Primitive roots

Let a and n be relatively prime positive integers. The smallest positive integer k so that $a^k = 1 \pmod{n}$ is called the *order of a modulo n*. The order of a modulo n is usually denoted by $\mathrm{ord}_n a$. If it happens that $\mathrm{ord}_n a = \phi(n)$, where ϕ is the Euler phi function, then a is said to be a *primitive root modulo n*.

It is well known which integers have primitive roots. It is also known that if the positive integer n has a primitive root, then it has a total of $\phi(\phi(n))$ incongruent primitive roots. The purpose of this lab is to attempt to discover which integers have primitive roots.

Exercises

1. Use the ring structure of $\mathbb{Z}_n$ and the software PascGaloisJE or one of the supporting Java applets to find a primitive root modulo each of the following integers.
   (a) 8
   (b) 14
   (c) 11
   (d) 25
   (e) 26
   (f) 6
2. Show that the integer 15 has no primitive roots.
3. Show that the integer 20 has no primitive roots.
4. How many incongruent roots does 11 have? Find a set of this many incongruent primitive roots modulo 11.
5. Make a conjecture about which positive integers n possess a primitive root.
6. Find all solutions of the congruences (express your answer as a congruence class).
   (a) $7^k = 13 \pmod{15}$
   (b) $5^k = 4 \pmod{19}$