More Uses of Pascal's Triangle
Combinations

1. Using the letters in the word HAT, list all the possible combinations of two different letters. How many are possible? (Order does NOT matter).

2. Billy has 4 different colors of marbles in a jar: red, yellow, blue, and green. If he draws out three marbles at a time then puts them back, how many different combinations are possible.

3. Suppose Billy only chooses two marbles. How many combinations are possible now?

4. Sally has five different coins: half-dollar, quarter, dime, nickel, and penny. If she can only spend two coins at a time, how many different amounts of change might she have?

5. Suppose Sally wants to spend three coins. How many combinations are possible now?

6. Suppose Sally wants to spend four coins. How many combinations are possible?

7. Construct 10 rows of Pascal’s triangle.

8. How do the numbers from the combination problems appear is Pascal’s triangle?

The notation for finding the number of combinations is written ${}_nC_r$ and read "n choose r". For example, if Billy has sixteen marbles in his jar and draws out 5 at a time, we would write ${}_{16}C_5$.

Using Pascal's triangle ________ and the previous examples, fill in the table below.

n	r	Total Combinations
3	2	
4	2	
5	2	
6	2	
4	3	
5	3	
6	3	
5	4	
6	4	
6	5	

9. What patterns do you see?

If the numbers get too large, using Pascal's triangle could be very tedious. Let's see if we can find a formula. First we need some background information. One way to write consecutive numbers that multiply is with an exclamation mark which is known as a factorial. For example, 5! = 5 x 4 x 3 x 2 x 1 = 120. Beginning with the given number, the factorial multiplies all the decreasing consecutive numbers down to 1. Evaluate the following factorials.(Hint: 0! is defined to be equal to 1)

10. 3!

11. 6!

12. 12!

Operations with factorials must be done carefully! For instance, $\frac{8!}{4!}$ does NOT equal 2!

The factorial must be expanded first then divided.

$$\frac{8!}{4!}=\frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{4\cdot 3\cdot 2\cdot 1}=8\cdot 7\cdot 6\cdot 5=1680$$

Evaluate the following quotients.

13. $\frac{4!}{3!}$

14. $\frac{5!}{3!}$

15. $\frac{9!}{6!}$

Fill in the following table.

n	r	**Total Combinations**	**n!**	**r!**	**$\frac{n!}{r!}$**
4	2				
5	3				
6	3				
7	4				
9	4				

16. What patterns do you see?

17. What needs to be done to $\frac{n!}{r!}$ in order to obtain the number of combinations in each case?

18. Using only factorials, write a formula for finding ${}_nC_r$.

Use the Pascal's triangle you constructed in #7 to complete the following activity. Another way to find the number of combinations in Pascal's triangle is to make smaller triangles. Consider ${}_7C_3$. Beginning with the 7, draw a diagonal line towards the top that connects 3 numbers: 7, 6, and 5. Draw two more lines that would complete an equilateral triangle. The answer to ${}_7C_3$ will be the bottom right vertex of the triangle, 35. Draw triangles to represent the following.

19. ${}_5C_4$

20. ${}_8C_2$

21. ${}_6C_3$

22. Why does this work?