Incorporating concept-based pedagogy into K-8 online mathematics tutorials

Xiaohong Wang* and Harel Barzilai

Department of Math and Computer Science,
Salisbury University,
Salisbury, MD 21801, USA
E-mail: xswang@salisbury.edu
E-mail: barzilai@gmail.edu
*Corresponding author

Abstract: One of the primary goals of contemporary K-8 mathematics education is to have children gain a conceptual understanding of mathematics. Teachers and learners of mathematics need to develop content knowledge that includes meaning and clarifies the *why* as well as *how* behind mathematical objects and processes into their knowledge processing. However, a preliminary review suggests that drill-based and calculational activities comprise most of online mathematical tutoring software, and only a small proportion a meaning-based (conceptual) approach. In this project, an online mathematics tutoring application is developed with an emphasis on concept-based tutorials. Furthermore, the online mathematics tutorials also incorporate various strategies for performing mathematical operations, which may help children better learn, understand, and retain arithmetic knowledge and skills.

Keywords: elementary math education; concept-based approach; online tutorial.

Reference to this paper should be made as follows: Wang, X. and Barzilai, H. (2013) 'Incorporating concept-based pedagogy into K-8 online mathematics tutorials', *Int. J. Teaching and Case Studies*, Vol. 4, No. 1, pp.70–82.

Biographical notes: Xiaohong (Sophie) Wang is a Professor of Computer Science at the Department of Computer Science, Salisbury University, Salisbury, Maryland, USA. Her areas of expertise are programming languages, software engineering, computer science and software engineering education and scientific computing. She has over 15 years of working experience in computer industry and academic environment.

Harel Barzilai is a Mathematician and Mathematics Educator who has held professorships at the University of Minnesota and Salisbury University (location of project described here) and a Visiting Associate Professorship at Cornell University (2006–2007). His interests include quantitative literacy for in childhood through adulthood for citizens; the mathematical preparation of future K12 teachers; active-learning, exploration-enriched pedagogical approaches focused on student understanding of underlying meaning; authoring curricular materials for such approaches; and environmental mathematics. More background is at http://barzilai.org/proj-index.shtml which also links to his Archive of Reform Calculus Resources, online since 1996.

1 Introduction

Looking over the past 40 years, from slide rulers to graphing calculators, from mainframes to personal computers and the internet, technological advances have made great impacts on mathematics education. In recent years, with the flourishing of the internet, many public schools have adopted web-based mathematics tutorial software in and out of the classroom to assist and enhance the mathematical skills of elementary and secondary students.

Elementary and secondary mathematics education is an important part of public education. Gaining a *conceptual understanding* of mathematics is one of the primary goals of the preparation of K-8 mathematics teachers (Bennett et al., 2010). It has been recognised that teachers and learners of mathematics must integrate their *process knowledge* of how to find the answer, with the content knowledge that includes meaning and which clarifies the *why* as well as *how* behind the mathematics. Educators of future mathematics teachers also note that "Being able to explain *why* gives one what we call mathematical power, which means that you can apply this knowledge to solve problems that are not just like the ones in the book" (Bassarear, 2001).

However, according to a preliminary survey conducted by a group of computer science students at Salisbury University in spring 2011, less than 10% of the existing online mathematics tutoring software emphasise conceptual understanding of mathematics. The great majority of them rely heavily on procedural knowledge and drill-based exercises. They incorporate very limited if any explorations and interactive learning experiences which build and support understanding of the meanings and the *why* behind the mathematics concepts.

To fill in the gap in this area, the web-based mathematics tutorials developed in this project place a heavy emphasis on meaning-based pedagogical approaches, instead of focusing on drill and 'memorised mechanical manipulation' based mathematics learning. Furthermore, these web-based tutorials incorporate multiple models (representations) and strategies for performing arithmetic operations. All these approaches are included in the design of the web-based tutorials to help children better learn, understand, and retain arithmetic knowledge and skills (Fuchs et al., 2010).

The remainder of this paper is organised as follows. Section 2 outlines the background of conceptual pedagogical approaches to be adopted in the web-based tutorials for elementary mathematics. Section 3 introduces the major components of the concept-based mathematics tutorials created, the tutorial development cycle, and the inter-disciplinary nature of the team for the project. Preliminary results and future work are discussed in Sections 4 and 5.

2 Meaning-based (conceptual) mathematics

While a detailed exposition of trends and research in mathematics education is not the focus of this paper, in this section, we sketch some key understandings which undergird current thinking about best-practice pedagogy in mathematics, particularly at the elementary school level. A consensus has been developing in the mathematics education community over the last ten to 20 years that truly effective learning cannot take place without teaching, and teachers, who possess a 'profound understanding of elementary mathematics'. In other words, the mathematics taught in elementary school grades is not so ‘elementary’ (or ‘simple’) after all, and a deeper understanding of elementary operations, including arithmetic, is critical: an understanding which is conceptual rather than strictly procedural (Ma, 1999). The National Council of Teachers of Mathematics (NCTM) states that “Psychological and educational research on the learning of mathematics has solidly established the important role of conceptual understanding in the knowledge and activity of persons who are proficient,” and “the union of factual knowledge, procedural proficiency, and conceptual understanding enhances all three components, making the resulting learning usable in powerful ways”. The NCTM concludes that “students must learn mathematics with understanding, actively building new knowledge. Learning mathematics *with understanding* is essential” (NCTM, 2009).

A conceptual analysis of what happens during subtraction, for example, reveals that the term ‘borrowing’ is a misnomer, since nothing is taken with the intention of returning it later. Rather, one object worth 10, for example, can be *replaced* by, *converted* into, or *exchanged* for, ten objects each of which is worth 1, or vice versa (a ‘regrouping’ process). Tutorials which bring out such interpretations, or guide the child to discover them through visualisations, virtual actions, story lines, or other meaning-based representations, are therefore better suited to conceptual learning. The modules developed by the software team in this project endeavoured to build up over time a suite of these types of tutorials.

As noted earlier, software has changed the landscape of mathematics teaching and learning, with many online mathematics tutorials available to help students learn and practice mathematics skills. A preliminary survey on existing free mathematics tutoring software over the internet was conducted by a group of computer science students in spring 201 to identify what types of software are currently available and to assess the degree to which they incorporate meaning-based contexts, explorations, visualisations, concept-based interactivity and word problems which emphasise underlying meaning, and *why* as well as *how* the mathematics works. The review found that less than 10% of the existing free mathematics tutoring software surveyed emphasises conceptual understanding of mathematics with most relying heavily on drill-based exercise, and incorporating very little if any explorations and interactive learning experiences which build and support conceptual understanding.

In this project, the web-based mathematics tutorials were developed to fill in the gap between the desperate need for meaning-based pedagogical approaches in elementary mathematics education and the lack of support for such pedagogical approaches in current available online mathematics tutorials.

3 Development of tutorials

We recognised that in order to produce high quality software, it was important to combine our expertise in mathematics, teacher education, and software development with that of practicing teachers who bring in years of ‘ground level’ experience, as well as to garner at least informal input from elementary students. This collaborative approach was the foundation for the development of the tutorials.

3.1 Building an interdisciplinary team

The collaboration started as a capstone software engineering project for four computer science major students from fall 2010 to spring 2011 under the supervision of two faculty advisors: one from computer science and one from mathematics. During the summer of 2011, a computer science student research assistant was hired to work on the tutorials. During that summer, seven K-8 mathematics teachers from Wicomico County in Maryland volunteered, and took part by providing evaluation and feedback on mathematical and pedagogical aspects of earlier versions of the tutorials; for this, we provided teachers with specific questions, and gave them the opportunity to give their input in open form.

Bringing together this interdisciplinary team allowed us to have expertise in both software development and in research-based, best-practice pedagogy for mathematics education as we moved forward. The computer science faculty guided the computer science students in the design, the development and testing of the online tutorials. The mathematics faculty, with a background in both mathematics and K-12 teacher education, provided several concept-based elementary arithmetic models and methods that have been incorporated in the tutorials, and detailed guidance for how the tutorials would flow step by step in order to flesh out meaning and 'why' for children. As noted, the practicing K-12 teachers provided professional expertise and classroom experience, and their professionally informed suggestions on the mathematical and pedagogical aspects of the tutorials. These teacher contacts in turn and at our suggestion, recruited a small number of elementary school children to volunteer to test-drive earlier versions of the tutorials. The teachers were present during these test-drives, monitored and recorded these students' interactions with the tutorials, and forwarded the children's reactions to us. This was done in the spring of 2012.

This interdisciplinary team therefore brought together key stakeholders and their professional experience as the framework within which high quality and authentically relevant tutorials could be developed for K-8 teachers and their students.

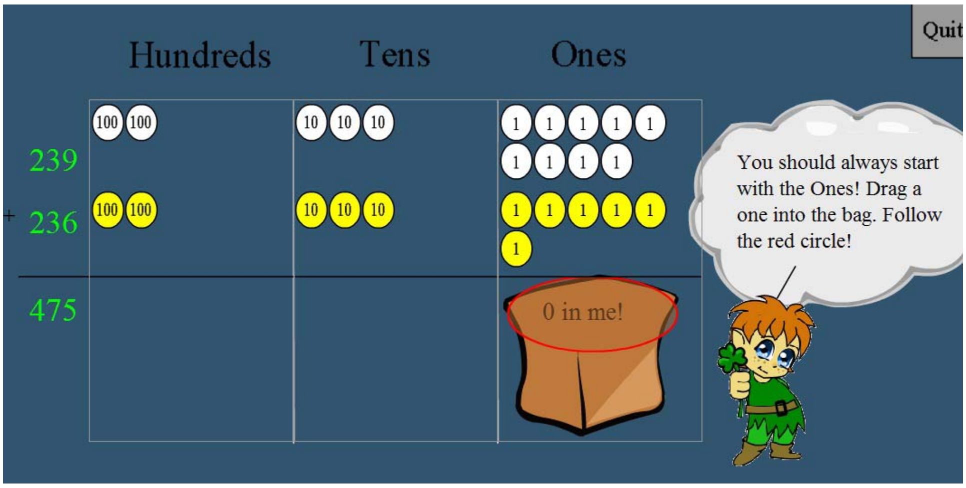
3.2 Design of concept-based tutorials

At this preliminary stage, this suite of online tutorials adopts concept-based approaches to the teaching and learning of addition and subtraction using engaging yet mathematically correct and precise animations. We would like to focus on helping children learn addition and subtraction in this present phase due to the consideration that further pilot testing should be done by school children and exhaustive survey on conceptual multiplication tutorials over the internet should be conducted before expanding.

Here are two design highlights of the concept-based tutorials:

- In the tutorials, context-specific (story-based) animations lead students step by step towards mastery of how to perform addition and subtraction, progressing from simpler problems in which regrouping is not necessary, to more complex ones in which regrouping in the ones or tens places, or both, is needed. Particular emphasis is placed on *how* and *why* regrouping works as is does through students' direct participation in the process. Children directly see where and why regrouping arises in common algorithms, and build in their everyday understanding that, for example, we can replace 10 out of 11 pennies by a dime. Although common but misleading terms like ‘borrowing’ are not entirely omitted, students are led to discover at what stage, and why, they need to exchange one 10-penny coin (a dime) for ten 1-penny coins (or vice versa), an exchange which is commonly called a ‘regrouping’. A smiling shamrock wielding character (perhaps a fabled leprechaun?) is to guide the child throughout the process of addition and subtraction. Here is an example of portions of the tutorial on addition, where regrouping in the 1’s column (place value) is required. Two numbers need to be added (344 + 439), where the sum of the two numerals (digits) in the ‘ones’ is 10 or more. This means there are 10 or more ‘penny’ coins. See Figure 1.

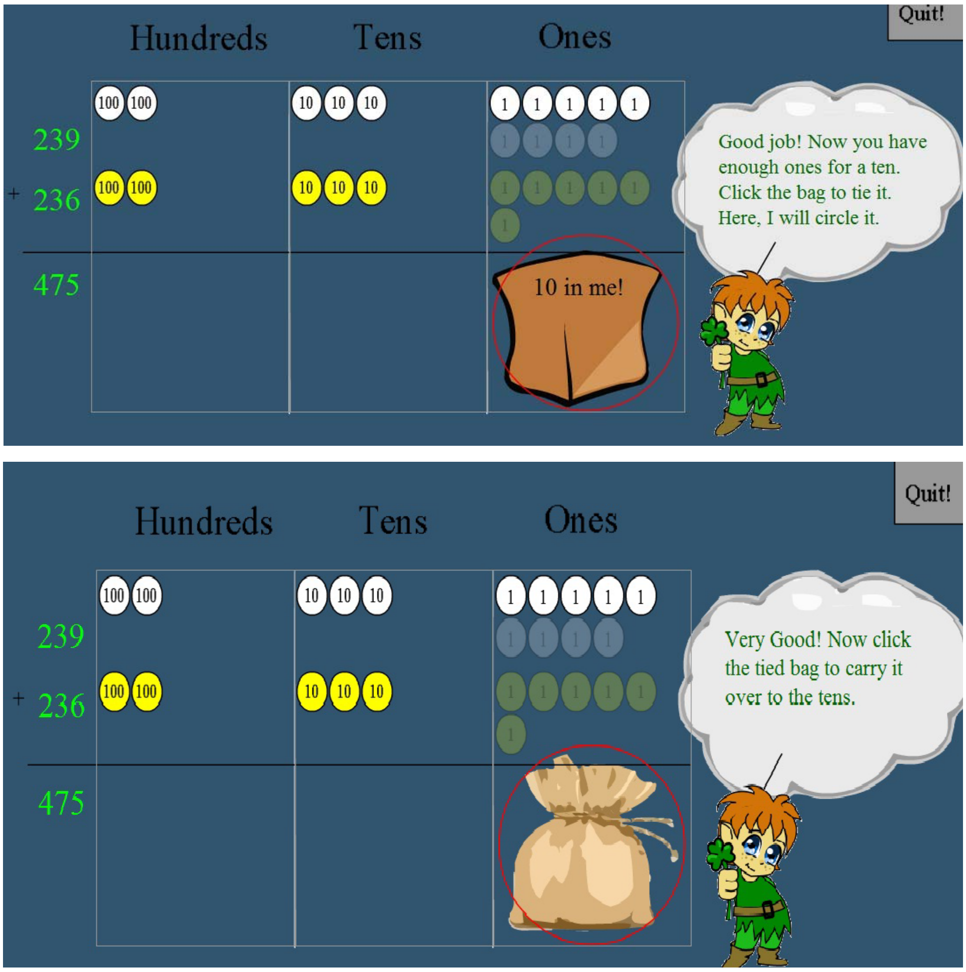
Figure 1 Exceeding ten in the 1’s column (see online version for colours)



Then the smiling shamrock guides the child to dragging exactly 10 of these pennies into a bag. The ‘magic’ bag keeps track of and displays for the child, a running count of how many pennies are in it, at each stage.

Once 10 pennies have been dragged into the bag, it is explained to the child that the bag now has enough ‘penny’ or ‘1’ type coins, to exchange for a ‘10’ coin. The child is to tie the bag so it is closed, representing cognitively a *single* entity now, one which can be traded for an entity representing the same value but with a different denomination. See Figure 2. Next, just as when going to a bank or with a cashier, this sealed bag can be exchanged. Although no bank graphics are used, the character leads the child to move the sealed bag (which has been in the 1s column below the two numbers ones), into the top of the 10s column. When this has been done, the sealed bag (with 10 pennies) is replaced by something else, albeit something of exactly the same value: exactly one ‘10’ coin (or a dime) in the 10s column (see the red coin marked 10 in Figure 3).

Figure 2 Placing ten 1s into the sack and tie it up (see online version for colours)



The individual 10s numerals and 1s numerals of the two numbers being summed, are then added and placed in their respective columns (Figure 4).

The subtraction process is similar. Figure 5 demonstrate the process to perform 975–325, which requires taking a coin of value 10-pennies in the 'tens' column (Step 1), taking it to the piggy bank and exchange it into ten coins with each of value 1-penny (Steps 2 and 3) and place them in the 'ones' column (Step 4).

Figure 3 Exchanging the ten1s into one 10 (see online version for colours)

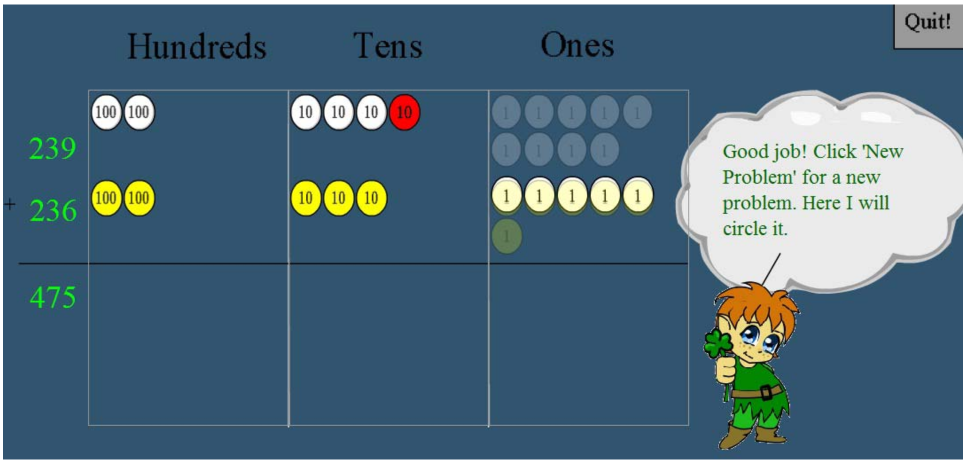
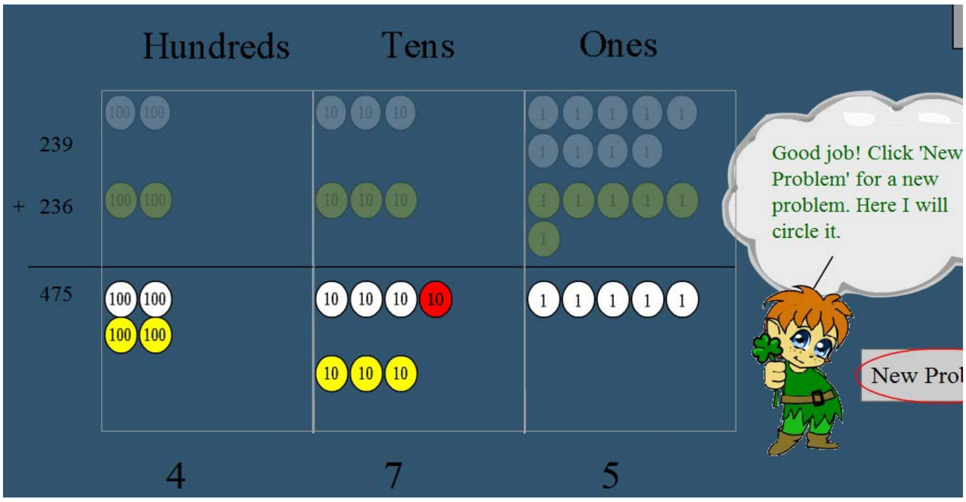
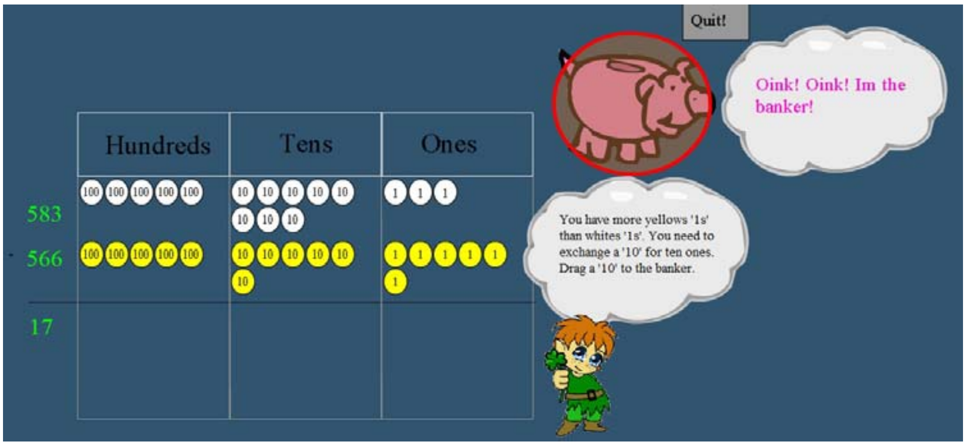


Figure 4 Proceed with normal addition at 10's and 1's place (see online version for colours)

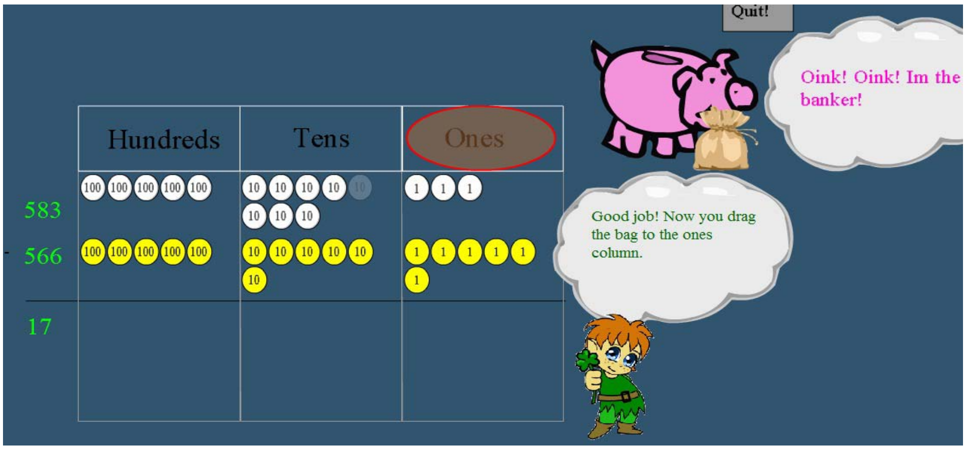


- Preliminary versions of the tutorials have been developed (see the screenshots in Figure 6) which look at subtraction from another angle. These incorporate aspects of subtraction such as the well known equal differences property (i.e., $a - b = (a + x) - (b + x)$) and techniques based upon it such as the algorithm sometimes known as the New Zealand method (Bassarear, 2001). These are well established ideas which nevertheless, as indicated by our K-12 contacts, are not often clearly explained or even taught in the schools, with the New Zealand method often entirely unfamiliar to US teachers. These subtraction variants have been taught in graduate courses for in-service elementary and middle school teachers by two mathematics education professors in the mathematics department; some in-service teachers in those courses have reported that such alternate methods have help them teach subtraction to struggling school children who previously were unable to master subtraction (see quoted comments in Section 4).

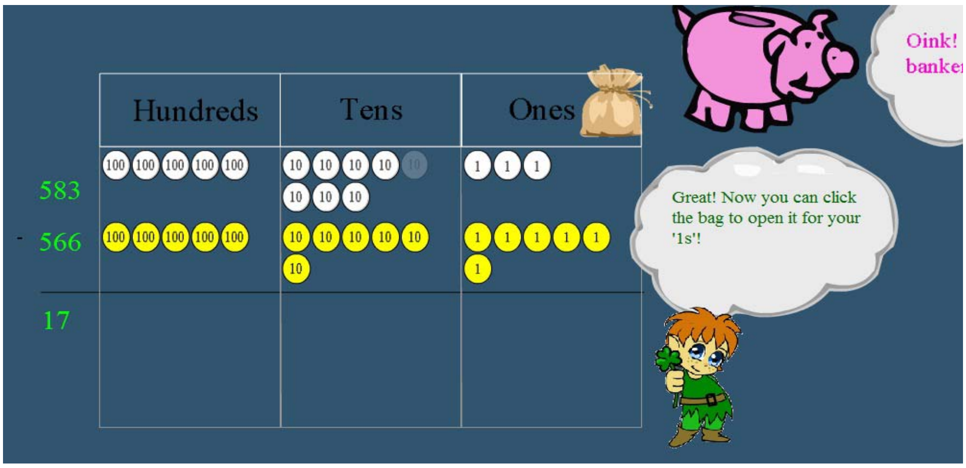
Figure 5 Subtraction (a) step 1, (b) step 2, (c) step 3 (d) step 4, (e) step 5, (f) step 6 (see online version for colours)



(a)

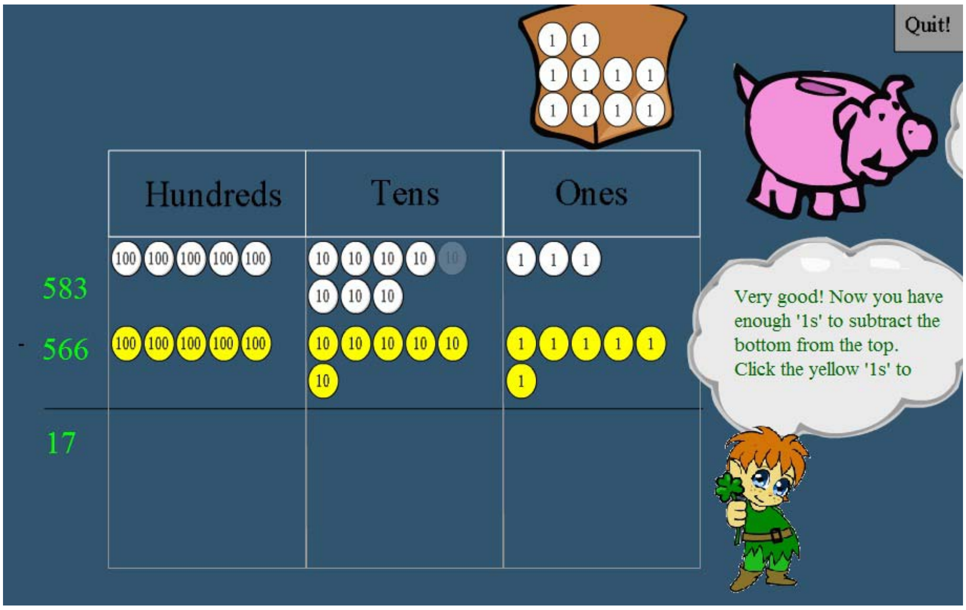


(b)

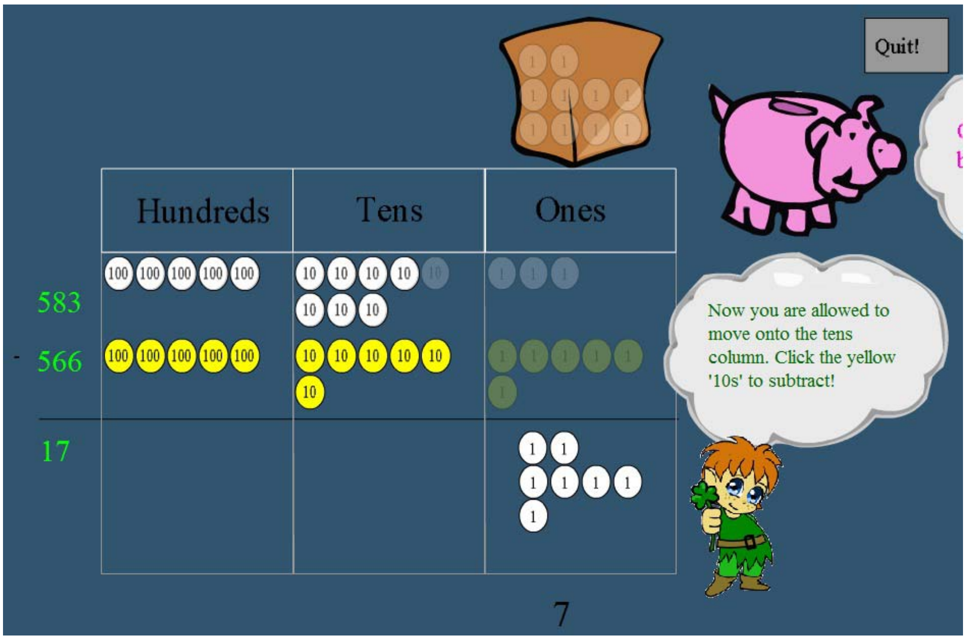


(c)

Figure 5 Subtraction (a) step 1, (b) step 2, (c) step 3 (d) step 4, (e) step 5, (f) step 6 (continued) (see online version for colours)

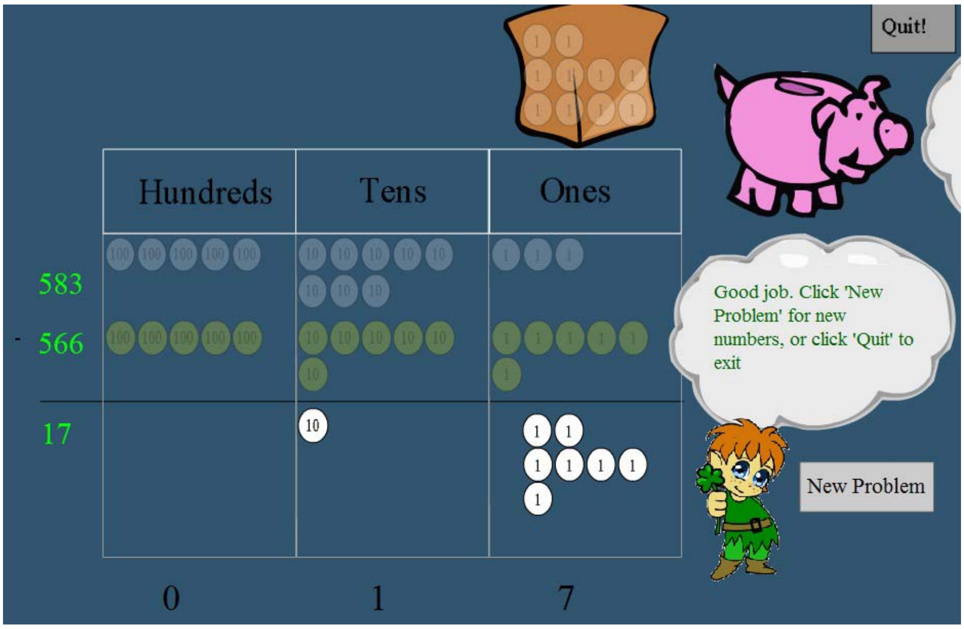


(d)



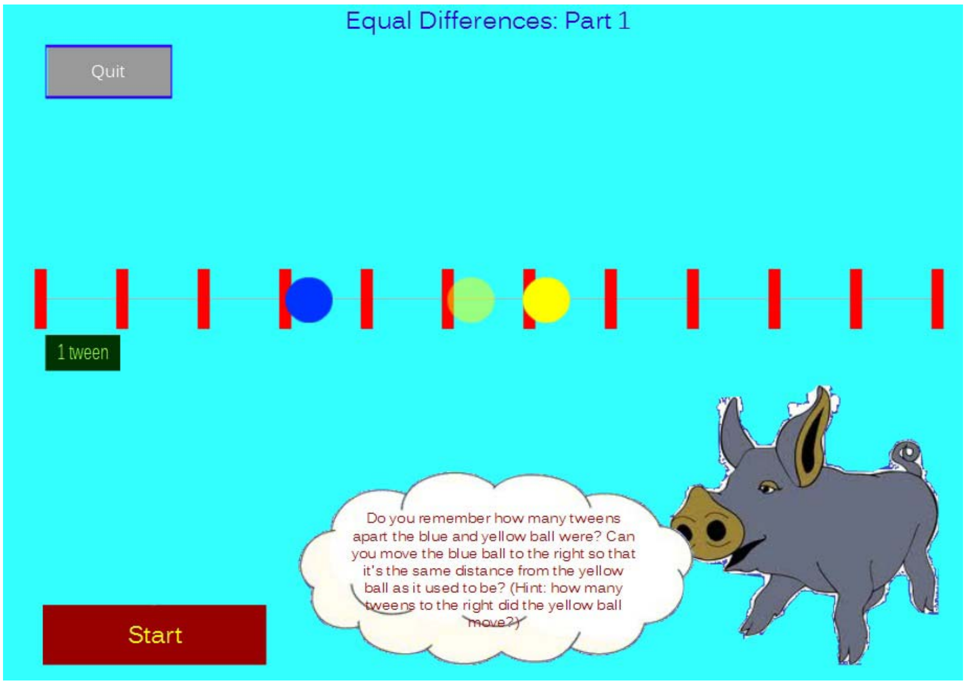
(e)

Figure 5 Subtraction (a) step 1, (b) step 2, (c) step 3 (d) step 4, (e) step 5, (f) step 6 (continued) (see online version for colours)



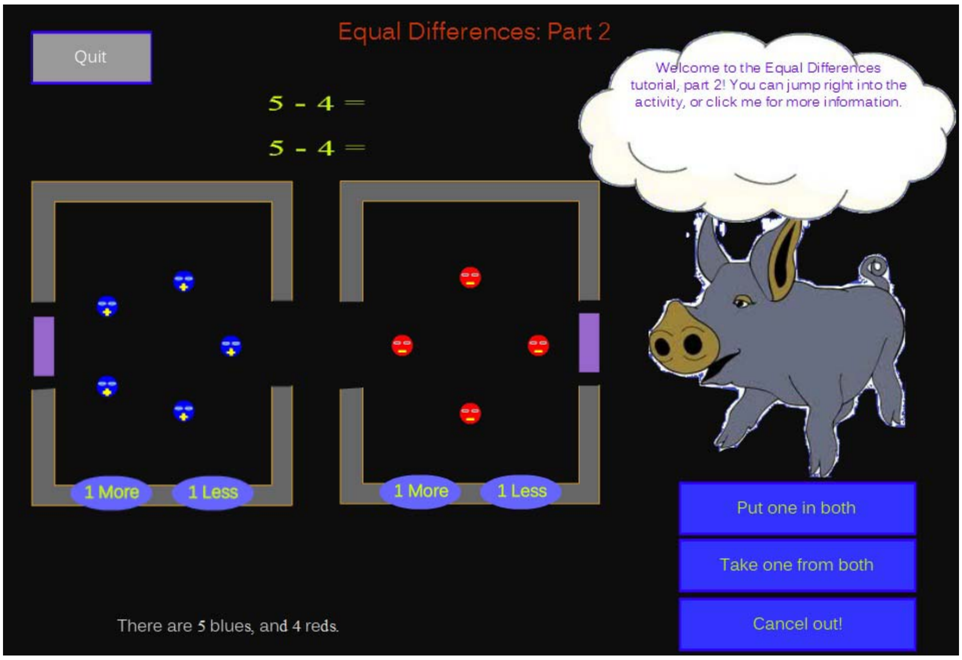
(f)

Figure 6 Subtraction with three equal difference approaches (a), (b), (c) (see online version for colours)

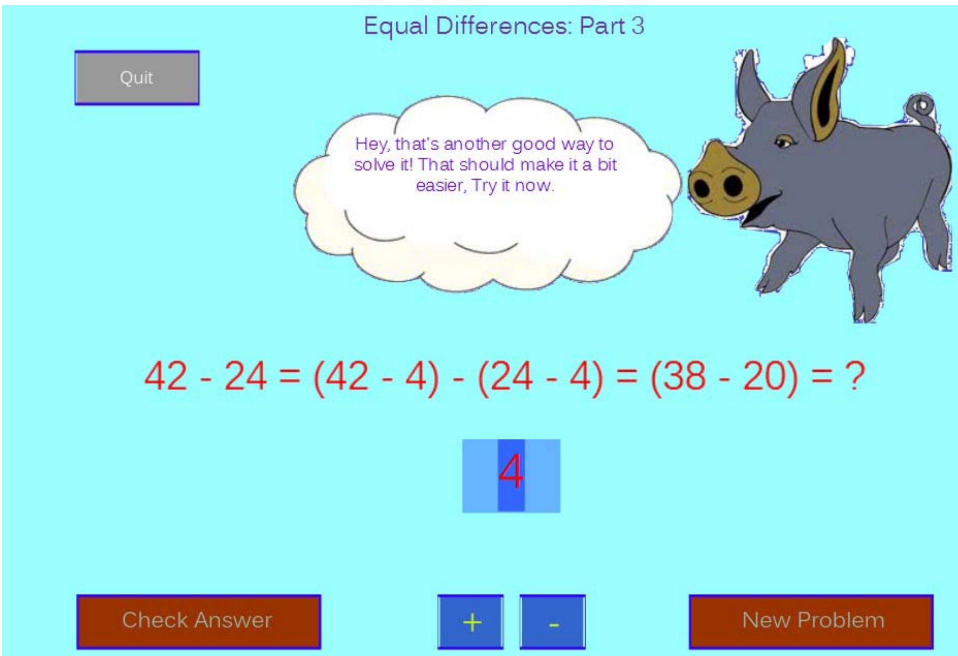


(a)

Figure 6 Subtraction with three equal difference approaches (a), (b), (c) (continued) (see online version for colours)



(b)



(c)

4 Preliminary results

Significant progress has been made toward building a complete suite of conceptual mathematics tutorials which can serve as a teaching and learning tool that can be readily incorporated by elementary teachers and parents of children in those grades.

So far, the tutorials on addition, subtraction and several equal difference-based subtraction tutorials have been developed. The addition and subtraction tutorials have been reviewed by K-8 teachers and several revisions have been made to those tutorials according to the feedback and suggestions from the teachers. A number of K-8 school children used addition and subtraction tutorials under the guidance of the participating teachers, in their classroom or at home under teacher supervision. Preliminary input from K-8 teachers and their students suggests that the conceptual approach, incorporating more than just 'how to find the answer', is very welcome, including the visual and story-based contexts in which the tutorials are grounded. One teacher commented, "I think that the graphics move at a good pace. So many times children don't *understand what's really happening and just memorize*." Another teacher's feedback was that "I love the idea of the children having to 'seal' the bag of 10 so that they truly understand that concept." Another teacher, who had access to the tutorial using the method based on equal differences (sometimes called the New Zealand method), said "I was not familiar with this approach. I have explored it now and love it I think my students would really benefit from this method."

In the project, K-8 teachers have had the opportunity to gain further exposure to research-based, active-learning pedagogies for mathematics instruction, and their further participation through piloting different tutorial versions and submitting 'wish list' suggestions and feedback, provided them with meaningful professional development, as attested in part by their willingness to continue participation with/without stipends.

During this project, the computer science students have gained hands-on experience in the design, development and implementation of web-based applications using Flash, php script language and mySql database. They also gained real-world experience in meeting demands from the real world clients and incorporating both software design and pedagogical design into their final product.

5 Future work

The preliminary outcomes of this project have been encouraging. We plan to expand the current suite of tutorials to explore arithmetic operations more broadly and make these tutorials available through a website. Besides hosting tutorials, the website will also be equipped with features such as progress tracking (for students) and discussion forums for teachers and parents. We also plan to organise workshops, give presentations, or provide written materials for local teachers on how to effectively use those tutorials in their mathematics instructions.

Acknowledgements

This project is partially supported by Bridge for Salisbury University's Connection to Careers for Every STEM student undergraduate research grant funded by the NSF's Science, Technology, Engineering, and Mathematics Talent Expansion Programme and Salisbury University faculty mini grant.

References

Bassarear, T. (2001) *Mathematics for Elementary School Teachers*, 2nd ed., Houghton Mifflin Company, Boston, MA.

Bennett, A.B., Burton, L.J. and Nelson, L.T. (2010) *Preface, Mathematics for Elementary Teachers: A Conceptual Approach*, 8th ed., McGraw Hill [online] http://www.chi2009.org/Authors/Guides/ArchiveGuide.html (accessed 02/27/2013).

Fuchs, L. et al. (2010) 'A framework for remediating number combination deficits', *Except Children*, Vol. 76, No. 2, pp.135–156. Cited at 'For students struggling with arithmetic, going back to counting strategies may be most effective intervention' [online] http://www.ernweb.com/public/1192.cfm (accessed 02/27/2013).

Ma, L. (1999) *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*, Lawrence Erlbaum Associates, Mahwah, NJ.

National Council of Teachers of Mathematics (NCTM) (2009) *Guiding Principles for Mathematics Curriculum and Assessment*, June [online] http://www.nctm.org/standards/content.aspx?id=23273 (accessed 02/27/2013).