircraft and also the complexity of the air traffic control system.

Taking into account the vortex that a leading plane leaves in its wake, the distance between the tail of one plane and the nose of the next plane should be at least 5.5 km or 3.4 mi. The minimum spacing between adjacent planes either to the side, above, or below should be at least 730 m or 0.45 miles. These distances were calculated using Bernoulli's principle, which states that the internal pressure of a fluid (such as air) decreases when its speed increases. Because the speed of a plane is very high, the pressure around the wings is low. The change in pressure associated with the Bernoulli factor, applied over the facing surface area, results in a force pushing the planes together; the force may alter the plane's flight pattern.

Finally, if two planes are heading towards each other, there must be enough space between them to perform evasive maneuvers. We find that 12 s is required; at normal flight speed, this translates to 2.9 km or 1.8 mi.

We define complexity of an airspace sector as the probability of a conflict occurring during a given period of time. To determine complexity, we assume that sectors are rectangular solids and that planes fly either in parallel or antiparallel directions. We calculate the probability that a plane enters a sector either too soon after another plane, or that two planes enter the same airway going in antiparallel directions.

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The weaknesses of this model include that all planes are assumed to be Boeing 767s. This model also does not take into account weather changes and multiple conflicts.

The strengths of this modeling include allowing for passenger safety while slightly shortening the Federal Aviation Administration (FAA) minimum distances, thereby increasing airspace capacity. The complexity model accounts for stress; stability analysis shows that a small change in the environment does not drastically change the model.

Background

According to current FAA separation guidelines, an aircraft must maintain a separation of 5 mi behind the plane and 2 mi adjacent to the plane [Gilbert 1973, 36–37].

Numerous benefits could come from reducing separation standards. Primarily, air space capacity would increase. Delays would be reduced because planes would not have to wait for an open airway. Finally, fuel costs would decrease because planes would be rerouted less frequently, with fewer delays.

Questions remain, however, as to whether other bottlenecks would mitigate these benefits. Potential conflicts can occur in one of two areas:

- Over 75% of collisions occur within the terminal area—the space within 30 mi of an airport—because traffic is dense and constantly changing.
- Other conflicts occur en-route; most en-route airplanes have a constant altitude and velocity [Gilbert 1973, 91].

Our model concentrates on en-route traffic.

The purpose of the air traffic control system is to avoid collisions. Avoidance depends on two basic sources of air traffic information:

- Air-derived collision avoidance involves either the visual detection of a conflict by the pilot or the radar detection of air disturbances around the plane.
- Ground-derived collision avoidance uses radar from ground-based radar antennae. Controllers monitor radar to detect potential conflicts and contact the pilots involved to give them new courses.

Ground-derived avoidance is the primary tool to maintain minimum spacing between planes. The controller of a certain air space gives continuous and detailed instructions to the pilot as to flight parameters that should be taken in the airspace, including heading and altitude. When all aircraft operate under the Ground Collision Avoidance System (GCAS), there is an extremely low rate of mid-air collisions [Collins 1977, 123].

The type of radar most commonly used by GCAS for en-route traffic monitoring is Air-Route Surveillance Radar (ARSR), long-range radar with a range of 200 mi and an altitude range of 40,000 ft. It has a slower rotation (3–6 rpm) than short-range radar (10–30 rpm); because the rotation is slower, accuracy and resolution are not as high [Gilbert 1973, 40].

Other disadvantages in the current radar system are [Federal Aviation Administration 1997, 4.2]:

- ARSR lacks sufficient low-altitude coverage because traffic is concentrated at higher altitudes.
- Radar equipment can be unreliable or malfunction.
- Blind spots exist in the radar pattern, behind large objects and other planes.
- Radar cannot differentiate between targets within 3° of each other from the radar antenna; these objects blur together on the screen.

The capacity of the airspace is limited by the minimum spacing between planes and also by the size of the workload placed on the controller team.

Airspace is broken down into *sectors*; one controller team manages each sector. The controllers must maintain radio contact with each aircraft located in the sector, and they must identify each aircraft on the radar screen. Each aircraft must be assigned a "travel plan" with a vector, heading, and altitude. Controllers must maintain constant surveillance on each aircraft flight pattern to identify potential conflicts. If the number of aircraft in a sector increases, more work is required by the controllers and more separation between planes is necessary to ensure that controllers spot potential conflicts. Controllers must also "transfer" planes that are exiting to a neighboring sector [Federal Aviation Administration 1997, 4.2].

Dividing the airspace into a greater number of smaller sectors would ease the workload of controllers, but the increased work in "sector transfers" and the increased cost in added controller teams and radar would reduce efficiency.

Assumptions in the Model

- All planes behave like Boeing 767s.
- All planes are considered cylindrical rings for physics calculations.
- Weather is not a factor in the safe distance between planes.
- All planes are en route; they are not landing or taking off.
- All planes are flying at about 35,000 ft.
- All planes have a constant speed of 857 km / hour.
- There is no energy loss due to friction.

Table 1.
Symbols in the model (with standard SI-MKS units).

Symbol	Meaning
a	Acceleration
α	Angular acceleration
$C(t)$	Number of planes in crucial beginning area of sector at time t
d	Distance between planes
$\Delta\theta$	Angular rotation
h	Height of sector
I	Rotational inertia
L	Length of sector
ω	Angular velocity
P	Pressure of air
$P(t)$	Probability of conflicts at time t
$R(t)$	Number of planes entering sector at time t
ρ	Density of air
t	Time
v	Velocity
w	Width of sector

Model Development

The model examines the distances that are required between planes from the front, back, above, below, and laterally; different forces and factors account for the different distances required.

When two planes in opposite directions approach each other, we assume that one of the planes descends to avoid a collision. Knowing that it takes 12 s to avoid a collision, we can find the minimum distances. Each plane creates a pair of vortices, areas of strong turbulence, that extend outward and around from the wingtips. A vortex affects the safe distance behind the plane (point c in **Figure 1**); the vortex from a large plane is large enough to damage seriously another plane that follows too closely.

The other important factor is Bernoulli's Principle, which states that the internal pressure of a fluid (liquid or gas) decreases at points where the speed of the fluid increases. The moving air around the wing reduces the air pressure and would cause a nearby plane to accelerate towards the first plane. The force affecting planes above, below, and to the sides (points d, b, e, and f) comes from a combination of the Bernoulli and vortex forces. Enough distance must be allowed between planes to overcome it.

Distance Required in Front of Plane

According to 241 Air Traffic Control Squadron (ATCS) [1999], 12 s is needed to steer clear of another object: 6 s for the controller to radio to the pilot, 4 s for the pilot to start the maneuver, and 2 s to gain enough space to clear. At a speed of 238 m / s, the corresponding distance is 2.9 km.

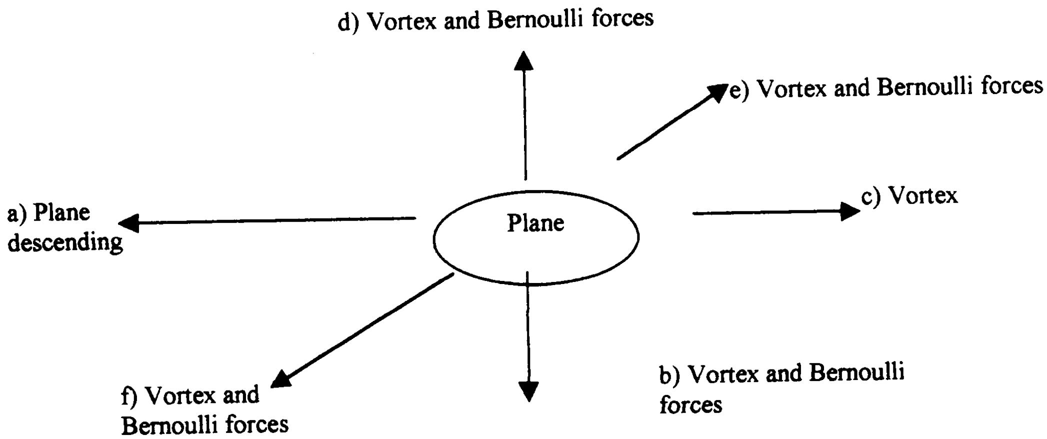

Figure 1. Plane and associated forces.

Distance Required Behind Plane

The minimum safe distance below and behind the plane is determined by the size of the vortex behind the plane. Each wing develops a vortex of air approximately 15.0 m in diameter, spinning at 42.7 m / s. The vortex sinks at 2.03 m / s until it is approximately 244 m below the level of the plane; it is usually 9,250 m long. This vortex is a great danger to following planes because it can cause them to roll [241 ATCS, 1999].

Because the Boeing 767 is 48.5 m long, and the vortex has a diameter of 15.0 m, a column of air with volume 8,850 m^3 acts upon a plane flying in a vortex from another plane. The air density at 10.7 km above the ground (cruising altitude for Boeing 767s) is 0.380 kg / m^3. Hence, the mass of air acting on the plane is 3,360 kg if the plane is flying into the vortex.

We assume that all of the angular momentum of the air is transferred to the plane:

$$I_{\text{air}}\,\omega_{\text{air}} = I_{\text{plane}}\,\omega_{\text{plane}}, \tag{1}$$

where I is rotational inertia and ω is angular velocity.

The air is a spinning disk, whose rotational inertia is given by

$$I_{\text{air}} = \frac{mr^2}{2} = 0.5 \times 3{,}360 \times (7.502)^2 = 9.77 \times 10^4 \text{ kg} \cdot \text{m}^2.$$

The angular velocity of the air at given distance d from the plane can be found by using the properties of the vortex. As it leaves the plane, winds near the edge of the vortex measure at 45.72 m / s. The circumference of the vortex is 47.1 m; it therefore takes 1.05 s for the air to travel through one rotation, which means that the initial angular velocity is 6.00 rad / s. The vortex disappears in 9,250 m; because the plane flies at 238 m / s, this is 38.8 s after the vortex was created. The angular deceleration, then, is found using the formula

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - 6.00}{38.8} = -0.155 \text{ m} / \text{s}^2.$$

The air decelerates at this rate. The plane moves at 238 m / s, so it will take $d/238$ s to travel distance d. The equation for angular velocity at distance d becomes:

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{\omega_f - 6.00}{d/238} = -0.155 \text{ m/s}^2.$$

We solve to find angular velocity ω_f in terms of d:

$$\omega_f = (-6.49 \times 10^{-4})d + 6.00 \text{ rad/s}.$$

Next, we consider the angular velocity and rotational inertia. We assume that the plane spins around its central axis. Because most of the plane's mass is located to the outside, we assume that the plane is a rotating ring. Using the mass and radius of the Boeing 767, we find the rotational inertia:

$$I = mr^2 = (156{,}500)(2.85)^2 = 1.27 \times 10^6 \text{ kg} \cdot \text{m}^2.$$

This calculation assumes that the plane is flying into the vortex, not across it. Across the vortex, its rotational inertia would be that of a pivoting rod, given by:

$$I = \frac{ml^2}{3} = \frac{(156{,}500)(48.5)^2}{3} = 1.23 \times 10^8 \text{ kg} \cdot \text{m}^2.$$

This is far greater inertia than for the plane flying into the vortex. Less inertia means that it takes less force to turn the plane, meaning that the situation is more dangerous. Therefore, to determine the safe distance, we consider further only the approach from behind.

The plane starts with zero angular velocity. We assume that it cannot turn more than 5° (0.0873 radians) in the crossing period (0.938 s) without discomfort and loss of control. Further, we assume that the plane is climbing at an angle of 5° or steeper as it goes through the vortex. At this angle, the plane, traveling at 238 m / s, would be in the vortex for no more than 0.938 s. The final angular velocity of the plane is found using these data and the equations

$$\Delta\theta = \omega_i t + \alpha t^2/2, \quad 0.0873 = 0 + \alpha \times (0.938)^2/2, \quad \alpha = 0.198 \text{ rad/s}^2;$$
$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta = 0 + 2 \times 0.198 \times 0.0873, \quad \omega_f = 0.186 \text{ rad/s}.$$

Substituting these values into **(1)** gives

$$(9.77 \times 10^4)[(-6.49 \times 10^{-4})d + 6.00] = (1.27 \times 10^6)(0.186),$$
$$d = 5,510 \text{ m}.$$

Adding 152 m for radar uncertainty, we get an unsafe zone 5.7 km long behind a plane.

Distance Required Vertically and Laterally

We use Bernoulli's Principle to find the minimum distance on the sides and up and down between planes. The distance between the two planes is d, the initial velocity of the vortex is 45.7 m / s, and the acceleration of the air is the same as the acceleration of the vortex, -1.766 m / s^2.

We apply the equation

$$v_f^2 = v_i^2 + 2ad = (45.7)^2 + 2(-1.77)d = 2.09 \times 10^3 - 3.53d.$$

Next, to determine the change in pressure, we apply Bernoulli's equation governing fluids:

$$P_i + \frac{\rho_i v_i^2}{2} = P_f + \frac{\rho_f v_f^2}{2},$$

where ρ is air density. The initial velocity is zero, and at 35,000 ft the density is 0.380 kg / m^3 and the atmospheric pressure is 2,340 Pa:

$$2.34 \times 10^3 = P_f + \frac{(.380)(2.09 \times 10^3 - 3.53d)}{2}, \qquad P_f = 1.95 \times 10^3 + 0.670d \text{ Pa}.$$

The change in pressure is

$$2.34 \times 10^3 - (1.95 \times 10^3 + 0.670d) = 397 - 0.670d.$$

Since pressure equals force divided by area, the force exerted can be found by using the surface area of the plane, which varies depending on whether the sides or the top and bottom are being considered.

Sides

The length of a Boeing 767 is 28.5 m and the width is 5.7 m. Assuming that the plane is a cylinder and half of it is facing the side, the surface area affected is $(28.5)(5.7)\pi/2 = 434$ m^2. (This is an overestimate, since we neglect the curvature of the body of the plane.) Since pressure = force / area, we have $397 - 0.670d = \text{force}/434$ and the force is $1.72 \times 10^5 - 291d$. We assume that the maximum lateral acceleration, before either the ride becomes too turbulent or the plane loses control, is 0.1 m / s^2. Newton's second law (force = mass $\times$ acceleration) becomes $1.72 \times 10^5 - 291d = (1.57 \times 10^5)(0.1)$, leading to d = 538 m.

The total distance on the sides now needs to be calculated:

$$\begin{aligned} \text{Distance } &= 538 + \text{Vortex Width} + .5(\text{wingspan}) + \text{Uncertainty factor for radar} \\ &= 538 + 15 + 47.6/2 + 152 = 729 \text{ m}. \end{aligned}$$

Therefore, 729 m must be allowed on each side of the plane.

Above and Below

For the vertical viewpoint, we have

$$\text{Surface area} = \text{half cylinder} + \text{wing area} = (48.5)(5.7)\pi/2 + 283 = 717\text{m}^2.$$

The pressure, calculated previously, is $396.8 - 0.670d$. We have

$$\begin{aligned} \text{Pressure} &= \text{Force / Area}, \\ 397 - .670d &= \text{Force}/717, \\ \text{Force} &= 2.85 \times 10^5 - 481d. \end{aligned}$$

Using Newton's Second Law as before, we have

$$F = ma, \qquad 2.85 \times 10^5 - 481d = (1.57 \times 10^5)(0.1), \qquad d = 559 \text{ m}.$$

The total vertical distance is

$$\text{Distance} = 559 + \text{Vortex Width} + \text{Uncertainty factor} = 559 + 15 + 152 = 727 \text{ m}.$$

Therefore, for safety, a plane needs 729 m on each side and 727 m above and below.

Complexity

Complexity, from a workload perspective, we define to be the probability of a conflict—and therefore the need for an air traffic controller (ATC) to intervene—in a certain period of time. A high likelihood of conflict in a short period of time means a lot of potential stress for the ATC. In addition, to accommodate time to recover from stress, the 5 min before the time being considered is also included in the definition of complexity.

We first find the probability of a conflict at a certain point in time. We assume that sectors are rectangular solids ($L \times w \times h$, in m). We divide a sector into *blocks* 727 m tall and 729 m wide, the minimum for Boeing 767 planes to be apart.

We assume that planes fly in either parallel or antiparallel directions. There are two possibilities for conflict.

- A plane enters a block too soon after another plane. The safe distance between planes following each other—the sum of the minimum distance in front of and behind a plane, 8,380 m—should not be violated; so planes entering a block should be 35.2 s apart.
- Two planes enter the same block from opposite ends, in antiparallel directions.

We assume that planes enter a block according to a random process, so that entries to the block are independent.[1]

[1] EDITOR'S NOTE: The presentation here has been adapted slightly from the authors'. The problem that they consider involves the composition of two independent *renewal processes*, one at each end of the block. Consider a block *available* if there is no plane in it; the limiting availability A of a block, as $t \to \infty$, depends only on the mean time between arrivals and the mean time to pass through the block [Trivedi 1982, 297–301 and 305, Problem 1]. The same kind of rectangular block model was used to calculate the probability of a mid-air collision over the North Atlantic [Machol 1975]; the editor thanks Antony Unwin of Universität Augsburg, Germany, for supplying this reference. See also the update at Machol [1995].

Let $m(t)$ be the probability density function for entry of a plane into a block, divided equally between the two directions.

Suppose that a plane enters the block at time t.

- The probability that another plane enters the block in the same direction as the first plane but too close behind it is

$$P_1(t) = \frac{1}{2} \int_t^{t+35.2} m(t)\, dt.$$

- The probability that another plane enters the block from the opposite direction while the first plane is in the block is

$$P_2(t) = \frac{1}{2} \int_t^{t+L/238} m(t)\, dt.$$

Hence, the conditional probability of conflict in a block, given entry of a plane into the block at time t, is the sum of the two probabilities:

$$P(\text{conflict} \mid \text{plane enters at } t) = \frac{1}{2} \left[\int_t^{t+35.2} m(t)\, dt + \int_t^{t+L/238} m(t)\, dt. \right].$$

The unconditional probability density of conflict, assuming independent arrivals of planes in the block, is

$$\begin{aligned} P(t) &= P(\text{conflict} \mid \text{plane enters at } t)\, P(\text{plane enters at } t) \\ &= P(\text{conflict} \mid \text{plane enters at } t)\, m(t). \end{aligned}$$

The most important component of complexity is the probability of a conflict; the most complex situation is a high likelihood of conflict over a sustained period of time.

However, a situation is more complex if the ATC is already stressed from previous problems, so in our definition of complexity for a block over an interval we add, weighted at 10%, the probability of a conflict during the 5 min (300 s) preceding:

$$\text{Complexity for a block} = P(t) + 0.1 \int_{t_1-300}^{t_1} P(t)\, dt.$$

While software to alert controllers of potential conflicts would add to the safety of flight, it would also add to the conflict. Most models and distance estimates, including this model, tend to overestimate the safe distance between planes. Even if this were remedied, there is also the uncertainty of the exact position of the plane due to the imprecision of the radar, which would cause the software to alert the ATC even when no conflict was likely to occur. This would add to the complexity, because the controller would have more potential conflicts through which to sift. However, this added complexity would add to the safety because most possible conflicts would receive ample warning.

Stability

We did a stability analysis in regard to changes in the mass of the planes, the velocity of the vortex, and the pilot's reaction time. The mass of the planes or the velocity of the vortex would be different for a different type of plane, and pilots' reaction times may vary.

With the lateral Bernoulli forces, if the initial velocity of the vortex is changed by 2.2%, the distance is changed by 4.8%. If the mass is changed by 6.4%, the distance is changed by 0.5%. With the vertical Bernoulli forces, if the initial velocity of the vortex is changed by 2.2%, the distance is changed by 4.5%.

The effect of the changes in the pilot's reaction time was analyzed in regard to the distance in front of the plane. If the time is changed by 8.3%, the distance also changes by 8.3%. Therefore, this model is stable with regard to all of the variables tested.

Strengths and Weaknesses

The model provides for safety. Parameters, such as the mass of the plane, can be changed as appropriate. However, the model does not accommodate two planes of different models. The complexity section accounts for controller stress. The model's minimum spacings for aircraft, including taking into account 500 ft of radar uncertainty, reflect FAA guidelines reasonably well.

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