Appendix: Brief Introduction to the PascalGT Software:

Welcome to PascalGT, a Windows-based program for drawing PascGalois triangles and hexagons, 1-D and 2-D automata and more. In this program, the cell values for automata generally correspond to group elements. Let's take a quick walk through the program and explore the basic features (this seven step tour can also be found in the quick start screen in the program's help files. That screen includes color images of what you should see.):

1) When you first start the program, you are in the Enter Data Window. Choose how you would like to enter the numbers into the group Multiplication Table using the Enter Group/Set Information in the Menu. Z_n, a cyclic group, with order of 12, for example, is a good start.

2) Choose Identity Element (any element added/multiplied times this number is itself -if you choose a preprogrammed group like Z_n then the identity element is 0) and Generators (Numbers down the side of a PascGalois Triangle)---Try 1,2. Note that in order to change the generators you must highlight the number in the box and type over it. Then click continue.

3) Now you are in the Image/Draw Window. To see the Triangle you have made, click Triangle from the PascGalois Menu button. To see what number each circle represents, Go into the Setup Conditions, move into the Choose color options list and choose Display current colors n<=16. (We choose this instead of n>16 because our group order is 12 from above).

4) This triangle doesn't tell us too much. Let's look at it with more rows. Under Setup Conditions, Click on Choose number of rows. The default for changing is 125 rows which should work for us. (Watch out, the larger the rows, the longer it takes to draw it)

5) Click again on Triangle to display the larger triangle (Don't worry, the screen automatically clears). Do you see any patterns inside this triangle? If not, Click on Choose Color Scheme(Rotate 16 colors) inside Choose Color Options under Setup Conditions to rotate the colors in the triangle. Then redraw the triangle with the new colors. Try different color schemes numbers and watch how some patterns jump out at you.

6) Okay, you still can't see some patterns, but there are some tricks to help you. You can white out the infrequent Colors. Click on Reset Popular Color Values inside Choose Color Options. There will be a prompt to enter a 1 if you want to white out the other values: Enter a 1. Now, click on Triangle to see your new triangle. It may look blurry, so increase the number of rows to 513 . (Warning, this may take a long time on older computers.)

7) We've finally drawn a nice pretty triangle. Why don't we add a caption to our triangle using the save menu. Be sure the Point-Up triangle option is clicked. Let's

save it as a .bmp or .jpg using the save menu.

Congratulations, you have successfully used the PascalGT program. There are a number of other functions which the program can perform but not all of them will be necessary to complete the laboratory exercises you will be using this semester. For the remainder of this section, we will go over the things you will need to use for the laboratory exercises to follow. You are in the interesting situation of trying to learn to use a package that supports inquiry into a field in mathematics at the same time you are learning the terminology of that field. Therefore the first time you read through this document, many of the terms may be unfamiliar to you. Don't worry! You will know what they mean by the time you need to use them. Refer back to this section as needed throughout the term while you are completing the labs.

The *Enter Data* Window:

Choosing a group or set: In the *Enter Data* window you will generally be choosing from the sub-menu items under *"Enter Group/Set Information*" one of the preprogramed groups. Those you will need are:

- Z_n: The cyclic group of order n. *This is also the choice you make in the first project for modulo n addition.*
- D_n: The dihedral group of order 2n, the symmetry group of a regular n-gon.
- $Z_n x Z_m$: The cross product of the cyclic group of order n with the cyclic group of order m.
- Z_n *Ring multiplication*: You will need this option for the *Rings and Fields* Project. As a set Z_n is the integers mod n. But this option uses multiplication mod n rather than addition as the operation. This option isn't a group multiplication (why?).

Group element identification: Although when you are introduced to these groups in class, the group elements may have various names and representations, for the purposes of the program, each group element is assigned an integer starting at 0 and continuing to one less than the order of the group. Thus the assignments for Z_n are 0, 1, 2, 3, . . . , n-1 (which does follow traditional notation). The assignments for D_n are 0, 1, 2, ..., 2n-1 and the assignments for $Z_n x Z_m$ are 0, 1, 2, ..., nm-1. Once you select a group from this list a multiplication table is displayed. If the order for your group is 16 or less the entire table is displayed; if the order of your group is more than 16 only part of the table is displayed. A good exercise in understanding what is going on would be to identify the symbols you are using to represent the elements of these groups with the numbers used by the program by examining the multiplication table. In all of the preprogramed **groups** the number 0 corresponds to the identity.

System Generators and Identity: The last thing you need to do before clicking on continue to move to the *Draw Screen* is to identify the generators and the identity. For all of the preprogramed groups the identity will be correctly entered for you (although the program will not keep you from changing this and some strange things may occur in some of your drawings if you do so). There are also default choices for the generators.

For the first two projects use the default settings. For Z_n the default setting is for both generators to be set to 1. For D_n, the default generators are a rotation through 360/n degrees and a reflection. For $Z_n x Z_m$ the default generators are (1,0) and (0,1) which are numbered 1 and n in the program. For some of your labs you may wish to change these; you may do so either by entering the number corresponding to the elements you wish to use instead in the identity and generators frame on the enter data screen or by choosing the *Change Generators* option under *Setup Conditions* on the *Draw Screen*. These generators, referred to as *"system generators*" will be of interest to you primarily because the PascGalois triangles that will be drawn by the program are:

- P_G(generator2, generator1) or P_G(a,b) .

Once you are satisfied with the generators, click on continue or press <ctrl>G to load the draw window.

The *Draw* Window

The draw window is where you will produce images. There are four menu headings with which you will need to be concerned. They are:
Setup Conditions, PascGalois, Color Subsets and Save.

***Setup Conditions*:** Under Setup Conditions the options you will need are found under:

- "Choose number of rows" this will change the number of rows that you will draw when you select from the PascGalois menu. You can also use the shortcut ctrl <R> to access this item. A dialogue box will appear where you can enter the number of rows, referred to hereafter as the *system number of rows.* Remember that drawing a large number of rows (more than 500) may take some time. If you do not exercise this option, the program defaults to 25 rows.
- "Choose color options."
    - The most important item in this submenu for you is the *"Display Current Colors, n<=16)* option which can also be accessed through the shortcut ctrl<D>. This will draw circles on the screen with the colors corresponding to each of the group elements, so long as the order of your group is $\leq$ 16. Next to each circle will be a number. The number is the one which corresponds to the same group element as the color. The numbers used are the same as the numbers used in the enter data window for the multiplication table. If you want to look at a group with more than 16 elements use the *"Show Current Colors (n>16)"* or ctrl<S> option.
    - For the first five labs, it is not necessary that you use any of the options that change color assignments. However, if you are having difficulty "seeing" what you are supposed to see, it is possible that a change of color scheme might help. The easiest way to do this is to select the "Choose Color Scheme (rotate 16 colors)" option from this sub menu. Simply enter a number between 0 (default colors) and 15. This will change the assignments.
    - Eventually you may want to choose colors specifically. The easiest way to do this is by choosing the "set colors manually" or ctrl<C> option. When you choose this, a grid with 16 text boxes and an ok button will

appear next to 16 circles of different colors. Simply enter the number of the group elements next to the colors to which you wish them to correspond. Any elements left out will be colored white and will disappear against the white background.

- There are many other options for changing colors, feel free to play with them at your leisure to draw pleasing pictures. Consult the system help for descriptions of these options.

- Change Generators: Choose this option if you want to draw images using different generators than those you previously entered in the enter data window. You will prompted for the group elements you want to use as the new generators. You must use the numbers corresponding to the group elements for the program to recognize them. This changes the "system generators."
- Display group order: If you forget which group you are using or want to check on its order this will write to the screen the number of elements in the group (entered in the enter data screen) the program is currently using.

***PascGalois*:** Under PascGalois you find the options that actually draw the images. *{If, when you choose from this menu, nothing gets drawn there are two possibilities. Most likely the program is taking a while because you have a large number of rows, in which case you cannot use any other menu options because the program is busy. The other possibility is that you have changed the colors so that everything is white. If nothing gets drawn and the program isn't still busy, try resetting to system colors from the menu: "Setup Conditions" , "Choose Color Options", "reset to system colors" or by pressing the F1 key.}* For the labs you will need to use:

- *Triangle PG(a,b)*, which can also be activated by ctrl<T>, draws the PascGalois Triangle $P_G(a,b)$ where a and b are the generators you either entered on the enter data screen or changed to in the setup conditions. If your group is Z_n and you use the default generators, then this is Pascal's triangle mod n with each number represented by a color. Pressing ctrl<D> will identify the color-number correspondences for you. The triangle has either 25 rows or the number of rows you entered in the setup options. Once the program has drawn the triangle you can use the mouse and click on a position in it and the program will display the row and column number of the position.
- *Superimposed Triangles*: This option, needed for the *Building a Group with PascGalois Triangles* project, prompts you for the numbers corresponding to two group elements you wish to use as a second set of generators. It then draws three triangles, one PascGalois Triangle using the system generators on the left, one PascGalois Triangle using the generators you just supplied on the right, and finally, below them, a triangle where each position is the color corresponding to the group product of the elements in the same position in the two triangles above it.
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In the course of your investigations you may also wish to look more closely at a portion of a triangle and thus use the PascGalois options:

- "*Bottom of Triangle*" prompts you for a row to begin on and draws the bottom portion of the triangle starting at the row you entered and ending at the row number of the current system number of rows.
- "*Subtriangle*" displays a dialogue box in which you enter the row and column number of the top vertex of the desired triangle and the row number in the PascGalois triangle that you wish to draw to (this needs to be larger than the row number of the vertex). When you press ok, if the numbers entered are legitimate, the program draws the portion of the PascGalois triangle that corresponds to the point up Subtriangle beginning at the vertex entered and ending at the row entered.
- "*Point Down Subtriangle*" prompts you for the beginning row and leftmost column of the triangle then prompts you for the width of the triangle. You must choose a width of greater than three and less than the difference between the beginning row and the beginning column so you need to choose a beginning column at least three less than your beginning row. Then the program draws the portion of the PascGalois triangle that constitutes the point down triangle starting at the given point with the given width.

Color Subsets: The items on the color subsets menu allow you to identify a number of group elements and color them the same or to select only some elements to color at all. These items alter the color identification that the program is using. It is best to reset to system colors (press F1 or see above) after using one of these to produce an image. Very strange colorings can occur if you use different options in this menu in sequence without resetting.

- *Select a few elements to color*: If you choose this option you will be prompted for the number of elements you want to color and the numbers corresponding to those elements. The system will then assign a color of white to all other elements so that when you draw images they will not show up.
- *Color a set of elements the same*: This allows you to assign to a group of elements the same color. It prompts you first for the number of elements you want to identify and then for the elements. You can repeat this if, for example you want to color 0, 2, and 4 one color and 1, 3, and 5 a different. You would choose this option, tell it you want 3 elements, then tell it the elements are 0,2, and 4. Then you would choose the option again and tell it you want 3 elements; then tell it the elements are 1,3, and 5. Try this with Z_6. You can also use the shortcut ctrl<l> to access this item.
- D_n *subsets*: If the group you are using is a dihedral group there are some preprogrammed identifications available to you. They should be self-explanatory from the submenu under D_n subsets.
- $Z_n x Z_m$ *subsets:* There are also some preprogramed identifications for $Z_n x Z_m$ which should be self explanatory.

In order to **change the group** you are working with you need to exit the draw screen and choose new group from the menu which appears.

To **exit the program** you can either choose exit from the draw screen and from the new group screen or you can simply click on the x at the top right of the screen.

If your instructor assigns the *Two Dimensional Automata* Project, then there will also be some items from the *"2-D Automata"* menu which you will need to use. These options draw and animate cellular automata (c.a.'s) which are defined in the project. Since this project is more open ended than many of the others, descriptions for all of the options in this menu are included below, even though you may not need to use them all:

Enter Starting Values:

For a one point square: This will prompt you for the side length n of the c.a. and will then internally define an nxn c.a. with cell values of 0 (or the identity) for all but one center cell whose value will be 1 (or the group element that corresponds to 1 in the program).

For a five point square: This will prompt you for the side length n of the c.a. and will then internally define an nxn c.a. with cell values of 0 (or the identity) for all but one center cell whose value will be the right generator (1 or b) defined on the enter data screen and the four corner cells whose value will be the left generator (2 or a). Note that as the sides are linked, the four corners are actually all neighbors.

Randomly: This will prompt you for the number of columns (width) m and the number of rows (height) n of the c.a. and will then internally define and randomly assign cell values to an mxn c.a.

Enter non-zero values: This will prompt you for the number of columns (width) m and the number of rows (height) n of the c.a. and will then prompt you for the number of non-zero (or non-identity) entries, their positions and their values. As always you must use the integer identification that the program uses to assign group element values to the cells.

By Hand: This will prompt you for the number of columns (width) m and the number of rows (height) n of the c.a. and will then ask you for the value for each cell. This is recommended only for small c.a.'s. As always you must use the integer identification that the program uses to assign group element values to the cells.

From file: This option can be used to reset the automata to a state that you had previously saved.

Animate

Define Rule for Automata: The program defaults to the rule described in the project. You can, however, define other rules for advancing the c.a. This option will display a 3x3 grid of buttons representing a cell (in the center) and all of its neighbors. You can click on the buttons in the order in which you would want to multiply the cell values to update the automata and thus define the rule. If you

want to return to the default rule, press the reset button. Click done when you have finished.

Animate Step by step: This will draw the automata and display a button to advance it. Every time you click on the button, the program will advance the automata and draw the new state. Use this when you want to carefully watch the steps.

Animate n Times: This will prompt you for a value of n and compute the states as the automata advances n times and display the result.

Animate continuously: This activates three buttons labeled *stop it, faster and slower* and will display, update and display the automata continuously until you click on the *stop it* button. You can speed up or slow down the display by clicking on the *faster* or *slower* buttons.

Make a 3-D file: This will save the automata states to a file which could then be used as data for a program that you write to do a 3 dimensional version of the cellular automata with states piled up vertically on top of one another. This option is not required for the project.

Save the current state to file: This will save the current state of the automata to a file which you can then use to reset the automata using the **Enter Starting Values, By Hand** option above.

Draw and Save: This will prompt you for a number of times n to animate. Then it will draw n iterations of the automata and save each as a bitmap titled ani01.bmp , ani02.bmp . . . ani*n*.bmp in the directory where the program was last pointing. In other words, it will save to the directory where the program itself is located unless you have saved or opened a file somewhere else, in which case it will save to that directory. The best thing to do is to save an image using the save options from the main menu before using this option. You can create animations using these images and a package like GIF Construction Set, which creates animated gifs or by writing a program to display them.

Counting and Shading: These menu options do not work very well but they shade the cells of the automata based on how often the cells take on values from the elements you identify.

Searching for steady state: Since the c.a.'s used in the program are finite, there are only finitely many possible states so that eventually the states must repeat and the c.a. will become periodic. These menu items are available to help you identify the periodic, steady state in which the c.a. eventually lands, the transient phase which occurs before the periodic cycle is reached, and their lengths in terms of the number of states included in each. (Note that for some automata, the steady state will simply be the state where all values are the identity or some

other constant state; in this case the length of the periodic cycle is one.):

Just Iterate: This option iterates the automata. It is used to try to get beyond the transient phase.

Find Period: This option saves the current state and iterates the automata until that state repeats. It will then report the length of the periodic cycle. If it fails, it will report that either the period is too long or the transient state had not been passed. A second try of this option will generally be successful if the period is not too long.

Save a State: Unlike the *Save the current state to file* option from the menu above, this option saves the current state internally and you can then reset to it using the next option or you can check to see if it recurs using the following option.

Reset to a Saved State: Reset to the state you last saved using the option above.

***Check for recurrence of a saved state*:** If you have saved a state using the option before last, you can check to see if it recurs using this option. You will be prompted for the maximum number of iterations the program should perform if the state has not recurred. If the state recurs within that number of iterations, it will report back which iteration the state was found on. This is useful for finding long periods.

Looking for the transient? Use this option when you know the length of the periodic cycle to which the c.a. eventually evolves. You will be prompted for the length of the periodic cycle and then the program will test to see how long the c.a. iterates before reaching that cycle.