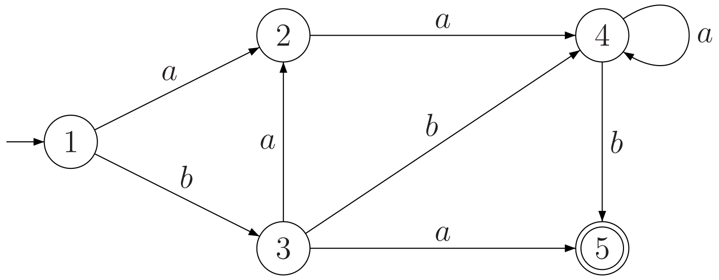
Fall 2013 # Exam #2 Key COSC 362

1. (*20 Points*) Convert the following NFA to a regular expression.



Solution: $ba + (aa + b(aa + b))a^*b$

2. (*15 Points Each*) Prove or disprove that each of the following languages are regular.

   (a) $L = \{w \mid n_a(w) \neq n_b(w)\}$
   Solution: If L is regular then so is $\bar{L}$, but $\bar{L} = \{w \mid n_a(w) = n_b(w)\}$ which has been proven to be nonregular. One can show that $\bar{L}$ is not regular by using a standard pumping lemma proof by contradiction with word $w = a^m b^m$.

   (b) $L = \{a^n b^n \mid n \geq 1\} \cup \{a^{n+5} b^n \mid n \geq 1\}$
   Solution: Assuming that L is regular, invoke the pumping lemma, let $w = a^m b^m$, then $w = xyz$ with $y = a^k$ for some $1 \leq k \leq m$. If we pump out a copy of y we have $xz = a^{m-k} b^m$ which is a word with fewer a's than b's and hence is not in L. This contradicts the pumping lemma proving that L is not regular.

3. (*15 Points*) Find a context-free grammar for the following language.

$$L = \{a^n b^m c^k \mid m = 3n + 2k\}$$

Solution:

$$\begin{aligned} S &\longrightarrow AB \\ A &\longrightarrow aAbbb \mid \lambda \\ B &\longrightarrow bbBc \mid \lambda \end{aligned}$$

4. (*15 Points*) Show that the following grammar is ambiguous.

$$\begin{aligned} S &\longrightarrow aABb \\ A &\longrightarrow bBA \mid aA \mid aAA \mid b \\ B &\longrightarrow abB \mid \lambda \end{aligned}$$

Solution: Consider the following two left-most derivations of $aabbabb$.

$S \to aABb \to aaAABb \to aabABb \to aabbBb \to aabbabBb \to aabbabb$

$S \to aABb \to aaABb \to aabBABb \to aabABb \to aabbBb \to aabbabBb \to aabbabb$

5. (*30 Points*) Consider the following grammar, G. In each conversion step below, follow the conversion or removal algorithm discussed in class.

$$\begin{aligned} S &\longrightarrow abAB \\ A &\longrightarrow bAB \mid \lambda \\ B &\longrightarrow BAa \mid A \mid \lambda \\ C &\longrightarrow aAD \\ D &\longrightarrow aAB \end{aligned}$$

(a) Remove all useless productions.
Solution: Using a dependency graph we find that C and D are useless.

$$\begin{aligned} S &\longrightarrow abAB \\ A &\longrightarrow bAB \mid \lambda \\ B &\longrightarrow BAa \mid A \mid \lambda \end{aligned}$$

(b) Remove all λ-productions from your result in 5a.
Solution: The variables A and B are nullable. So we get,

$$\begin{aligned} S &\longrightarrow abAB \mid abB \mid abA \mid ab \\ A &\longrightarrow bAB \mid bA \mid bB \mid b \\ B &\longrightarrow BAa \mid A \mid Aa \mid Ba \mid a \end{aligned}$$

(c) Remove all unit-productions from your result in 5b.
Solution: Using another dependency graph we have that $B \Rightarrow A$, so using the replacement we have

$$\begin{aligned} S &\longrightarrow abAB \mid abB \mid abA \mid ab \\ A &\longrightarrow bAB \mid bA \mid bB \mid b \\ B &\longrightarrow BAa \mid Aa \mid Ba \mid a \mid bAB \mid bA \mid bB \mid b \end{aligned}$$

(d) Convert the grammar into Chomsky Normal Form from your result in 5c.
Solution: We first do the preliminary step of terminal replacement,

$$\begin{aligned} S &\longrightarrow B_aB_bAB \mid B_aB_bB \mid B_aB_bA \mid B_aB_b \\ A &\longrightarrow B_bAB \mid B_bA \mid B_bB \mid b \\ B &\longrightarrow BAB_a \mid AB_a \mid BB_a \mid a \mid B_bAB \mid B_bA \mid B_bB \mid b \\ B_a &\longrightarrow a \\ B_b &\longrightarrow b \end{aligned}$$

Then for each production with more than two variables on the right we do our final step of adding variables and productions,

$$\begin{aligned} S &\longrightarrow B_aD_1 \mid B_aD_3 \mid B_aD_4 \mid B_aB_b \\ A &\longrightarrow B_bD_2 \mid B_bA \mid B_bB \mid b \\ B &\longrightarrow BD_5 \mid AB_a \mid BB_a \mid a \mid B_bD_2 \mid B_bA \mid B_bB \mid b \\ D_1 &\longrightarrow B_bD_2 \\ D_2 &\longrightarrow AB \\ D_3 &\longrightarrow B_bA \\ D_4 &\longrightarrow B_bB \\ D_5 &\longrightarrow AB_a \end{aligned}$$