Problem 4-5-41

$$y'' + 2y' + 5y = \begin{cases} 10 & 0 \le t \le 3\pi/2 \\ 0 & t > 3\pi/2 \end{cases} \qquad y(0) = y'(0) = 0$$

$y_p = A \Rightarrow 5A = 10$

$y_p' = y_p'' = 0 \qquad \Downarrow A = 2$

$r^2 + 2r + 5 = 0 \Rightarrow r = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$

$y_h = e^{-t}[c_1 \sin 2t + c_2 \cos 2t] \qquad y_h' = -e^{-t}[c_1 \sin 2t + c_2 \cos 2t] + e^{-t}[2c_1 \cos 2t - 2c_2 \sin 2t]$

$y_{g_1} = 2 + e^{-t}[\,''\,] \qquad y_{g_1}(0) = 2 + c_2 \qquad = e^{-t}[(-c_1 + 2c_2)\sin 2t + (2c_1 - c_2)\cos 2t]$

$y_{g_1}'(0) = 2c_1 - c_2 \qquad \text{so} \quad 0 = 2 + c_2 \Rightarrow c_2 = -2 \text{ and } 2c_1 = c_2 \Rightarrow c_1 = -1$

$y_1(t) = 2 - e^{-t}[\sin 2t + 2\cos 2t]$

$y_1(3\pi/2) = 2 - e^{-3\pi/2}[2(-1)] = 2 + 2e^{-3\pi/2}$

$y_1'(t) = e^{-t}[(1+4)\sin 2t + (-2+2)\cos 2t] = 5e^{-t}\sin 2t$

$y'(\frac{3\pi}{2}) = 0.$

$y_{g_2} = e^{-t}[\alpha_1 \sin 2t + \alpha_2 \cos 2t] \qquad y_{g_2}(\frac{3\pi}{2}) = e^{-\frac{3\pi}{2}}[-\alpha_2] = 2 + 2e^{-3\pi/2}$

$y_{g_2}' = -e^{-t}[\,''\,] + e^{-t}[2\alpha_1 \cos 2t - 2\alpha_2 \sin 2t] \Rightarrow \alpha_2 = -2(e^{\frac{3\pi}{2}} + 1)$

$y_{g_2}'(\frac{3\pi}{2}) = -(2 + 2e^{-3\pi/2}) + e^{-\frac{3\pi}{2}}[-2\alpha_1] = 0$

$\text{so } e^{-3\pi/2}\cancel{2}\alpha_1 = -\cancel{2}(1 + e^{-3\pi/2}) \Rightarrow \alpha_1 = -e^{3\pi/2} - 1$

$$\text{so } y(t) = \begin{cases} 2 - e^{-t}[\sin 2t + 2\cos 2t] & 0 \le t \le 3\pi/2 \\ -e^{-t}[(e^{3\pi/2}+1)\sin 2t + 2(e^{3\pi/2}+1)\cos 2t] & t > 3\pi/2 \end{cases}$$

Let's Plot this to check...

Let's do a graphical check of our solution for 4-5-41:

```
In[6]:= y[t_] := Piecewise[
   {{2 - e^-t (Sin[2 t] + 2 Cos[2 t]), t < 3 π / 2}, {-e^-t (e^(3 π/2) + 1) (Sin[2 t] + 2 Cos[2 t]), t > 3 π / 2}}]
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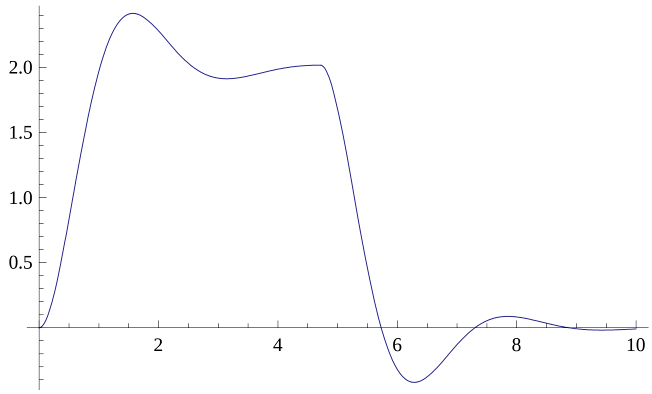
In[7]:= Plot[y[t], {t, 0, 10}]
```

Out[7]=



Let's take a closer look to check on the derivative match:

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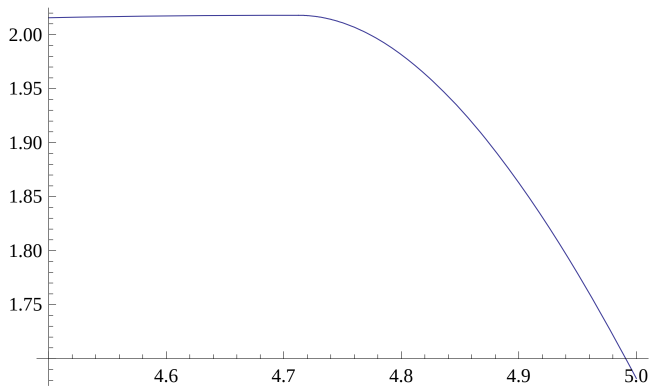
In[9]:= Plot[y[t], {t, 4.5, 5}]
```

Out[9]=



Looks good to me:-)