Using Computer Graphics to Investigate Group Structure

Towson Talk

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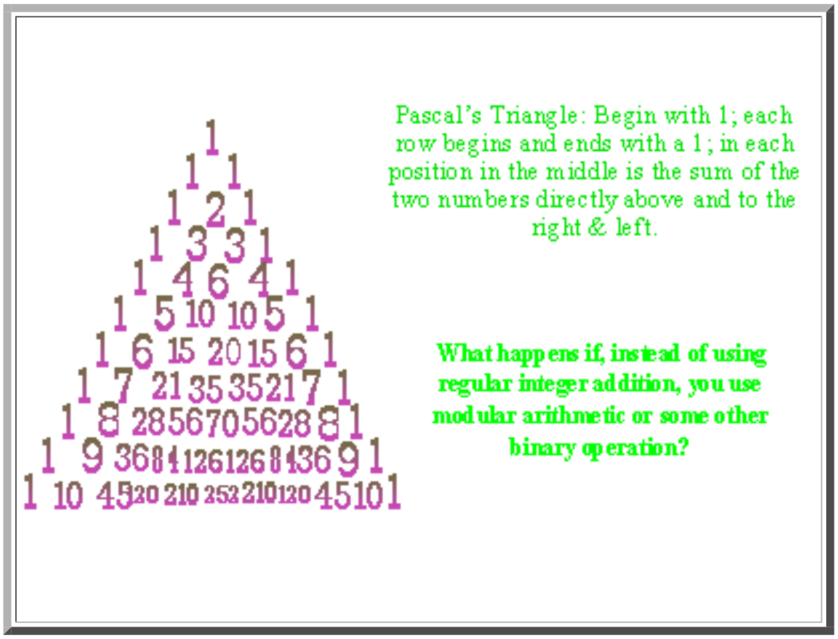
Comments or Questions?


1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
Pascal's Triangle: Begin with 1; each row begins and ends with a 1; in each position in the middle is the sum of the two numbers directly above and to the right & left.
What happens if, instead of using regular integer addition, you use modular arithmetic or some other binary operation?

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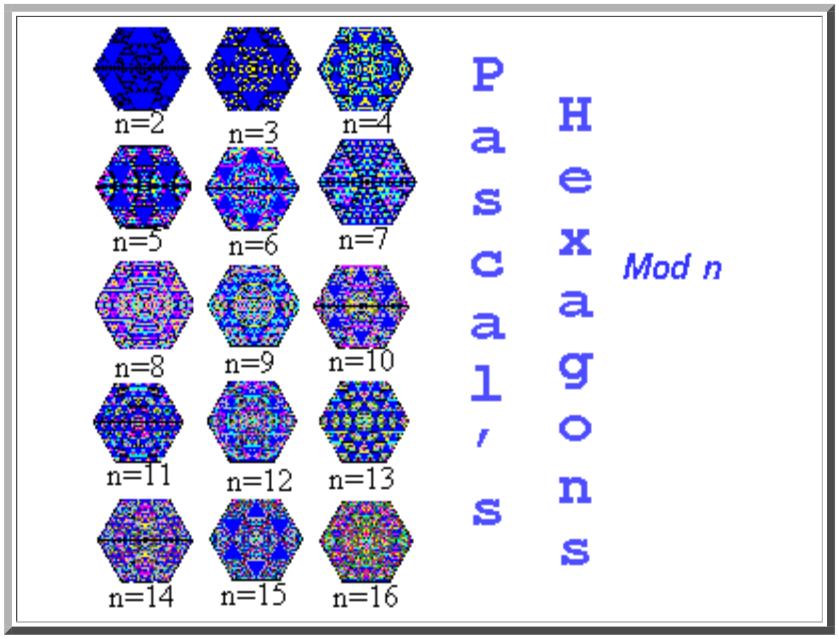
n=2
n=3
n=4
n=5
n=6
n=7
n=8
n=9
n=10
n=11
n=12
n=13
n=14
n=15
n=16
Pascal's
Hexagons
Mod n

A New Multiplication Table for the Klein-4 group Z_2 X Z_2

The Klein-4 group G is the non-cyclic group of order 4. It is abelian and each group element is its own inverse. Note that G = { (0,0) , (0,1) , (1,0) , (1,1) } is generated by (1,0) and (0,1). Now assign 4 distinct colors to the group elements. The image below is produced by placing one generator down the left side of the triangle and the other down the right. The pattern you see is a consequence of the group multiplication for Z_2 X Z_2.

Question: Reflection about the axis down the center of the triangle induces a map f: G-> G. What is special about this map ?

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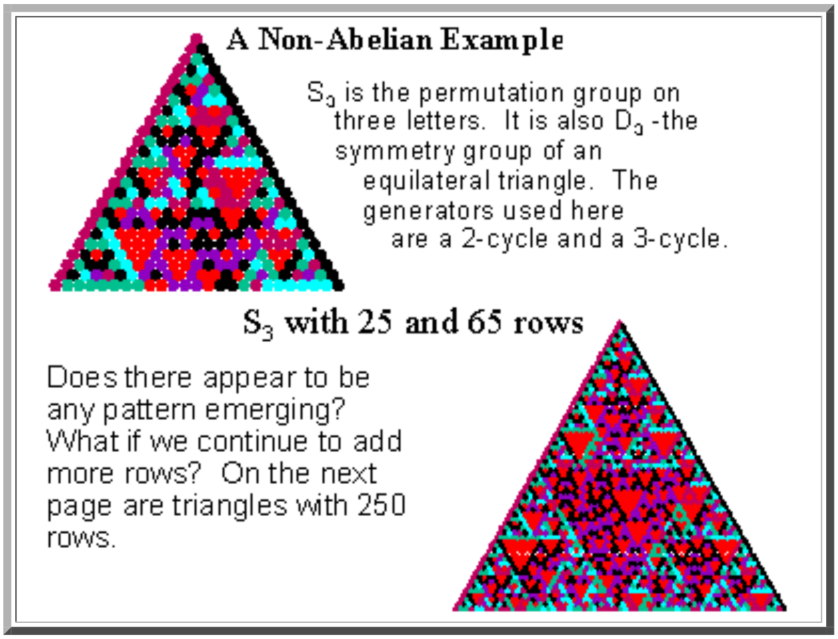
A Non-Abelian Example
S3 is the permutation group on three letters. It is also D3 -the symmetry group of an equilateral triangle. The generators used here are a 2-cycle and a 3-cycle.
S3 with 25 and 65 rows
Does there appear to be any pattern emerging? What if we continue to add more rows? On the next page are triangles with 250 rows.

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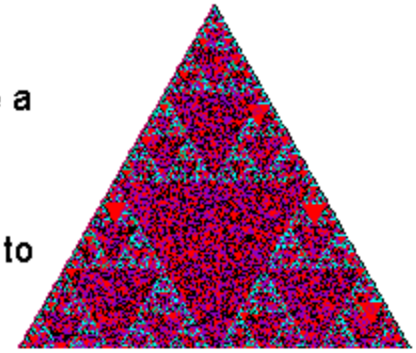
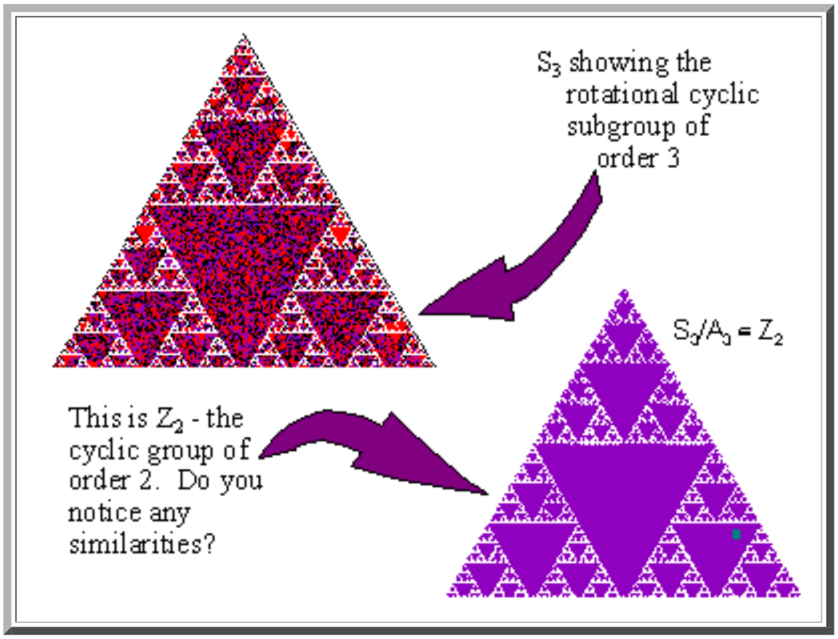
S_3 with 250 Rows

Now we begin to see a pronounced pattern emerging.

How does this relate to the group structure?




S_3 showing the rotational cyclic subgroup of order 3
$S_3/A_3 = Z_2$
This is Z_2 - the cyclic group of order 2. Do you notice any similarities?

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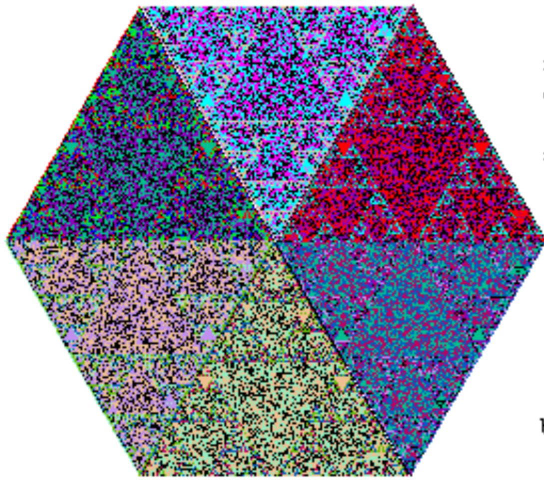

The degree to which we recognize patterns in these triangles is clearly influenced by the choice of color scheme. All six triangles in the hexagon shown here were created using the Pascal's Triangle Rule and S_3 group multiplication. Only the color schemes differ. This raises interesting questions about perception and indicates that there may be some interdisciplinary questions to be addressed.

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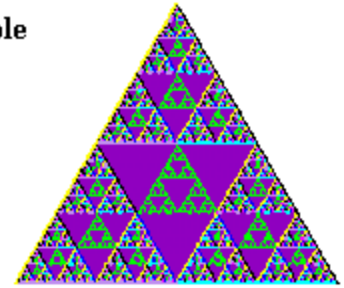
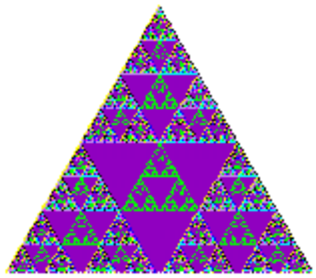
Another Non-Abelian Example

D_4 - the symmetry group of a square - with 129 and 250 rows





D_n is the symmetry group of a regular n-gon. It has order 2n and a cyclic subgroup of order n generated by rotation through $2\pi/n$. The remaining elements correspond to reflections.

These triangles are generated by one rotation and one reflection.

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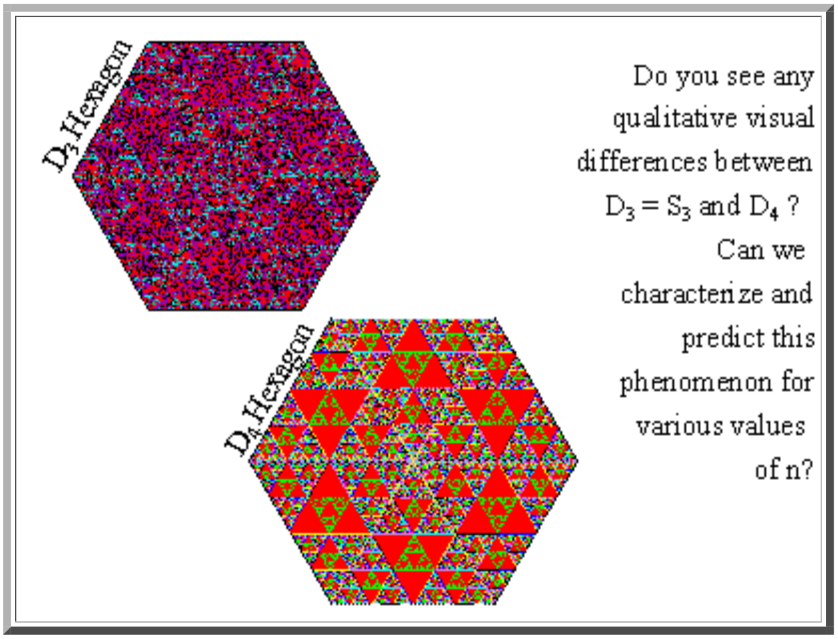
D_3 Hexagon
D_4 Hexagon
Do you see any qualitative visual differences between $D_3 = S_3$ and D_4 ?
Can we characterize and predict this phenomenon for various values of n?

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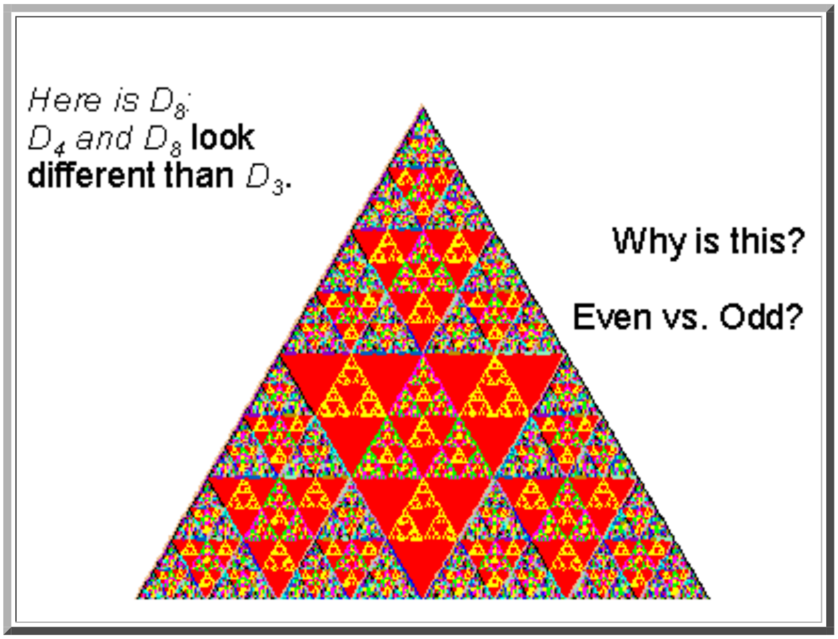
Here is D_8:
D_4 and D_8 **look different than** *D_3*.
Why is this?
Even vs. Odd?

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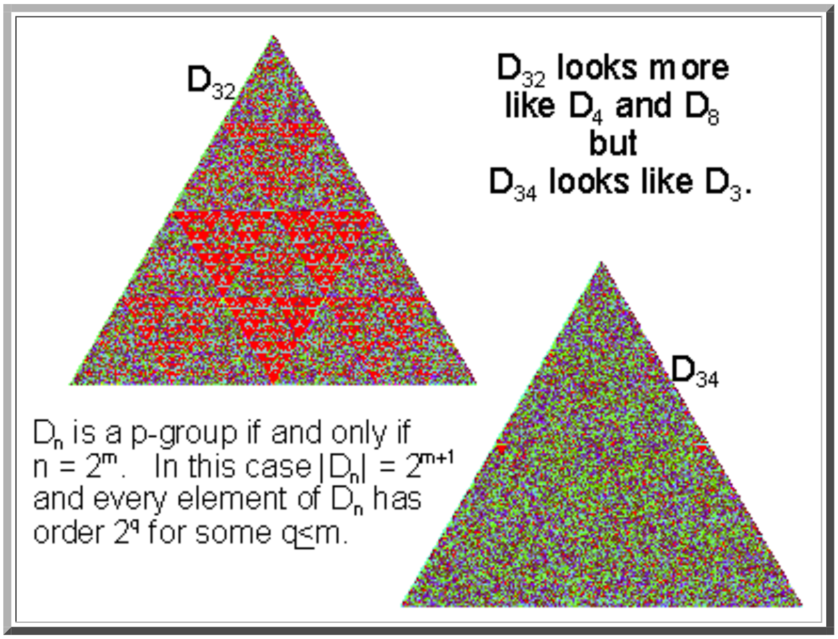
D_{32}
D_{32} looks more
like D_4 and D_8
but
D_{34} looks like D_3.
D_{34}
D_n is a p-group if and only if
$n = 2^m$. In this case $|D_n| = 2^{m+1}$
and every element of D_n has
order 2^q for some $q \leq m$.

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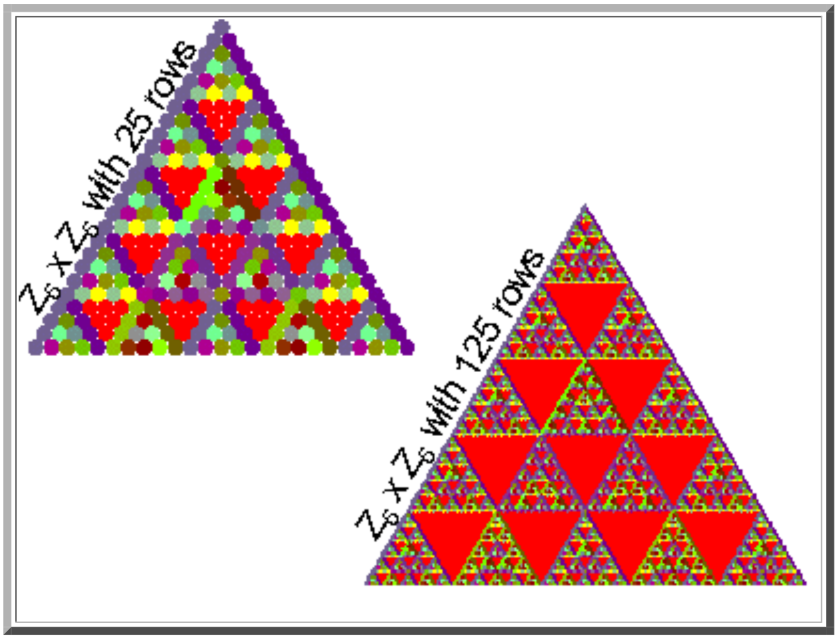
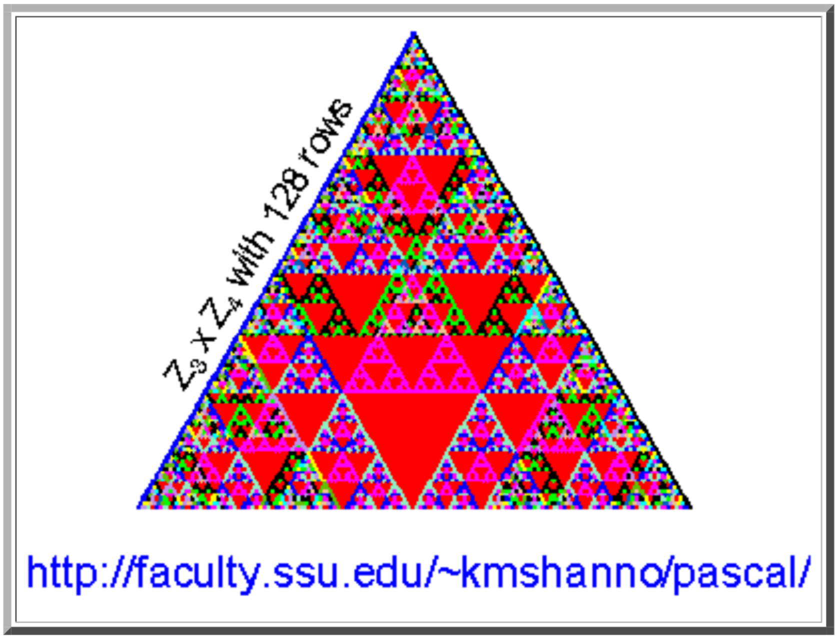

Z6 x Z6 with 25 rows
Z6 x Z6 with 125 rows



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