Section 2.3 problem 39

$$\frac{dT}{dt} = \begin{cases} 1-T & \text{when the heat is on} \\ -T & \text{" " " " off} \end{cases}$$

$$\frac{dT}{dt} = 1-T \Rightarrow \int \frac{1}{1-T}\,dT = \int dt$$

I'll assume for now $T \leq 100$

$$-\ln(1-T) = t + C$$

$$\ln(1-T) = -t + C$$

$$1-T = Ce^{-t}$$

$$T = 1 - Ce^{-t}$$

$$T(0) = 1-C \Rightarrow C = 1-T(0)$$

$$T(1) = 1 - \frac{1-T(0)}{e} = 1 - \frac{1}{e} + \frac{T(0)}{e} = \frac{T(0)}{e} + 1 - \frac{1}{e}.$$

When

$$\frac{dT}{dt} = -T \Rightarrow T(t) = Ce^{-t} = T_0 e^{-t}$$

$$T(1) = T(0)\frac{1}{e}$$

So we alternate

time	temp/100	mult by; add
9 am	0	$(*\frac{1}{e}) + 1 - \frac{1}{e}$
10		$*\frac{1}{e}$
		$*\frac{1}{e} + 1 - \frac{1}{e}$
		etc

looks like a great time for a spread sheet ☺

Temp/100	1/e	heat on add 1-1/e	Time	Temp	
0	0.367879	0.632121	9	0	
0.632121	0.367879	0	10	63.2	
0.232544	0.367879	0.632121	11	23.3	
0.717669	0.367879	0	12	71.8	Noon
0.264016	0.367879	0.632121	1	26.4	
0.729246	0.367879	0	2	72.9	
0.268275	0.367879	0.632121	3	26.8	
0.730813	0.367879	0	4	73.1	
0.268851	0.367879	0.632121	5	26.9	
0.731025	0.367879	0	6	73.1	
0.268929	0.367879	0.632121	7	26.9	
0.731054	0.367879	0	8	73.1	
0.26894	0.367879	0.632121	9	26.9	
0.731058	0.367879	0	10	73.1	
0.268941	0.367879	0.632121	11	26.9	
0.731058	0.367879	0	12	73.1	Midnight
0.268941	0.367879	0.632121	1	26.9	
0.731059	0.367879	0	2	73.1	
0.268941	0.367879	0.632121	3	26.9	
0.731059	0.367879	0	4	73.1	
0.268941	0.367879	0.632121	5	26.9	
0.731059	0.367879	0	6	73.1	