Section 1.1 problem 17

Let a_k be Kevin's Acceleration and a_a be Alisons's, T_k be the time it takes Kevin to finish and T_a the time it takes Alison. Then, if K(t) and A(t) are the distance functions for Kevin and Alison respectively:
Since both accelerate at a constant rate:
$K'' = a_k \Rightarrow K' = a_k t + C$ and $K = ½a_k t^2 + Ct + C+$.
$A'' = a_a \Rightarrow A' = a_a t + C$ and $A = ½a_a t^2 + C*t + C\^{}$.

For simplicity's sake let's let the length of the track be 12.

Since for both, initial velocity and position are 0 all the C's are 0 so:
$K = ½a_k t^2$ and $A = ½a_a t^2$
$K(T_k) = ½\ a_k\ T_k^2 = 12 \Rightarrow a_k = 24/T_k^2$
$A(T_a) = ½\ a_a\ T_a^2 = 12 \Rightarrow a_a = 24/T_a^2$

We also know that Alison covers the last ¼ of the distance in 3 seconds so at T_a-3 Alison's position is 12(1-¼) = 9. Similarly Kevin covers the last 1/3 of the race in 4 seconds so at T_k-4 Kevin's position is 9.
That is:
$K(T_k-4) = ½\ a_k\ (T_k-4)^2 = 8 = ½\ a_k\ T_k^2 - 4\ a_k\ T_k + 8\ a_k = 8$
$A(T_a-3) = ½\ a_a\ (T_a-3)^2 = 9 = ½\ a_a\ T_a^2 - 3\ a_a\ T_a + 9/2\ a_a = 9$

$½\ a_k\ T_k^2 - 4\ a_k\ T_k + 8\ a_k = 8$

so $½\ (24/T_k^2)\ T_k^2 - 4\ (24/T_k^2)\ T_k + 8\ (24/T_k^2) = 8$

and $12 - 96/T_k + 192/T_k^2 = 8 \Rightarrow 12\ T_k^2 - 96\ T_k + 192 = 8\ T_k^2 \Rightarrow T_k^2 - 24T_k + 48 = 0$.
$T_k \approx 2.2$ or 21.8 since 2.2 < 4, Kevin finishes in about 21.8 seconds.

$½\ a_a\ T_a^2 - 3\ a_a\ T_a + 9/2\ a_a = 9$
so $½\ (24/T_a^2)\ T_a^2 - 3\ (24/T_a^2)\ T_a + 9/2\ (24/T_a^2) = 9 \Rightarrow 12 - 72/T_a + 108/T_a^2 = 9$
$\Rightarrow 3\ T_a^2 - 72\ T_a + 108 = 0$ or $T_a^2 - 24\ T_a + 36 = 0 \Rightarrow T_a \approx 1.6$ or 22.4 Since 1.6 < 3, Alison finishes in about 22.4 seconds.

So, Kevin wins by about 22.4 - 21.8 or .6 seconds.