

Name: _____

Write all of your responses on these exam pages. If you need more space please use the backs. Make sure that you show all of your work.

1. **Definitions:** (*4 Points Each*) Give a definition for each of the following.

(a) An Elementary Matrix

(b) The Null Space of a matrix

(c) A Subspace of \mathbb{R}^n

(d) The Rank of a matrix

(e) The (i, j) -Cofactor, C_{ij} , of a matrix A .

2. **True and False:** (3 Points Each) Mark each of the following as either true or false. If the statement is false either give a counterexample or correct the statement so that it is true.

(a) _____ $\det(A + B) = \det(A) + \det(B)$, where both A and B are $n \times n$ matrices.

(b) _____ The determinant of a triangular matrix is the sum of the entries on the main diagonal.

(c) _____ If A is an invertible matrix then $\det(A^{-1}) = (\det(A))^{-1}$.

(d) _____ If A is a square matrix such that $A^T A = I$ then $\det(A) = 1$.

(e) _____ $(AB)^T = A^T B^T$ for matrices A and B such that AB is defined.

(f) _____ If $AB = 0$ for non-zero $n \times n$ matrices A and B , then neither A nor B can be invertible.

(g) _____ If A is an $n \times n$ matrix such that the columns of A span \mathbb{R}^n then the columns of A are linearly independent and form a basis for \mathbb{R}^n .

(h) _____ If $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p\} \subset \mathbb{R}^n$ then $\text{Span}(\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p\})$ is a subspace of \mathbb{R}^n .

(i) _____ If T is a linear transformation from \mathbb{R}^n to \mathbb{R}^m with $T\mathbf{x} = A\mathbf{x}$ and the dimension of the Null Space of A is 0 then T is an onto map.

(j) _____ If B is the reduced echelon form of the matrix A then $B = MA$ for some invertible matrix M .

3. **Calculations:** (10 Points Each) Do each of the following.

(a) Given that

$$A = \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ -2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 3 & 5 \\ 2 & -2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

find the following if they exist. If the operation is not defined state why?

i. $2A - 3B$

ii. AB

iii. BC

iv. BAC

v. $2A - 3C^T$

- (b) Find the determinant of the following matrix, A . You may use a short-cut method if it is applicable. Is the matrix A invertible? If so find its inverse and if not state why. Show all of the steps in the derivations and keep your answers in exact form.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 3 \\ -2 & 1 & -2 \end{bmatrix}$$

- (c) Find the determinant of the following matrix using cofactor expansion. You may use a short-cut method if and when it is applicable. Show all of the steps in the derivations and keep your answers in exact form.

$$A = \begin{bmatrix} -4 & 0 & -9 & -2 \\ -5 & -3 & -4 & 0 \\ 8 & 1 & 10 & 5 \\ -4 & -10 & -1 & 3 \end{bmatrix}$$

- (d) Find the determinant of the following matrix using the reduction method. You may not use a short-cut method nor a combination of reduction and cofactor expansion. The reduction method should be carried out to at least echelon form. Show all of the steps in the derivations and keep your answers in exact form.

$$A = \begin{bmatrix} -4 & 0 & -9 & -2 \\ -5 & -3 & -4 & 0 \\ 8 & 1 & 10 & 5 \\ -4 & -10 & -1 & 3 \end{bmatrix}$$

- (e) For the following matrix A find a basis for $\text{Nul}(A)$ and $\text{Col}(A)$. Show all of the steps in the derivations and keep your answers in exact form.

$$A = \begin{bmatrix} 2 & 4 & -2 & 2 & 3 \\ -1 & 1 & -2 & 1 & 1 \\ 3 & 5 & -2 & -1 & 3 \end{bmatrix}$$

(f) Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ where

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{b}_3 = \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix}$$

be a basis for \mathbb{R}^3 and let

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

find $[\mathbf{x}]_{\mathcal{B}}$. Show all of the steps in the derivations and keep your answers in exact form.