f.10 Problem 11

$$m = \frac{8\text{ lbs}}{32} = \frac{1}{4}$$

$$\frac{1}{4}y'' + y' + 10y = 2\cos 2t$$

$$y'' + 4y' + 40y = 8\cos 2t$$

$$r = \frac{-4 \pm \sqrt{16 - 160}}{2} = -2 \pm \frac{12}{2}i = -2 \pm 6i$$

$$y_h = e^{-2t}(c_1 \sin 6t + c_2 \cos 6t)$$

$$y_p = A\cos 2t + B\sin 2t$$

$$y_p' = -2A\sin 2t + 2B\cos 2t$$

$$y_p'' = -4A\cos 2t - 4B\sin 2t$$

$$-A\cos 2t + 2B\cos 2t + 10A\cos 2t - B\sin 2t - 2A\sin 2t + 10B\sin 2t = 2\cos 2t$$

$$-A + 2B + 10A = 2 \Rightarrow 9A + 2B = 2$$

$$-B - 2A + 10B = 0 \qquad 9B - 2A = 0$$

$$4B + 18A = 4$$

$$81B - 18A = 0$$

$$85B = 4 \Rightarrow B = 4/85$$

$$A = \frac{9}{2}(4/85) = 18/85$$

$$y_p = C\sin(2t + \Theta)$$

$$= \sqrt{\frac{16 + 324}{(85)^2}}\sin(2t + \Theta)$$

$$= \frac{2}{\sqrt{85}}\sin(2t + \Theta)$$

$$\tan\Theta = \frac{A}{B} = \frac{18}{4} = \frac{9}{2} \qquad \Theta = \tan^{-1}\left(\frac{9}{2}\right) \approx 1.35213$$

$$y_p = \frac{2}{\sqrt{85}}\sin(2t + 1.35)$$

$$y = e^{-2t}(c_1 \sin 6t + c_2 \cos 6t) + \frac{2}{\sqrt{85}}\sin(2t + 1.35)$$

$$y' = e^{-2t}((-2c_1 - 6c_2)\sin 6t + (6c_1 - 2c_2)\cos 6t) + \frac{4}{\sqrt{85}}\cos(2t + 1.35)$$

$$y(0) = c_2 + \frac{2}{\sqrt{85}}\sin(1.35) = c_2 + \frac{18}{85} = 0$$

$$\Rightarrow c_2 = -\frac{18}{85}$$

$$y'(0) = 6c_1 - 2c_2 + \frac{4}{\sqrt{85}}\cos(1.35)$$

$$= 6c_1 + \frac{36}{85} + \frac{8}{85} = 6c_1 + \frac{44}{85} = 0$$

$$\Rightarrow c_1 = \frac{-22}{3(85)} = -\frac{22}{255}$$

$$y = e^{-2t}\left(\frac{2}{255}\sin 6t - \frac{18}{85}\cos 6t\right) + \frac{2}{\sqrt{85}}\sin(2t + 1.35)$$

$$y = Ae^{-2t}\sin(6t + \phi) + \frac{2}{\sqrt{85}}\sin(2t + 1.35)$$

$$= e^{-2t}(c_1 \sin 6t + c_2 \cos 6t) + \frac{2}{\sqrt{85}}\sin(2t + 1.35)$$

$$y' = e^{-2t}((-2c_1 - 6c_2)\sin 6t + (6c_1 - 2c_2)\cos 6t) + \frac{4}{\sqrt{85}}\cos(2t + 1.35)$$

$$\gamma_r = \sqrt{\frac{k}{m} - \frac{b^2}{2m^2}}$$

$$= \sqrt{40 - 8} = \sqrt{32} = 4\sqrt{2} \approx 5.657$$

$$\sqrt{c_1^2 + c_2^2} = \frac{\sqrt{(22)^2 + (9)(18)^2}}{255} \approx .2287$$

$$\tan\phi = +c_2/c_1 = -\frac{18(3)}{22} \sim 2.455$$

$$\phi \approx +1.184 + \pi = 4.326$$

$$y = .2287e^{-2t}\sin(6t + 4.325) + \frac{2}{\sqrt{85}}\sin(2t + 1.35)$$

$$\approx .2287e^{-2t}\sin(6t - 4.325) + \frac{2}{\sqrt{85}}\sin(2t + 1.35)$$

Resonanay Frequency $= \gamma_r/2\pi$

$$= 2\sqrt{2}/\pi.$$