Blaise Pascal & Evariste Galois

Pascal

Galois

Through space the universe grasps me and swallows me up like a speck; through thought I grasp it. *Pensees. 1670.*

Unfortunately what is little recognized is that the most worthwhile scientific books are those in which the author clearly indicates what he does not know; for an author most hurts his readers by concealing difficulties.
Quoted in N Rose, Mathematical Maxims and Minims (Raleigh N C 1988)

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Blaise Pascal

Blaise Pascal 1623 to 1662

Pascal worked on conic sections and produced important theorems in projective geometry. In correspondence with Fermat he laid the foundation for the theory of probability.

Find out more at:
http://www-history.mcs.st-andrews.ac.uk/history/Mathematicians/Pascal.html

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Evariste Galois

Evariste Galois 1811 to 1832

Famous for his contributions to group theory, **Galois** produced a method of determining when a general equation could be solved by radicals.

Find out more at:

http://www- history.mcs.st-andrews.ac.uk/history/Mathematicians/Galois.html

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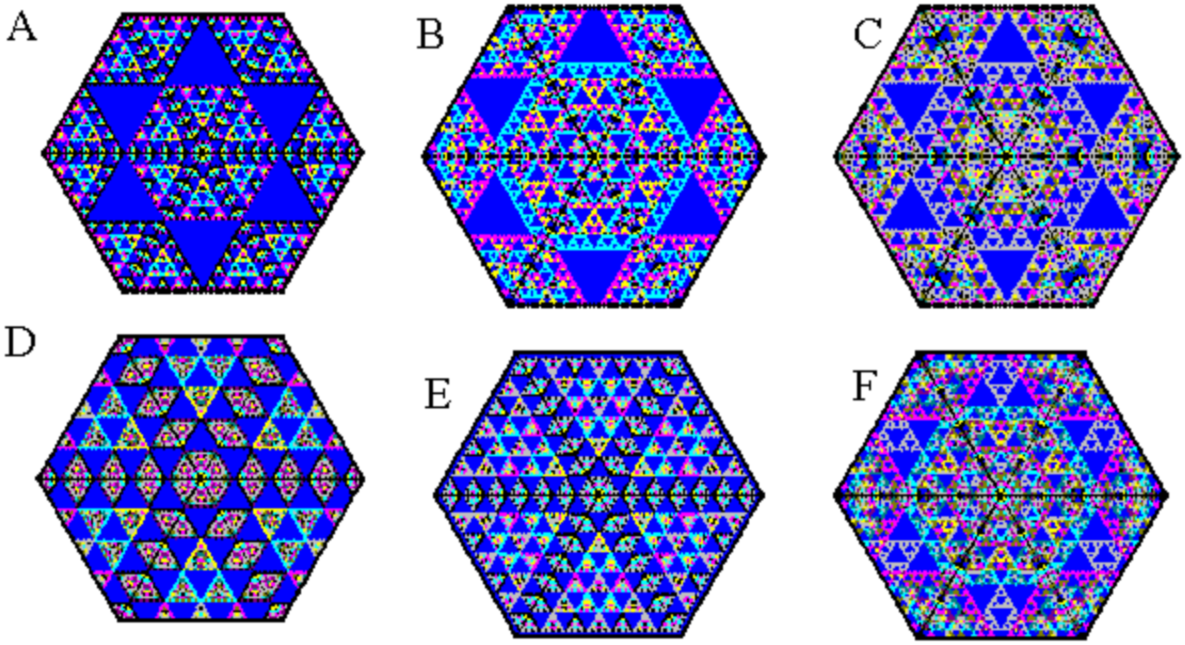
Prime or Composite? That is the Question.



The algorithm that generated these hexagon patterns depends on a whole number. Three of these were generated by prime numbers and three by composite numbers (with relatively prime factors). Can you group these six figures into two sets of three, with similar properties? Which group consists of the simpler (cooresponding to primes)patterns?

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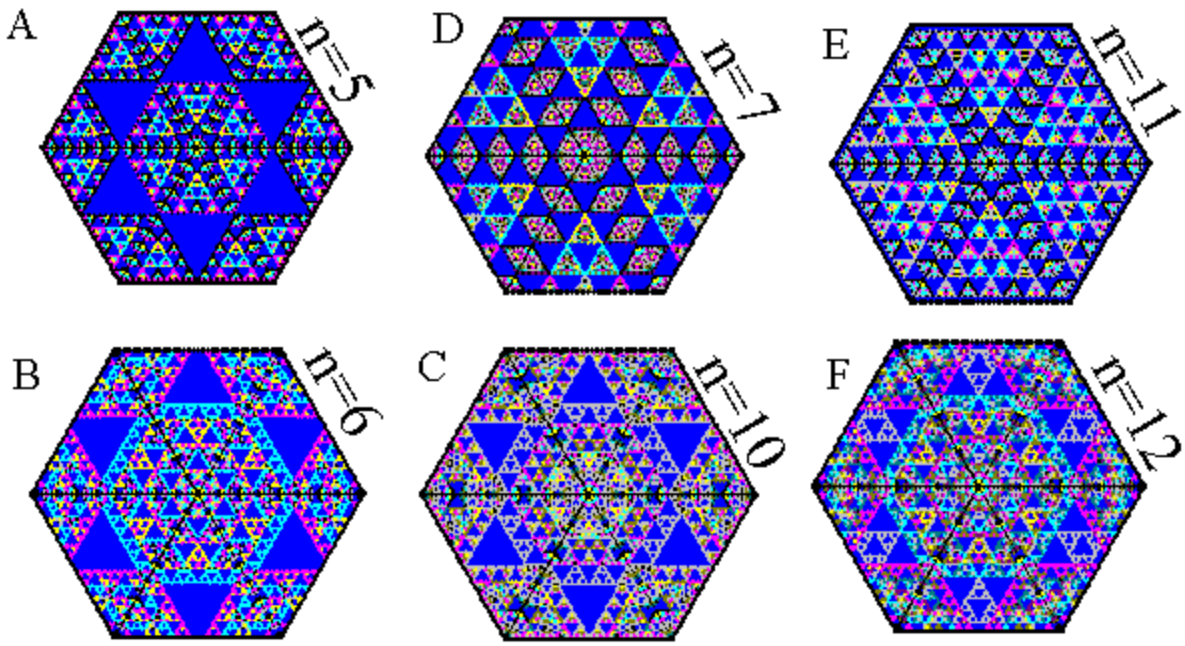
Prime or Composite? Here are the answers.



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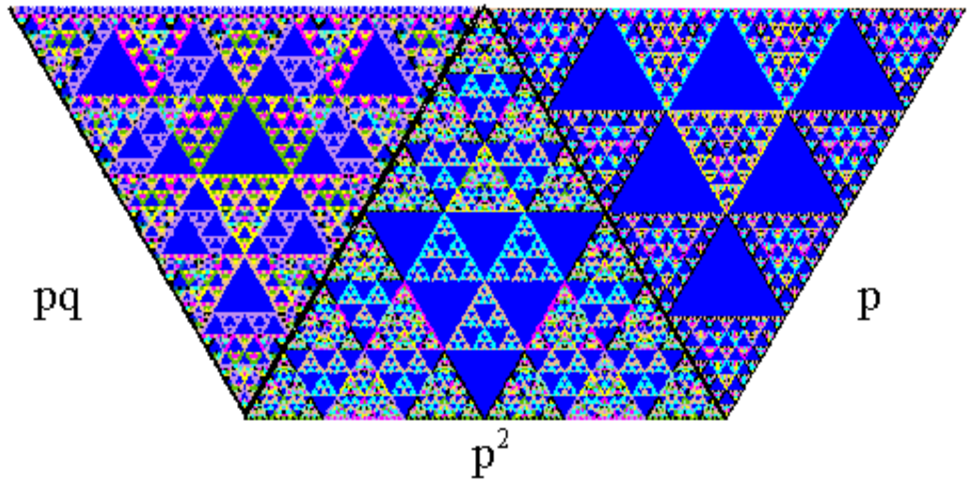
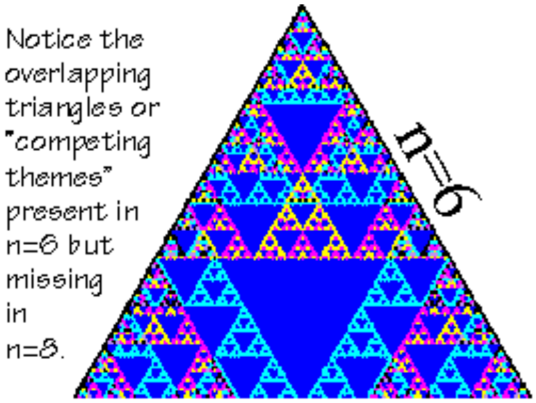
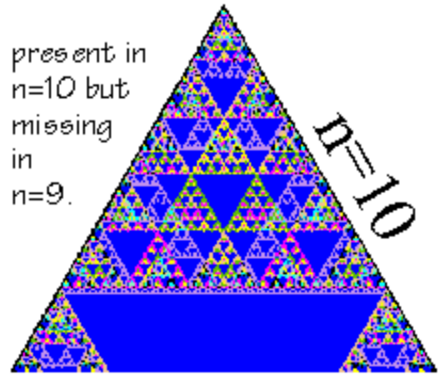
Aesthetics?

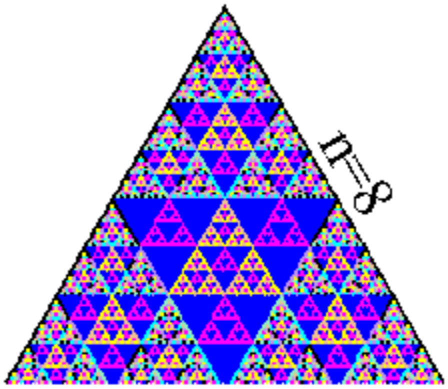


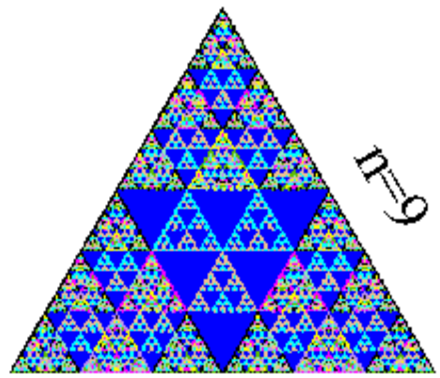
Which kind of pattern is most aesthetically pleasing?

Competing Themes

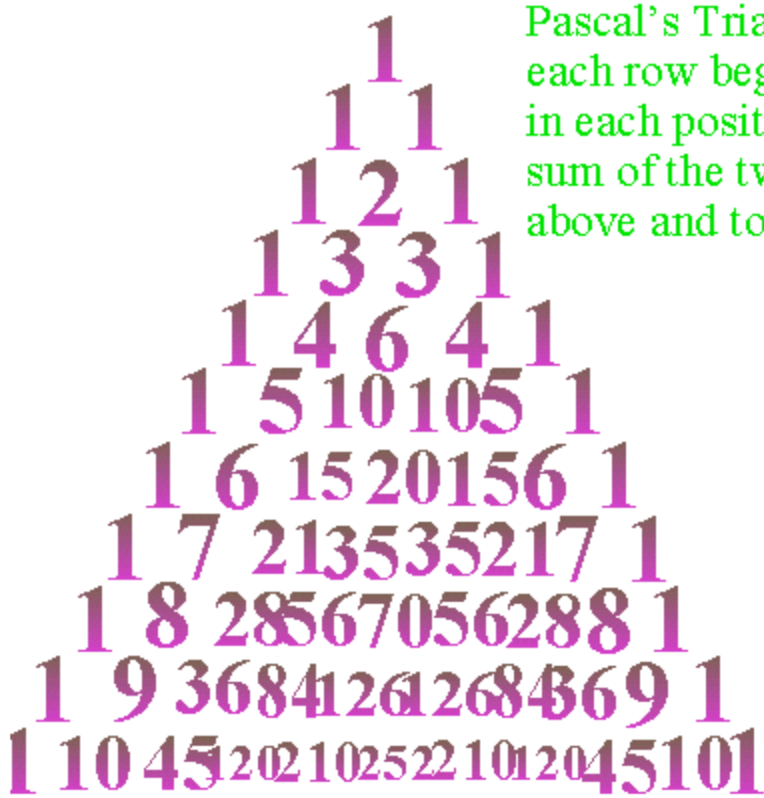








Pascal's Triangle



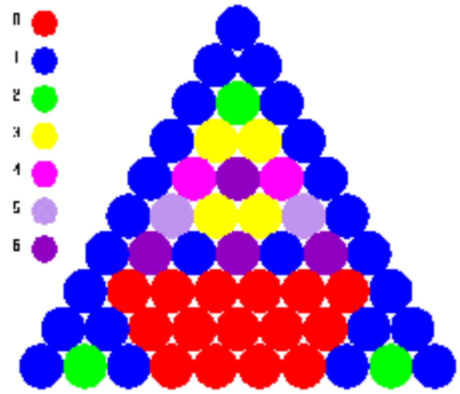
Pascal's Triangle: Begin with 1; each row begins and ends with a 1; in each position in the middle is the sum of the two numbers directly above and to the right & left.

Modular Arithmetic

Modular Arithemtic

$(a + b)_{\text{mod } n}$ = the remainder of a + b when divided by n.

The numbers 0, 1, ...,n-1 form a cyclic group under addition mod n. The identity element is 0. For any number a, $0 < a < n$; $0 < n\text{-}a < n$ and $(a + (n\text{-}a))_{\text{mod } n} = 0$ so a^{-1} is (n-a).

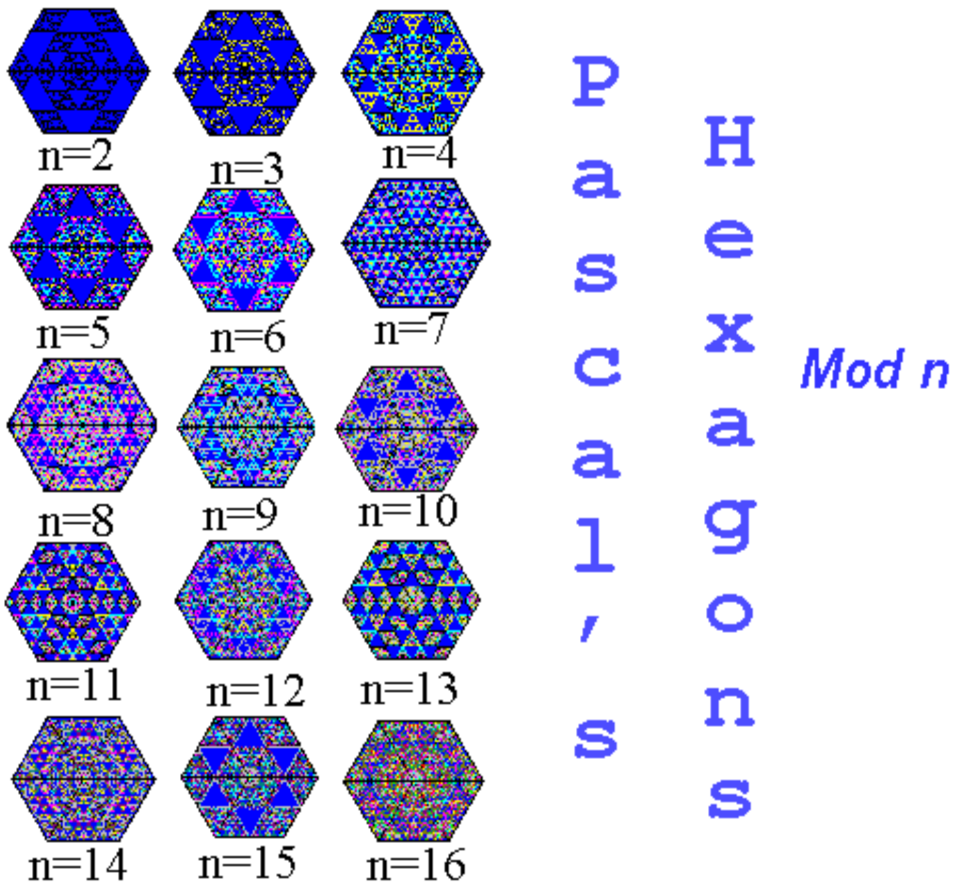


Suppose you were to draw Pascal's Triangle using addition mod n and representing each of the n possible entries by a different color.

For example, consider addition mod 7 and the set {0,1,2,3,4,5,6}. The group "multiplication" table for this opperation is:

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

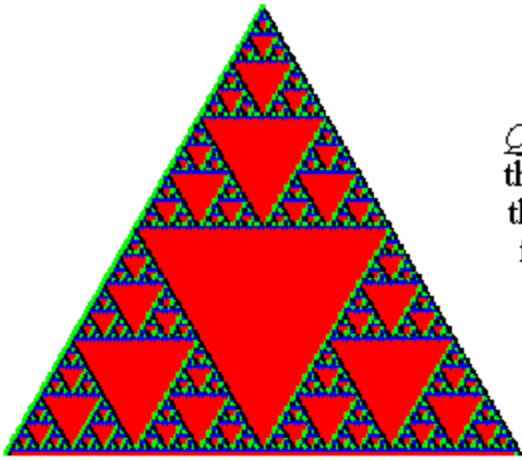
Hexagons Mod N



What about other binary operations? A non-cyclic example.

A New Multiplication Table for the Klein-4 group Z_2 X Z_2

The Klein-4 group G is the non-cyclic group of order 4. It is abelian and each group element is its own inverse. Note that G = { (0,0) , (0,1) , (1,0) , (1,1) } is generated by (1,0) and (0,1). Now assign 4 distinct colors to the group elements. The image below is produced by placing one generator down the left side of the triangle and the other down the right. The pattern you see is a consequence of the group multiplication for Z_2 X Z_2.

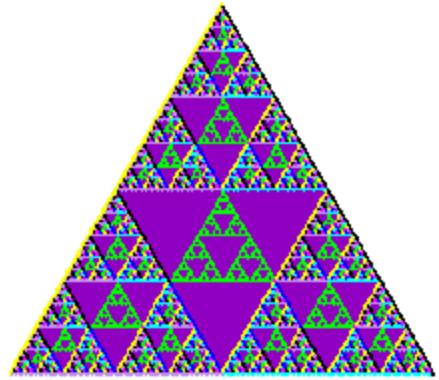


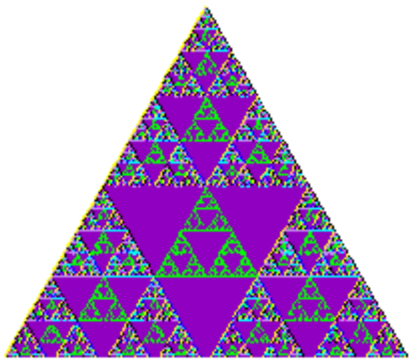
Question: **Reflection about the axis down the center of the triangle induces a map f: G-> G. What is special about this map ?**

Or even a non-abelian (communative) one?

A Non-Abelian Example

D_4 - the symmetry group of a square - with 129 and 250 rows





D_n is the symmetry group of a regular n-gon. It has order 2n and a cyclic subgroup of order n generated by rotation through $2\pi/n$. The remaining elements correspond to reflections.

These triangles are generated by one rotation and one reflection.

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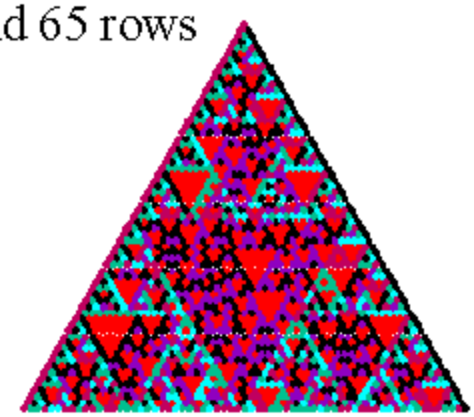
Where patterns get harder to spot: D3

Another Non-Abelian Example

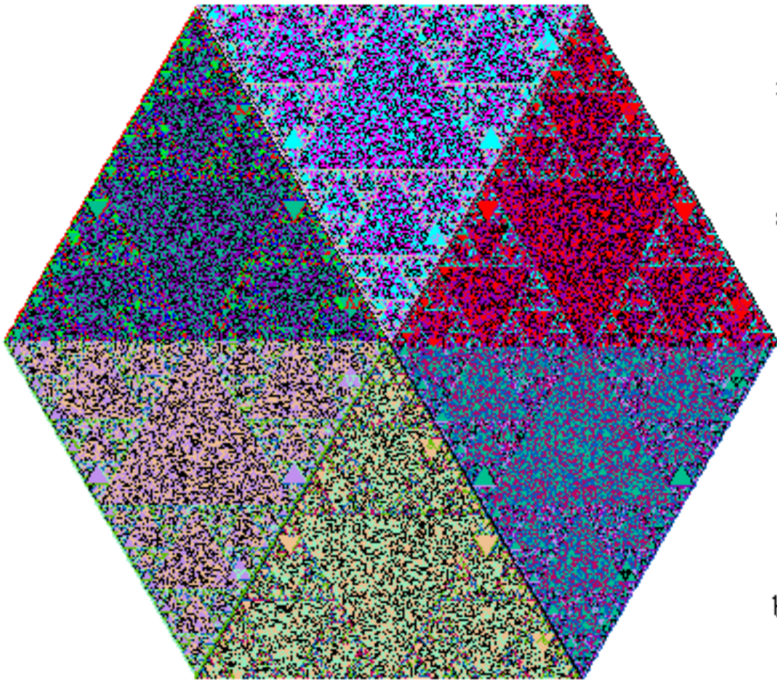
S_3 is the permutation group on three letters. It is also D_3 -the symmetry group of an equilateral triangle. The generators used here are a 2-cycle and a 3-cycle.

S_3 with 25 and 65 rows

Does there appear to be any pattern emerging? What if we continue to add more rows? On the next page are triangles with 250 rows.



But there are patterns; you just need to know how to look . . .



The degree to which we recognize patterns in these triangles is clearly influenced by the choice of color scheme. All six triangles in the hexagon shown here were created using the Pascal's Triangle Rule and S_3 group multiplication. Only the color schemes differ. This raises interesting questions about perception and indicates that there may be some interdisciplinary questions to be addressed.

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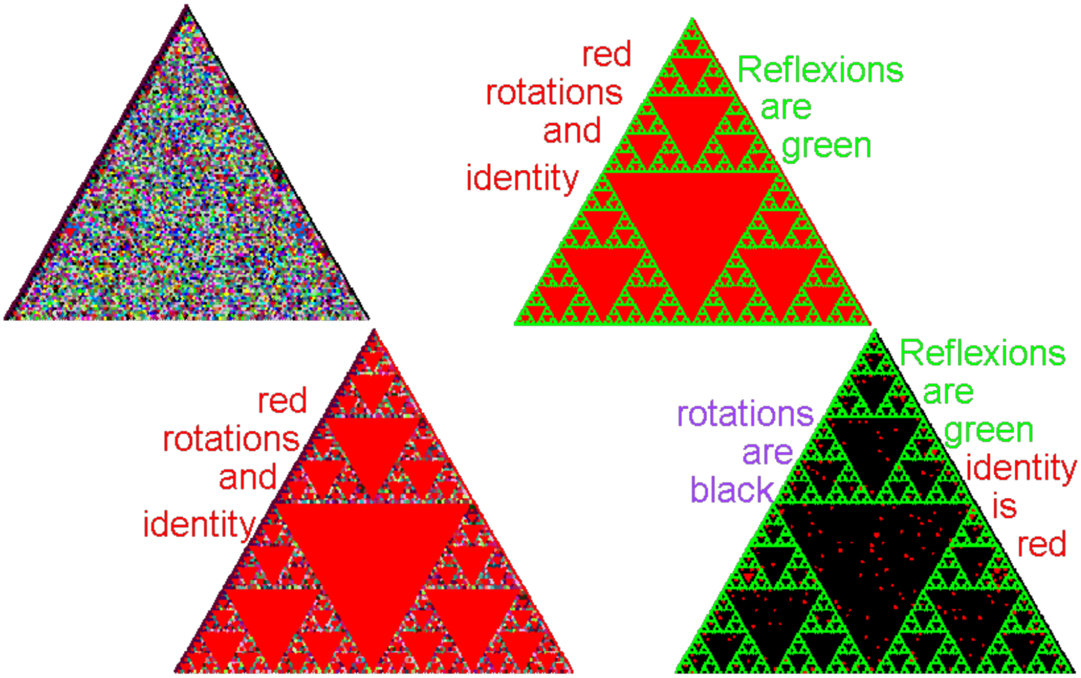
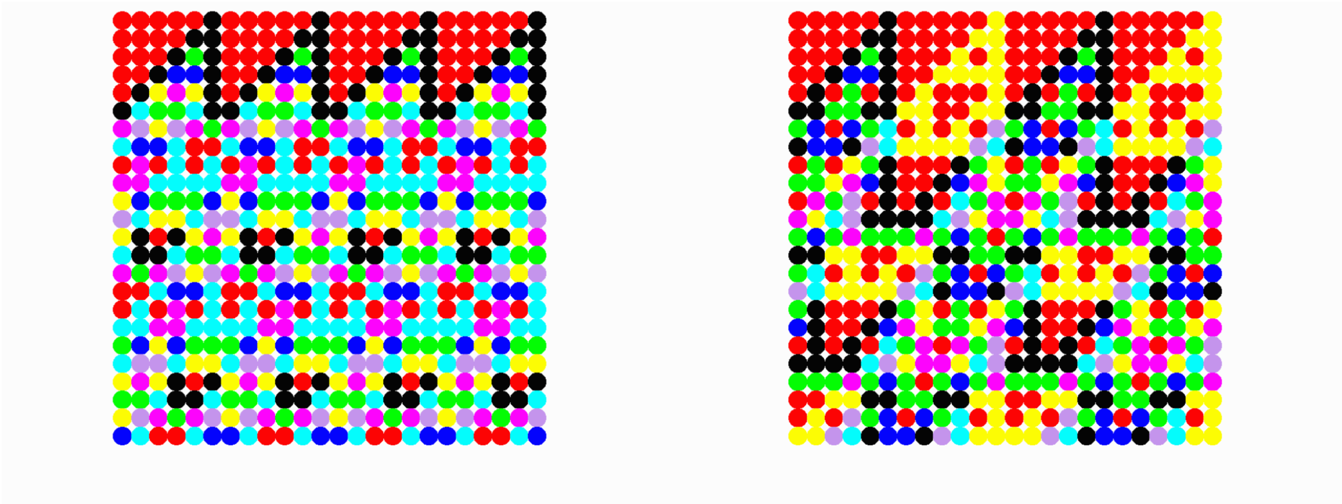
D_{33}



<-- **animate 3 choices** **Animate - disolve** **-->**



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Note: This page is, for the most part, unoriginal. The original can be found at: http://www.tech.org/~stuart/life/rules.html

John Conway's Game of Life

The Rules

The Game of Life was invented by John Conway (as you might have gathered). The game is played on a field of cells, each of which has eight neighbors (adjacent cells). A cell is either occupied (by an organism) or not. The rules for deriving a generation from the previous one are these:

Death	If an occupied cell has 0, 1, 4, 5, 6, 7, or 8 occupied neighbors, the organism dies (0, 1 neighbors: of loneliness; 4 thru 8: of overcrowding).
Survival	If an occupied cell has two or three neighbors, the organism survives to the next generation.
Birth	If an unoccupied cell has three occupied neighbors, it becomes occupied.

Where You Can Find More

The original article describing the game can be found in the April 1970 issue of Scientific American, page 120.

C language implementation of the algorithm as a solution to a homework problem. Formerly found at http://babbage.sosu.edu/cs/courses/cs2123/solutions/chap5-18.txt.

Java implementation of the game. *Much* cooler than mine.

Another Java implementation of the game. Also *much* cooler than mine.

If you go to a search engine like AltaVista and issue a query such as *john conway game life scientific american*, you should find lots of stuff (including the three references above).