Introduction to the Professor

Welcome to the PascGalois Project. The PascGalois Project is devoted to the exploration and visualization of abstract mathematical concepts. Although our main focus is on the teaching of introductory abstract algebra at the undergraduate level, our materials have also been used in the instruction of Discreet Mathematics, Number Theory, Dynamical Systems, Combinatorics and Mathematics for Elementary Education. The project has also had a significant number of undergraduate research students in these fields as well as computer science.

The materials included are designed as supplements to a modern algebra course. Algebra is regarded by many students as the least visual course in the undergraduate mathematics curriculum. The first goal of these activities is to help your students visualize many of the important concepts from a first semester undergraduate abstract algebra course, particularly group theory. These activities are not intended to replace a textbook, nor the standard homework problems that are found in any traditional text. Rather, they should serve as a supplement allowing your students to attempt different types of problems that rely more on exploration and visualization using computer graphics. It is our hope that approaching algebra from multiple perspectives will help create a deeper understanding of the subject. We also hope that your students will see connections with other areas of mathematics. The target audience for these materials is both junior and senior mathematics and mathematics secondary education majors.

Our set of algebra labs are divided into two groups, one that uses the set of PascGalois JE applets and the other that is intended for use with the PascGalois JE application. Each group is further divided into two categories, a core that addresses many of the main course topics using $\mathbb{Z}_n$ as the primary group of study and a set of supplemental labs, some that extend the core to other group structures such as $\mathbb{D}_n$ and $\mathbb{S}_n$ and others that focus on more advanced topics.

The PascGalois JE application can be downloaded from the software page and contains all of the facilities you will need to complete the labs. The labs that use the applets have the applets built into the labs and hence you do not need to download or install any software. The applets are designed to give the user just enough functionality to complete the exercises and so they tend to be less intimidating to students then the full program. Both the applets and the application have a very similar look and feel so switching between them should not present too many difficulties.

What follows is a list of our abstract algebra labs, each with a prerequisite list, follow-up list and comments we hope you will find helpful.

Core Labs:

Introduction to PascGalois JE or to the PascGalois JE Applets: We suggest that the students do this lab in its entirety before proceeding on to the other labs. First it will give the student a basic idea of what the program does and it will give them some experience in navigating the program. The lab is lengthy but fairly mechanical. Since this lab simply introduces the software there are no prerequisites (Aside from knowing about modular arithmetic). Also since this is an introduction to the software we consider this lab to be a prerequisite for all of the other labs.

Pattern Recognition and Pascal's Triangle Modulo n: As its name implies this lab concentrates on pattern recognition for the groups $\mathbb{Z}_n$. In the subsequent labs we frequently ask the student if they recognize a particular pattern, of course suggesting an isomorphism. Many of the patterns they see will be those they encounter in this lab. They will get a feel for what $\mathbb{Z}_2$, $\mathbb{Z}_3$, ... "look like" just as they have a feeling for what $y = x^2$, $y = x^3$, $y = \sin(x)$, ... look like. The only prerequisite lab is the software introduction lab and a good follow-up lab would be *Pattern Recognition Using Dihedral Groups*.

An Introduction to Isomorphism: Groups of Order 4: This lab is a nice lab if you discuss isomorphism early in the course. If you hold off on the concept of isomorphism until later you may wish to

skip this lab and just do the Isomorphism lab. This lab has the student create all possible groups of order 4 by using only the basic rules for constructing the operation table. The student then inputs the tables into the program to obtain images of their groups and form conjectures to the equivalence of some of their group tables. We would suggest both the software introduction and the pattern recognition (mod n) labs as prerequisites. A good follow-up would be the *Isomorphism* lab which is part of the core.

Subgroups and Generators of $\mathbb{Z}_n$: As with the other labs in the core this lab focuses on $\mathbb{Z}_n$. We would suggest both the software introduction and the pattern recognition (mod n) labs as prerequisites. A good follow-up would be the *Subgroups and Generators of $\mathbb{D}_n$ and $\mathbb{S}_n$* lab which investigates non-cyclic groups.

Isomorphism: This lab approaches the isomorphism map as a changing of colors between two images such that the resulting triangles look the same. The student will be dragging and dropping the colors from one image to another and hence be constructing the isomorphism via the computer mouse. The main question that is posed is when $\mathbb{Z}_n \times \mathbb{Z}_m$ is a cyclic group. We would suggest both the software introduction and the pattern recognition (mod n) labs as prerequisites. A good prelab would be the *Groups of Order 4* lab but it is not required.

Quotient Groups of $\mathbb{Z}_n$: This is a quick introduction to the concept of a quotient group. The visualization of quotient groups is a particular strength of the PascGalois JE software. One of the main difficulties students have with quotient groups is the idea that a set of elements is a single element in the quotient structure, i.e. considering a coset as a point. This lab focuses on this concept by having the student color all of the elements of a coset the same color, hence seeing the set as a point. We would suggest both the software introduction and the pattern recognition (mod n) labs as prerequisites. A good follow-up would be the *Quotient Groups of $\mathbb{D}_n$ and $\mathbb{S}_n$* lab which investigates non-abelian groups and subgroup normality.

Supplemental Labs:

Building A Group with PascGalois Triangles: This is a nice lab to help the student think more abstractly. The idea is to think of PascGalois triangles as single elements of a set and define an operation on them. This lab uses both $\mathbb{Z}_n$ and $\mathbb{D}_n$ as base structures so we would suggest that you do at least the software introduction and the pattern recognition (mod n) labs and either discuss the $\mathbb{D}_n$ element notation of the PascGalois JE program or do the Pattern Recognition Using Dihedral Groups lab.

Pattern Recognition Using Dihedral Groups: This is simply a continuation of the *Pattern Recognition and Pascal's Triangle Modulo n* lab. The software introduction and the pattern recognition (mod n) labs should be done before this one.

Subgroups and Generators of $\mathbb{D}_n$ and $\mathbb{S}_n$: This is simply a continuation of the *Subgroups and Generators of $\mathbb{Z}_n$* lab. The software introduction, pattern recognition (mod n) and Subgroups and Generators of $\mathbb{Z}_n$ labs should be done before this one.

Direct Products and Automorphisms: The main question here is when does reflecting a PascGalois triangle over the vertical bisector induce an automorphism on $\mathbb{Z}_n \times \mathbb{Z}_m$? We would suggest that you do at least the software introduction and the pattern recognition (mod n) labs prior to this one and have discussed the concept of isomorphism.

Quotient Groups of $\mathbb{D}_n$ and $\mathbb{S}_n$: This is a continuation of the Quotient Groups of $\mathbb{Z}_n$ lab. It also discusses the visualization of a normal subgroup. We would suggest that you do at least the software introduction, the pattern recognition (mod n) and Quotient Groups of $\mathbb{Z}_n$ labs prior to this one. You should have also used the PascGalois JE program (or applets) to view PascGalois triangles over $\mathbb{D}_n$ and $\mathbb{S}_n$ in some capacity.

Subnormal Series: This lab focuses on subnormal series over both $\mathbb{Z}_n$ and $\mathbb{D}_n$. We would suggest that you do at least the software introduction and the pattern recognition (mod n) labs prior to this one in addition to using the PascGalois JE program (or applets) to view PascGalois triangles over $\mathbb{D}_n$.

Self-Similarity and the Klein-4 Group: This lab investigates the growth triangles of $\mathbb{Z}_n$ and the Klein-4 group. We would suggest that you do at least the software introduction and the pattern recognition (mod n) labs. The *An Introduction to Isomorphism: Groups of Order 4* lab would be useful but not a necessity.

Rings and Fields: This lab gives a quick look into integral domains, fields and zero divisors. This lab only requires the software introduction and the pattern recognition (mod n) labs.

Discovering a Binomial Identity: This lab gives a quick introduction to using PascGalois triangles to develop identities consisting of binomial coefficients modulo n. This lab only requires the software introduction and the pattern recognition (mod n) labs.