Problem 4-4-26

$y'' + 2y' + 2y = 4te^{-t}\cos t$

check homogenius: $r^2 + 2r + 2 = 0 \Rightarrow r = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$

so $e^{-t}\cos t$ is a soln to the homogenius...

Guess $y_p = (At^2 + Bt)e^{-t}\sin t + (Ct^2 + Dt)e^{-t}\cos t$

$= e^{-t}[(At^2 + Bt)\sin t + (Ct^2 + Dt)\cos t]$

✓ $y_p' = -e^{-t}[\,''\,] + e^{-t}[(2At + B)\sin t + (At^2 + Bt)\cos t + (2Ct + D)\cos t - (Ct^2 + Dt)\sin t]$

$= e^{-t}[(2At + B - At^2 - Bt - Ct^2 - Dt)\sin t + (At^2 + Bt + 2Ct + D - Ct^2 - Dt)\cos t]$

$= e^{-t}\{[-(A+C)t^2 + (2A - B - D)t + B]\sin t + [(A-C)t^2 + (2C + B - D)t + D]\cos t\}$

$y_p'' = -e^{-t}\{\ \} + e^{-t}\{[-2(A+C)t + (2A - B - D) - (A-C)t^2 - (2C + B - D)t - D]\sin t$

$+ [-(A+C)t^2 + (2A - B - D)t + B + 2(A-C)t + (2C + B - D)]\cos t\}$

$= e^{-t}\{[(A+C)t^2 - (2A - \cancel{B} - D)t - B - (A-C)t^2 - (2A + 4C + \cancel{B} - D)t + 2A - C - 2D]\sin t$

$+ [(C-A)t^2 - (2C + B - \cancel{D})t - D - (A+C)t^2 + (4A - 2C - \overset{B}{\cancel{D}})t + B + \cancel{2A - C}) + 2C + B - D]\cos t\}$

$= e^{-t}\{[2Ct^2 - (4A - 4C - 2D + \cancel{B})t + (2A - 2D - \overset{B}{\cancel{B}} - C)]\sin t$

$+ [-2At^2 + (4A - \cancel{4}C - 2B)t + 2C + 2B - 2D]\cos(t)\}$

Why am I doing this by hand? Use Mathematica!

$y_p'' + 2y_p' + 2y_p = 2e^{-t}\{[B + C + 2At]\cos(t) + (A - D - 2Ct)\sin(t)\}$

$= 4te^{-t}\cos t$

$\Rightarrow 4A = 4 \;;\; B + C = 0 \quad 2C = 0 \quad A - D = 0$

$\Rightarrow A = 1 \quad D = 1 \quad B = C = 0$

So $y_p = (t^2\sin t + t\cos t)e^{-t}$

Define general form for yp then differentiate, simplify and differentiate to check my hand work:

```
yp[t_] := ⅇ^-t ((A t^2 + B t) Sin[t] + (C t^2 + D t) Cos[t])
```

```
yp'[t]
```

```
-ⅇ^-t ((D t + C t^2) Cos[t] + (B t + A t^2) Sin[t]) +
 ⅇ^-t ((D + 2 C t) Cos[t] + (B t + A t^2) Cos[t] + (B + 2 A t) Sin[t] - (D t + C t^2) Sin[t])
```

```
yp''[t]
```

```
ⅇ^-t (2 C Cos[t] + 2 (B + 2 A t) Cos[t] - (D t + C t^2) Cos[t] + 2 A Sin[t] -
    2 (D + 2 C t) Sin[t] - (B t + A t^2) Sin[t]) + ⅇ^-t ((D t + C t^2) Cos[t] + (B t + A t^2) Sin[t]) -
 2 ⅇ^-t ((D + 2 C t) Cos[t] + (B t + A t^2) Cos[t] + (B + 2 A t) Sin[t] - (D t + C t^2) Sin[t])
```

```
Simplify[%]
```

```
-2 ⅇ^-t ((-C + D + B (-1 + t) - 2 A t + 2 C t + A t^2) Cos[t] + (B + D + 2 C t - D t - C t^2 + A (-1 + 2 t)) Sin[t])
```

```
Simplify[-ⅇ^-t ((D t + C t^2) Cos[t] + (B t + A t^2) Sin[t]) +
  ⅇ^-t ((D + 2 C t) Cos[t] + (B t + A t^2) Cos[t] + (B + 2 A t) Sin[t] - (D t + C t^2) Sin[t])]
```

```
ⅇ^-t ((D - D t + t (B + 2 C + A t - C t)) Cos[t] - (B (-1 + t) + t (D + A (-2 + t) + C t)) Sin[t])
```

But this is *Mathematica*, let's just plug it into the DE and simplify:

```
yp''[t] + 2 yp'[t] + 2 yp[t]
```

```
ⅇ^-t (2 C Cos[t] + 2 (B + 2 A t) Cos[t] - (D t + C t^2) Cos[t] + 2 A Sin[t] -
    2 (D + 2 C t) Sin[t] - (B t + A t^2) Sin[t]) + 3 ⅇ^-t ((D t + C t^2) Cos[t] + (B t + A t^2) Sin[t]) -
 2 ⅇ^-t ((D + 2 C t) Cos[t] + (B t + A t^2) Cos[t] + (B + 2 A t) Sin[t] - (D t + C t^2) Sin[t]) +
 2 (-ⅇ^-t ((D t + C t^2) Cos[t] + (B t + A t^2) Sin[t]) +
    ⅇ^-t ((D + 2 C t) Cos[t] + (B t + A t^2) Cos[t] + (B + 2 A t) Sin[t] - (D t + C t^2) Sin[t]))
```

```
Simplify[%]
```

```
2 ⅇ^-t ((B + C + 2 A t) Cos[t] + (A - D - 2 C t) Sin[t])
```

Now we see what our solution is. Let's double check it:

```
ypp[t_] := ⅇ^-t (t^2 Sin[t] + t Cos[t])
```

```
ypp''[t] + 2 ypp'[t] + 2 ypp[t]
```

```
-2 ⅇ^-t (Cos[t] + t^2 Cos[t] + t Sin[t]) + ⅇ^-t (3 t Cos[t] - t^2 Sin[t]) +
 3 ⅇ^-t (t Cos[t] + t^2 Sin[t]) + 2 (ⅇ^-t (Cos[t] + t^2 Cos[t] + t Sin[t]) - ⅇ^-t (t Cos[t] + t^2 Sin[t]))
```

```
Simplify[%]
```

```
4 ⅇ^-t t Cos[t]
```