I. Product Rule:

$$\frac{d}{dx}[f(x) \cdot g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + \frac{d}{dx}[g(x)] \cdot f(x).$$

$$(f \cdot g)' = f' \cdot g + g' \cdot f$$

II. Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)] \cdot g(x) - \frac{d}{dx}[g(x)] \cdot f(x)}{[g(x)]^2}.$$

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - g' \cdot f}{g^2}$$

III. Elasticity of Demand:

$$E(x) = -\frac{pf'(p)}{f(p)}.$$

IV. Chain Rule:

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

SCORE_____

1. Let $f(x) = x^2$ and $g(x) = x^2 - 5$. Find each of the following:

(a) $f(1) + g(1)$

[3 points]

(b) $f(0) - g(0)$

[3 points]

(c) $f(1) \cdot g(1)$

[3 points]

(d) $\dfrac{f(1)}{g(0)}$

[3 points]

(e) $(g(f(2))$

[3 points]

(f) $g(1 - a)$

[3 points]

2. Find the domain of $f(x) = \sqrt{4 - x^2}$.

[6 points]

3. Solve the following equation for x:

$$3^{2x} = 3^4$$

[6 points]

4. Solve the following compound inequality for x::

$$-6 < 2x - 4 < 4$$

[6 points]

5. Find the following limits, if the limit fails to exist, explain why:

(a) $\lim_{x \to -2} 4x^2 + x - 16$

[6 points]

(b) $\lim_{x \to 5} \dfrac{x^2 - 8x + 15}{x^2 - 3x - 10}$

[6 points]

(c) $\lim_{x \to -1} \dfrac{2x^2 + 11x + 5}{x^2 + 2x - 3}$

[6 points]

6. Find the derivative of $f(x) = \pi^6$.

[5 points]

7. Use the table given to estimate $\lim_{x \to 1} f(x)$

x	f(x)
0.9	4.19
0.99	4.0199
0.999	4.001999
1.001	3.997999
1.01	3.9799
1.1	3.79

[6 points]

8. Find the derivative of $f(x) = 3x^2 + 4x$.

[6 points]

9. Using the limit definition, find the derivative of $f(x) = 3x^2 + 4x$.

[6 points]

10. It is estimated that t years from now, the enrollment at Oakwood College will be $E(t) = 100t^2 + 375t + 7200$.

(a) Find the average rate at which enrollment will be changing over the period $t = 1$ and $t = 5$.

[6 points]

(b) Find the instantaneous rate of change at year $t = 5$

[6 points]

10. Find an equation of the tangent line to graph of $f(x) = 2x^3 + 5x - 1$ at the point $(1, 6)$.

[8 points]

11. Write as a single logarithm: $\ln 3x + \ln(x - 1) - 2\ln y$.

[6 points]

12. Calculate the derivative of $f(x) = \sqrt[3]{(7 - 3x)}$.

[5 points]

13. Calculate the derivative of $f(x) = \dfrac{x-3}{x^2-x}$.

[5 points]

14. Calculate the derivative of $f(x) = x^2 e^x$

[5 points]

15. A company's profit is $P(x) = -x^2 + 6x - 5$ thousand dollars for selling x hundred units of its product. Find and interpret the rate of change of profit when the company is selling 400 units.

[6 points]

16. Let $V(t) = 16e^{-0.5t}$ be the amount of coffee (in ounces) left in Star's cup t minutes after she receives her drink. Find and interpret $V'(2)$.

[6 points]

17. Find $f'''(x)$ of the following function:

$$f(x) = \frac{2}{x^2}$$

[8 points]

18. Let $f(x) = x^3 - 6x^2 + 15$.

(a) Find all critical numbers.

[5 points]

(b) Find all relative extrema.

[5 points]

(c) Find the largest open interval(s) on which $f(x)$ is concave up and those on which $f(x)$ is concave down.

[5 points]

(d) Find all inflection points.

[5 points]

19. Esther's Desi Vegan Restaurant has been very busy recently. The demand for *chana masala*, one of its popular dishes is $p = -\frac{1}{2}x + 33$ (where p is price and x is the number demanded). What should Esther charge to maximize revenue for this dish?

[8 points]

20. Find the absolute extrema for $f(x) = 2x^2 - 8x$ on the closed interval $[0, 5]$.

[6 points]

21. For a Toyolla Terrain SUV,

$$M(x) = -0.015x^2 + 1.31x - 7.3,\ 30 \leq x \leq 60$$

represents the miles per gallon obtained at a speed of x mph. Find the absolute maximum miles per gallon and the absolute minimum, and the speeds at which they occur.

[6 points]

22. The demand for motorcycle tires imported from Rossi MotoGP Import-Export is 40,000 per year and may be assumed to be uniform throughout the year. The cost of ordering a shipment of tires is \$400, and the cost of storing each tire for a year is \$2, find the order size and the number of orders that minimize inventory costs (storage and reorder costs).

[6 points]

23. Evaluate the integral $\int \left(\frac{1}{3}x^3 - 2x + 8\right)\ dx$

[5 points]

24. Evaluate the integral $\int \left(x^2 - x + \frac{2}{x} + 5 + e^x\right)\ dx$

[5 points]

25. Evaluate the integral $\int \left(3x^2(x^3+2)^{3/2}\right)\ dx$

[5 points]

Bonus. A local club is arranging a charter flight to Hawaii. The cost of the trip is \$425 each for 75 passengers, with a refund of \$5 per passenger for each passenger in excess of 75.

(a) Find the number of passengers that will maximize the revenue received from the flight.

[8 points]

(b) Find the maximum revenue.

[7 points]