

1. (20 Points) Do one and only one of the following integrals.

(a) $\int \frac{2x-3}{(x+1)(x-2)^2} dx = -\frac{5}{9} \ln|x+1| + \frac{5}{9} \ln|x-2| - \frac{1}{3(x-2)} + C$

(b) $\int \frac{x^2+1}{(x-1)(x^2+4)} dx = \frac{2}{5} \ln|x-1| + \frac{3}{10} \ln|x^2+4| + \frac{3}{10} \tan^{-1}\left(\frac{x}{2}\right) + C$

2. (20 Points) Do one and only one of the following integrals.

(a) $\int \tan^3(x) \cos^2(x) dx = -\ln|\cos(x)| + \frac{1}{2} \cos^2(x) + C$

(b) $\int \sec^4(x) \tan^3(x) dx = \frac{1}{6} \sec^6(x) - \frac{1}{4} \sec^4(x) + C$

3. (20 Points) Do one and only one of the following integrals.

(a) $\int 7x^2 \sqrt{3-x^2} dx = \frac{63}{8} \left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) - \frac{1}{9} x \sqrt{3-x^2}(3-2x^2) \right) + C$

(b) $\int \frac{x^3}{\sqrt{1+x^2}} dx = \frac{1}{3} (1+x^2)^{3/2} - \sqrt{1+x^2} + C$

4. (20 Points) Do one and only one of the following integrals. Determine whether the integral is convergent or divergent. If it is convergent, evaluate it.

(a) $\int_0^\infty x e^{-x^2} dx = \frac{1}{2}$

(b) $\int_0^7 \frac{x-1}{\sqrt{x}} dx = \frac{8\sqrt{7}}{3}$

5. (20 Points) Do one and only one of the following integrals.

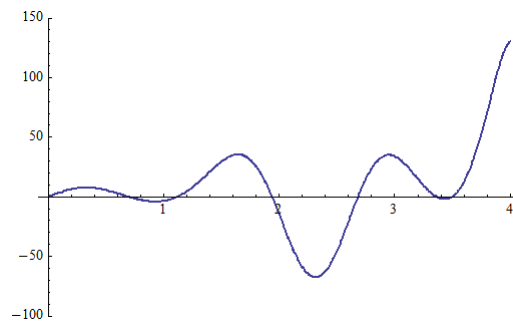
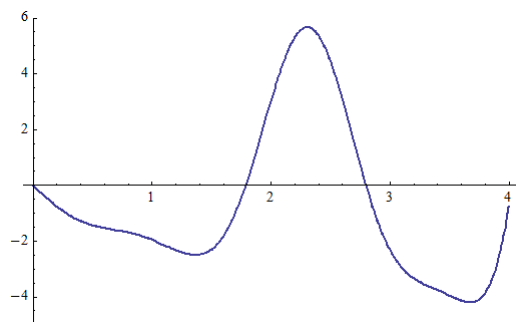
(a) $\int x \sin(2x) dx = -\frac{1}{2} x \cos(2x) + \frac{1}{4} \sin(2x) + C$

(b) $\int x^2 e^{3x} dx = \frac{1}{3} x^2 e^{3x} - \frac{2}{9} x e^{3x} + \frac{2}{27} e^{3x} + C$

6. (10 Points) This set of exercises all deal with the integral,

$$\int_0^4 \sin(x \cos(x)) dx$$

- (a) Use the Trapezoidal Rule with $n = 4$ to approximate the integral. Your answer must be correct to at least 5 decimal places. **Solution:** $T_4 = -0.64733529987223330646$
- (b) Find the error bound for this approximation. Your answer must be correct to at least 5 decimal places. The graphs of the second and fourth derivatives of $f(x) = \sin(x \cos(x))$ respectively are below.



Solution: $|E_T| \leq \frac{K(b-a)^3}{12n^2} \leq \frac{6 \cdot 4^3}{12 \cdot 4^2} = 2$