

Name: _____

- Write all of your responses on these exam pages. If you need more space for your answers please use the backs of the exam pages.
- Make sure that you show all of your work, answers without supporting work will receive no credit.
- **No calculation devices are to be used on this exam.**

1. (10 Points) Find $\frac{dy}{dx}$ of

$$\tan(x - y) = 2xy^3 + 1$$

You do not need to simplify your answer.

2. (15 Points) Find the equation of the tangent line, in slope-intercept form, to the conchoid of Nicomedes,

$$x^2y^2 = (y + 1)^2(4 - y^2)$$

at the point $(2\sqrt{3}, 1)$. Keep your answer in exact form

3. (10 Points Each) Find $\frac{dy}{dx}$ of the following functions. You do not need to simplify your answer.

(a) $y = (\sin(x))^{\ln(x)}$

(b) $y = \ln(\tan^{-1}(x^4))$

(c) $y = \sqrt{x}e^{x^2-x}(x+1)^{2/3}$

4. (15 Points) The number of yeast cells in a laboratory culture increases rapidly initially but levels off eventually. The population is modeled by the logistic function

$$n = f(t) = \frac{a}{1 + be^{-0.7t}}$$

where t is measured in hours. At time $t = 0$ the population is 20 cells and is increasing at a rate of 12 cells/hour. Find the values of a and b . According to this model, what happens to the yeast population in the long run?

5. (*10 Points*) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 4 ft/s, how fast is the angle between the ladder and the ground changing when the bottom of the ladder is 6 ft from the wall? Keep your answer in exact form but simplify it down to an “easy” number.

6. (10 Points) Find the differential dy of

$$y = \frac{x+1}{x-1}$$

and evaluate dy when $x = 2$ and $dx = 0.05$. Keep your answer in exact form but simplify it down to an “easy” number.

7. (10 Points) Use a linear approximation (or differentials) to estimate the value of $e^{0.1}$.