Problem 13 from 1.4

$$\lim_{n\to\infty} \frac{e - y_n}{1/n} = \frac{e}{2}$$

$$\frac{d}{dn}\left(1+\frac{1}{n}\right)^n = \frac{d}{dn}\left(e^{n\ln(1+\frac{1}{n})}\right)$$
$$= e^{n\ln(1+\frac{1}{n})}\left(\ln\left(1+\frac{1}{n}\right) + \frac{n}{1+1/n}\cdot\left(-\frac{1}{n^2}\right)\right)$$
$$= \left(1+\frac{1}{n}\right)^n\left[\ln\left(1+\frac{1}{n}\right) - \frac{1}{n+1}\right]$$

$$\lim_{n\to\infty} \frac{e - (1+\frac{1}{n})^n}{1/n} =$$

$$= \lim_{n\to\infty} \frac{(1+\frac{1}{n})^n\left[\frac{1}{n+1} - \ln(1+\frac{1}{n})\right]}{-1/n^2}$$

$$= \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n\left[n^2\ln\left(1+\frac{1}{n}\right) - \frac{n^2}{n+1}\right]$$

$$= \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n\left[n^2\left[\frac{1}{n} - \frac{1}{2n^2} + \frac{1}{3n^3} - \frac{1}{4n^4}\cdots - \frac{1}{n+1}\right]\right]$$

$$= \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n\left[n^2\left(\frac{1}{n} - \frac{1}{n+1}\right) - n^2\left(\frac{1}{2n^2} - \frac{1}{3n^3} + \frac{1}{4n^4} - \cdots\right)\right]$$

$$= \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n\left[\frac{n^2(\cancel{n}+1-\cancel{n})}{n^2+n} - n^2\left(\frac{1}{2n^2} - \cdots\right)\right]$$

$$= e\left(1 - \frac{1}{2}\right) = \frac{e}{2}$$

MacLaurin series for

$\ln(1+t) = f(t)$ $\qquad f(0) = 0$

$\frac{1}{1+t} = f'(t)$ $\qquad f'(0) = 1$

$-\frac{1}{(1+t)^2} = f''(t)$ $\qquad f''(0) = 1$

$+\frac{2!}{(1+t)^3} = f'''(t)$ $\qquad f''(0) = 2$

$\vdots$

$-\frac{(n-1)!}{(1+t)^n} = f^{(n)}(t)$ $\qquad f^{(n)}(0) = (n-1)!$

$$f(t) = 0 + t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} \text{ etc} \ldots$$