

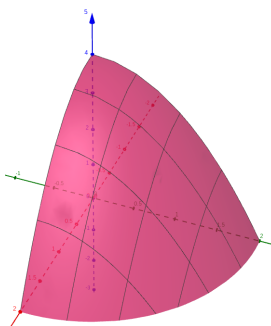
**Name:** \_\_\_\_\_

Write all your work and solutions on the extra paper provided and hand it in with this exam paper. Please put your name on each page in case the pages get separated.

1. (15 Points): Use cylindrical coordinates to set up **but do not evaluate**

$$\iiint_E x + y + z \, dV$$

where  $E$  is the solid in the first octant that lies under the paraboloid  $z = 4 - x^2 - y^2$ .



2. (15 Points): Use spherical coordinates to set up **but do not evaluate**

$$\iiint_E y^2 \, dV$$

where  $E$  is the solid hemisphere  $x^2 + y^2 + z^2 \leq 9$  with  $y \geq 0$ .

3. (15 Points): Find the Jacobian of the transformation,  $x = u^2 + uv$ ,  $y = uv^2$ .  
 4. (15 Points): Find the gradient vector field  $\nabla f$  of  $f(x, y) = y \sin(xy)$ .  
 5. (20 Points): Do one and only one of the following.

(a) Evaluate

$$\int_C z \, dx + xy \, dy + y^2 \, dz$$

where  $C : x = \sin(t), y = \cos(t), z = \tan(t), -\pi/4 \leq t \leq \pi/4$ .

(b) Use Green's Theorem to evaluate

$$\int_C y^3 \, dx - x^3 \, dy$$

where  $C$  is the positively oriented circle  $x^2 + y^2 = 4$ .

6. (20 Points): Do one and only one of the following.

(a) Find a function  $f$  such that  $\mathbf{F} = \nabla f$

$$\mathbf{F} = \langle 2x \sin(yz) - y \sin(x) + 4, x^2 z \cos(yz) - 1/y + \cos(x), x^2 y \cos(yz) - 3z^2 \rangle$$

Use it to evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$  along the curve  $C : \mathbf{r}(t) = \langle 2t^2, t, e^t \rangle$  with  $1 \leq t \leq 2$ .  
Don't worry about simplifying the result.

(b) Find the Curl and Divergence of the vector field

$$\mathbf{F}(x, y, z) = zx^2 \cos(y) \mathbf{i} + xy^3 \sin(z) \mathbf{j} + yz^4 e^x \mathbf{k}$$

7. **Extra Credit:** (10 Points): Do either of the two exercises above that you did not choose for the previous problems. Do one and only one of these.