Introduction to the Student

"… A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas."   *G.H. Hardy*

Welcome to the PascGalois Project. The materials included are designed as supplements to a modern algebra course. Algebra is regarded by many students as the least visual course in the undergraduate mathematics curriculum. The first goal of these activities is to help you visualize many of the important concepts from a first semester abstract algebra course, particularly group theory. These activities are not intended to replace a textbook, nor the standard homework problems that are found in any traditional text. Rather, they should serve as a supplement allowing you to attempt different types of problems that rely more on exploration and visualization using computer graphics. It is our hope that approaching algebra from multiple perspectives will help create a deeper understanding of the subject. We also hope that you will see connections with other areas of mathematics. The structures studied in these projects have applications with other disciplines including dynamical systems and number theory. However, no prior knowledge of those subjects is assumed to use these materials.

A second goal of these materials is to allow you opportunities to examine more "open-ended" problems. For example, many of the exercises ask you to look for a pattern in a given image. Actually, these images contain numerous patterns. You may find one pattern whereas a classmate finds a completely different one. However, the algebraic structures under consideration play a crucial role in producing the patterns found in these exercises. You may even find a pattern that nobody else has noticed yet! In fact, these projects can serve as a good starting point for an undergraduate research project. Although the exercises are designed for course work, they do have an exploratory flavor that makes them amenable to further research.

One of your tasks will be to make connections between the appearance of the computer generated images and the underlying concepts from abstract algebra. Even once these connections have been found, you will see how difficult it is to describe the relationships using precise terminology while writing up solutions. Mathematics majors certainly need to learn how to solve problems. However, it is equally important that they learn how to put their solutions clearly and accurately in writing so that others can read and understand their work. Avoiding vague and undefined terms is crucial to this effort. As you write solutions to these projects ask yourself whether the reader should already know the terms you are using. Or, have you defined them sufficiently in the write-up so that the reader has a precise understanding of what you are trying to convey?

The PascalGT software (included) is used for all of the projects. Documentation is included. The best way to get started with PascalGT is just to sit down and "play" with it for awhile. As you work the projects feel free to experiment a bit along the way. Ask your own questions and follow up with an investigation. You may very well soon start making your own discoveries!