Successor/Predecessor Rules in Binary Trees

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Introduction

Binary tree traversals are commonly made in one of three patterns, inorder, preorder, and postorder. These traversals are easy to describe recursively as follows, for a subtree rooted at n:

```c
void inorder(node * n)
{
    inorder(leftChild(n);
    visit(n);
    inorder(rightChild(n);
}
void preorder(node * n)
{
    visit(n);
    preorder(leftChild(n));
    preorder(rightChild(n));
}
void postorder(node * n)
{
    postorder(leftChild(n));
    postorder(rightChild(n));
    visit(n);
}
```

Unfortunately, these easy recursive functions are not useful for building iterators for binary trees. We need functions that can take one step at a time through the traversal. Each step can be thought of as finding the next or previous node for forward-moving or backwards-moving iterators, respectively.

This paper describes the “rules” for determining the next (successor) or previous (predecessor) nodes for any node in a binary tree for each of the traversal patterns.
Inorder Successor

To find the inorder successor of node $u$:
If $u$ has a right child, $r$, then $\text{succ}(u)$ is the leftmost descendent of $r$
Otherwise, $\text{succ}(u)$ is the closest ancestor, $v$, of $u$ (if any) such that $u$ is descended from the left child of $v$. If there is no such ancestor, then $\text{succ}(u)$ is undefined.

An iterator would start with the leftmost node.

For example, an inorder traversal of the following binary tree yields the sequence DBFGEAC.

![Binary Tree Diagram]

Taking the nodes one at a time and applying the rule:

node $D$: Does not have a right child. Its successor is the closest ancestor, $v$ such that node-D is descended from the left child of $v$. Node-D is descended from the left child of node-B, so $\text{succ}(D)$ is node-B.

node $B$: Has a right child (node-E), so successor is the leftmost descendent of node-E, namely node-F.

node $F$: Has a right child (node-G), so successor is the leftmost descendent of node-G, namely node-G itself.

node $G$: Does not have a right child. Its successor is the closest ancestor, $v$ such that node-G is descended from the left child of $v$. Node-G is descended from the left child of node-E, so $\text{succ}(G)$ is node-E.

node $E$: Does not have a right child. Its successor is the closest ancestor, $v$ such that node-E is descended from the left child of $v$. Node-E is descended from the left child of node-A, so $\text{succ}(E)$ is node-A.
node A: Has a right child (node C), so successor is the leftmost descendent of node C, namely node C itself.

node C: Does not have a right child. Its successor would be the closest ancestor, v such that node C is descended from the left child of v. However, there is no such ancestor, so succ(C) is undefined (node C has no successor).

Preorder Successor

To find the preorder successor of node u:
If u has a left child, l, then succ(u) is l.
Otherwise, if u has a right child, r, then succ(u) is r.
Otherwise, u is a leaf and the following rules apply:
   if u is a left child and has a right sibling, rs, then succ(u) is rs.
   otherwise, if u has an ancestor, v, which is a left-child and v has a right sibling, vrs, then succ(u) is vrs
      If there is no such ancestor, then succ(u) is undefined.

An iterator would start with the root of the tree.

For example, a preorder traversal of the following binary tree yields the sequence ABDEFC.

Taking the nodes one at a time and applying the rule:

node A: Has a left child, node B, so successor is node B.
node B: Has a right child, node D, so successor is node D.
node D: Has a left child, node E, so successor is node E.
node E: Is a leaf. It is a left child and has a right sibling, node-F, so successor is node-F.

node F: Is a leaf. Is not a left child. It has an ancestor, node-B, that is a left child and that has a right sibling, node-C, so successor of node-F is node-C.

node C: Is a leaf. Is not a left child. Does not have an ancestor that is a left child. Therefore, the successor of node-C is undefined.

**Postorder successor**

To find the postorder successor of node \( u \):

- If \( u \) is the root of the tree, \( \text{succ}(u) \) is undefined.
- Otherwise, if \( u \) is a right child, \( \text{succ}(u) \) is \( \text{parent}(u) \).
- Otherwise \( u \) is a left child and the following applies:
  - if \( u \) has a right sibling, \( r \), \( \text{succ}(u) \) is the leftmost leaf in \( r \)'s subtree
  - otherwise \( \text{succ}(u) \) is \( \text{parent}(u) \).

An iterator would start with the leftmost leaf (not necessarily the leftmost node).

For example, a postorder traversal of the following binary tree yields the sequence EDBCA. Notice that it starts with the leftmost leaf, node-E, not the leftmost node, node-B.

![Binary Tree Diagram]

Taking the nodes one at a time and applying the rule:

node E: Is a left child and does not have a right sibling. Therefore, the successor of node-E is its parent, node-D.

node D: Is a right child. The successor of node-D is its parent, node-B.
**node B:** Is a left child and does have a right sibling, node-C. Therefore the successor of node-B is node-C.

**node C:** Is a right child. The successor of node-C is its parent, node-A.

**node A:** Is the root of the tree, so its successor is undefined.

### Inorder Predecessor

To find the inorder predecessor of node u
If u has a left child, l, then \( \text{pred}(u) \) is the rightmost descendent of l
Otherwise, \( \text{pred}(u) \) is the closest ancestor, v, of u (if any) such that u is descended from the right child of v.
If there is no such ancestor, then \( \text{pred}(u) \) is undefined.

An iterator would start with the rightmost node.

For example, a reverse inorder traversal of the following binary tree yields the sequence **CAEGFBD**, Notice that it starts with the rightmost node-C.

```
  A  
 /   
B     C
 /   
D   E
 /  
F   G
```

Taking the nodes one at a time and applying the rule:

**node C:** Does not have a left child. Closest ancestor such that node-C is descended from the right child is node-A. Therefore, the predecessor of node-C is node-A.

**node A:** Has a left child, node-B. Rightmost descendent of node-B is node-E.

**node E:** Has a left child, node-F. Rightmost descendent of node-F is node-G.
**node G:** Does not have a left child. Closest ancestor such that node-$G$ is descended from the right child is node-$F$.

**node F:** Does not have a left child. Closest ancestor such that node-$F$ is descended from the right child is node-$B$.

**node B:** Has a left child, node-$D$. Rightmost descendent of node-$D$ is node-$D$ itself.

**node D:** Does not have a left child. There is no ancestor such that node-$D$ is descended from the right child. Therefore, the predecessor of node-$D$ is undefined.

**Preorder Predecessor**

To find the preorder predecessor of node $u$:
If $u$ is the root of the tree, then $\text{pred}(u)$ is undefined
If $u$ has a left sibling, $l_s$, then $\text{pred}(u)$ is the rightmost descendent of $l_s$
Otherwise, $\text{pred}(u)$ is $\text{parent}(u)$.

An iterator would start with the rightmost node.

For example, a reverse preorder traversal of the following binary tree yields the sequence $CDFEBA$. Notice that it starts with the rightmost node-$C$.

```
    A
   / \
  B   C
 /     \
D       E
 /     \
F       
```

Taking the nodes one at a time and applying the rule:

**node C:** Has left sibling, node-$B$. Predecessor of node-$C$ is rightmost descendent of node-$B$, namely node-$F$.

**node F:** Has left sibling, node-$E$. Rightmost descendent of node-$E$ is node-$E$ itself.
**Postorder Predecessor**

To find the postorder predecessor of node $u$:
If $u$ has a right child, $r$, then $\text{pred}(u)$ is $r$.
Otherwise If $u$ has a left child, $l$, then $\text{pred}(u)$ is $l$.
Otherwise if $u$ has a left sibling, $ls$, then $\text{pred}(u)$ is $ls$.
Otherwise if $u$ has an ancestor, $v$, which is a right child and has a left sibling, $vls$, then $\text{pred}(u)$ is $vls$.
Otherwise, $\text{pred}(u)$ is undefined.

An iterator would start with the root of the tree.

For example, a reverse postorder traversal of the following binary tree yields the sequence ACBDFE. Notice that it starts with the root node-A.

Taking the nodes one at a time and applying the rule:

**node A:** Has a right child, node-C.

**node C:** Has a left sibling, node-B.

**node B:** Has a right child, node-D.

**node D:** Has a left child, node-F.
node **F**: Has an ancestor, node-D, that is a right child and that has a left sibling, node-E. Therefore, the postorder predecessor of node-F is node-E.

node **E**: No right child, no left child, no suitable ancestor. Therefore, the postorder predecessor of node-E is undefined.